Plane wave limits in gauge theory

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$$\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$$

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Which string theory?

A concrete proposal

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- *spectrum* of, say, single trace operators; e.g., chiral primaries (in short multiplets)

$$\mathrm{Tr}'\,\phi\otimes\cdots\otimes\phi$$

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However...recent progress in this direction has been made using a *different* large N limit, the so-called *plane wave limit*.

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Boost along γ and zoom in on γ while rescaling g

- Choose coordinates U, V, Y^i such that

$$g = 2dUdV + AdV^2 + B_i dY^i dV + C_{ij} dY^i dY^j$$

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- “parabolic blow-up” ($\Omega > 0$)

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- rescale

$$g_{\text{plane wave}} = \lim_{\Omega \to 0} \Omega^{-2} g(\Omega)$$

In “Brinkmann coordinates”

$$g_{\text{plane wave}} = 2dx^- dx^+ + \sum_{i,j} A_{ij}(x^+) x^i x^j (dx^+)^2 + \sum_i dx^i dx^i$$

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- "lightlike gauge"

$$\imath_{\partial/\partial U} A^{(p)} = 0$$

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Note: field strength is *null*:

$$F_{\text{plane wave}} = dA_{\text{plane wave}} = dx^+ \wedge \Theta(x^+)$$

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It is useful in classifying the different plane wave limits of a given background.

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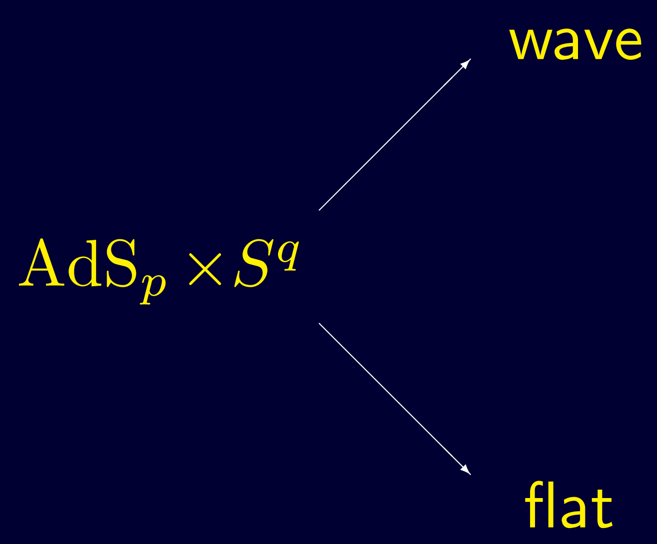

depending on whether or not the component of $\dot{\gamma}$ tangent to S^q vanishes. [Blau–FO–Hull–Papadopoulos (2002)]

Some examples

Eleven-dimensional supergravity vacua: [FO–Papadopoulos (2002)]

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KG

↗ ↖

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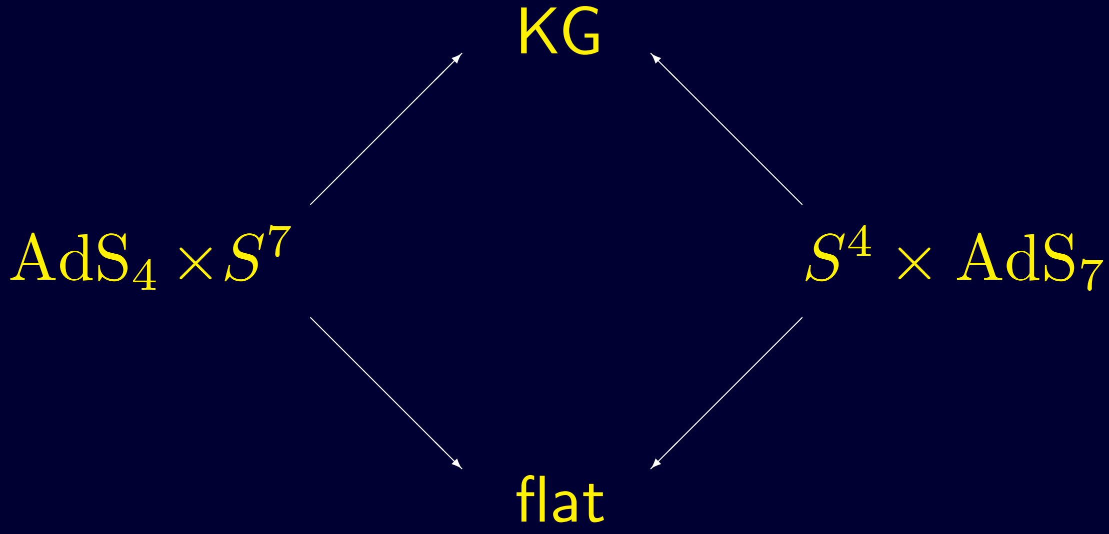

