M2-branes and 3-algebras

José Miguel Figueroa-O'Farrill

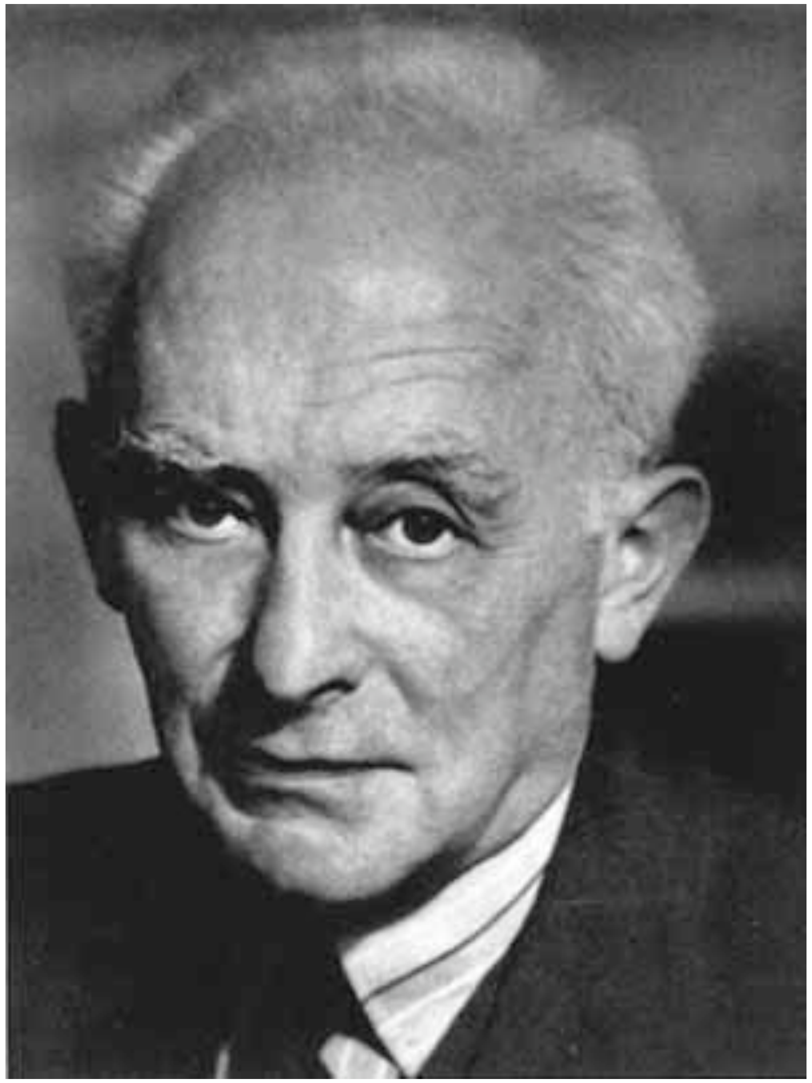
IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Wrocław, 1 July 2009

http://www.maths.ed.ac.uk/~jmf/Research/Talks/Wroclaw.pdf