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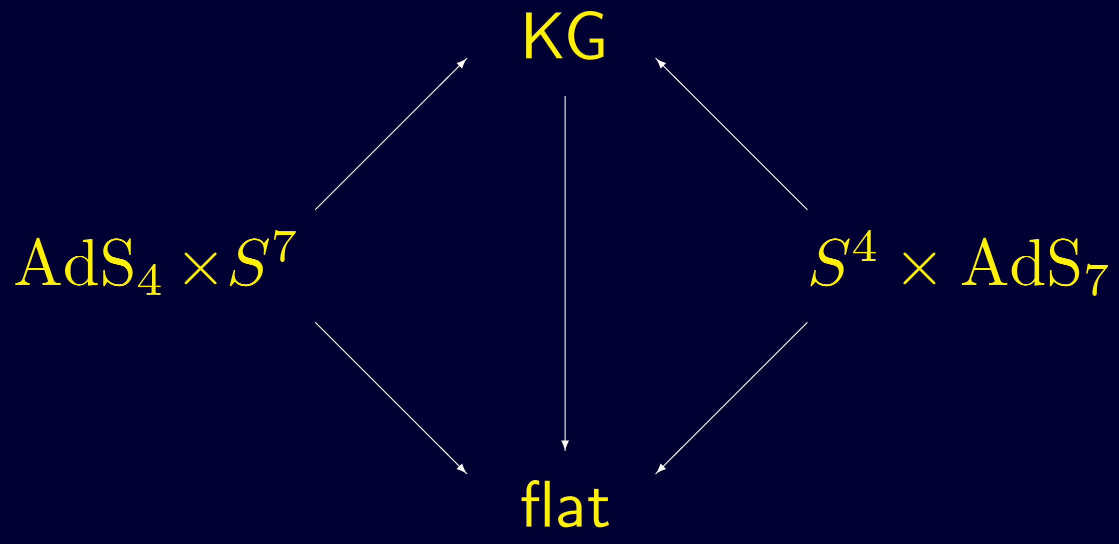

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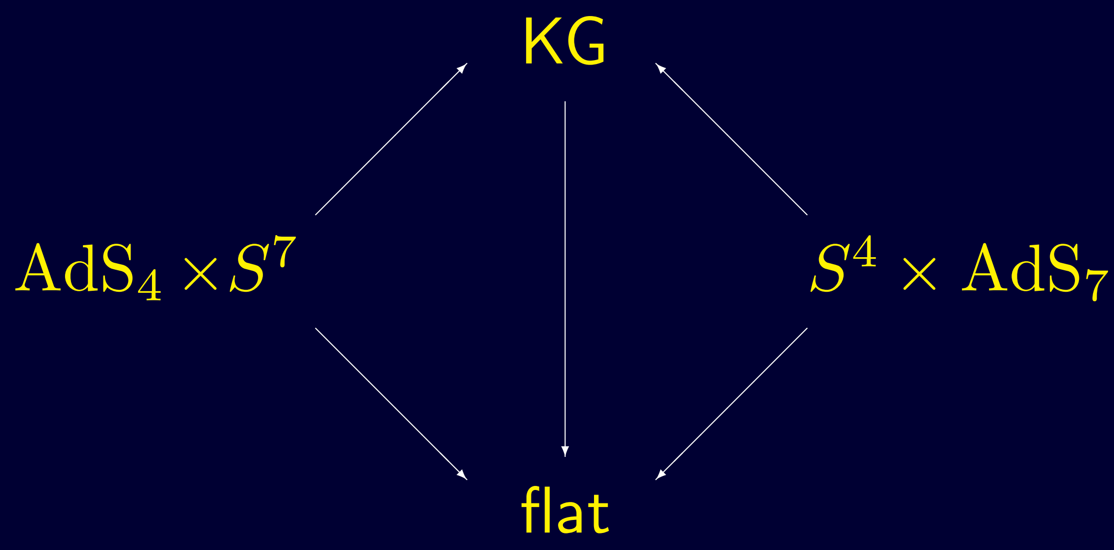

KG: 11-dimensional symmetric plane wave [Kowalski-Glikman (1984)]

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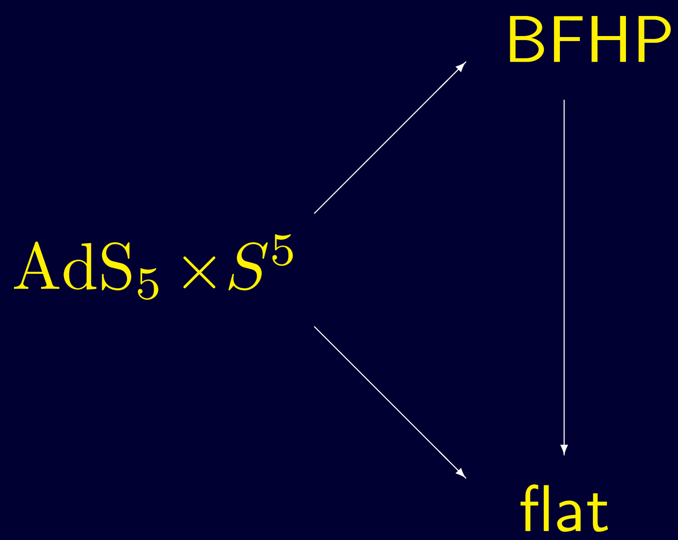
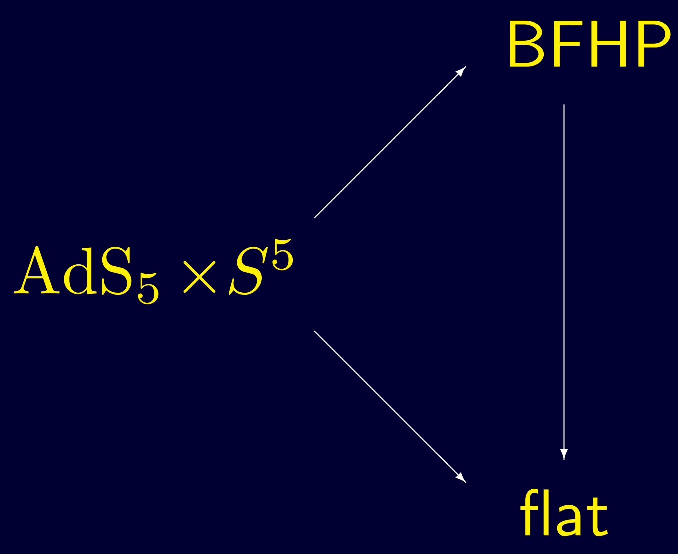


In ten-dimensional IIB supergravity: [FO–Papadopoulos (2002)]



BFHP: 10-dimensional symmetric plane wave
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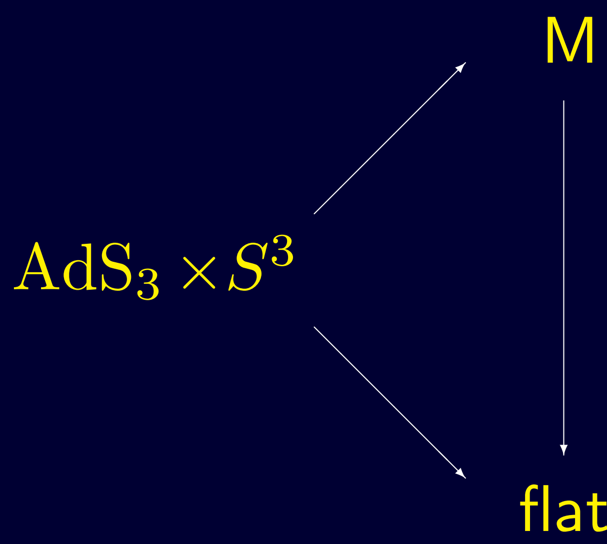

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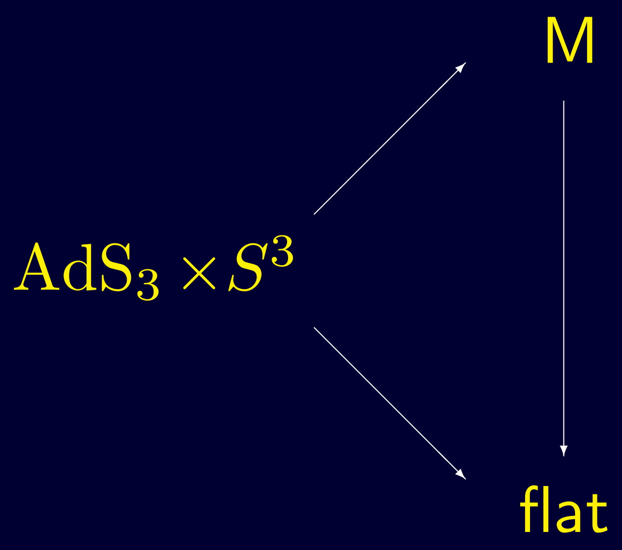
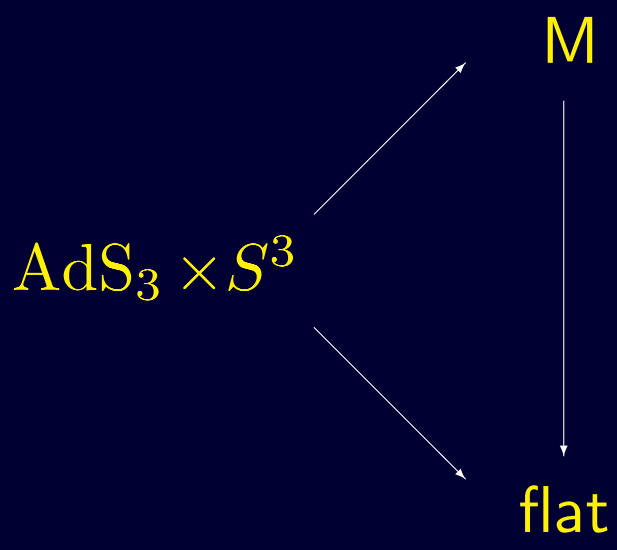


M: 6-dimensional symmetric plane wave [Meessen (2001)]

In six-dimensional (1,0) supergravity:

[Chamseddine–FO–Sabra (to appear in 2003)]



M: 6-dimensional symmetric plane wave [Meessen (2001)]

All solutions are Lie groups and all plane wave limits are group contractions. [Stanciu–FO (2003)]

A word on the wave geometry

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We will now see that the spectrum can be recovered from the gauge theory but we will first need to understand how the plane wave limit is manifested in the gauge theory.

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Notice: As $N \to \infty$ we focus on states with larger and larger J. Thus observables are not held fixed in this limit.

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- they are dual to the *free* string excitations on the plane wave background

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BMN operators are single trace operators consisting in a string of J Z's and a finite number of *impurities*.

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etc...

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In other words...

there is a kinematical correspondence

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How about interactions?

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[Kristjansen et al., Gross et al., Constable et al. (2002)]

- theory develops a different effective coupling constant

$$\lambda' = \frac{g_{\mathsf{YM}}^2 N}{J^2} = \frac{1}{(\mu p^+ \alpha')^2}$$

- a different genus-counting parameter

$$g_2^2 = \left(\frac{J^2}{N}\right)^2 = 16\pi^2 g_s^2 (\mu p^+ \alpha')^4$$

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- string field theory

  [..., Chu–Khoze, Gomis et al. (2003)]

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- the correspondence has been extended to theories with less supersymmetry; but as usual QCD remains elusive.

Thank you.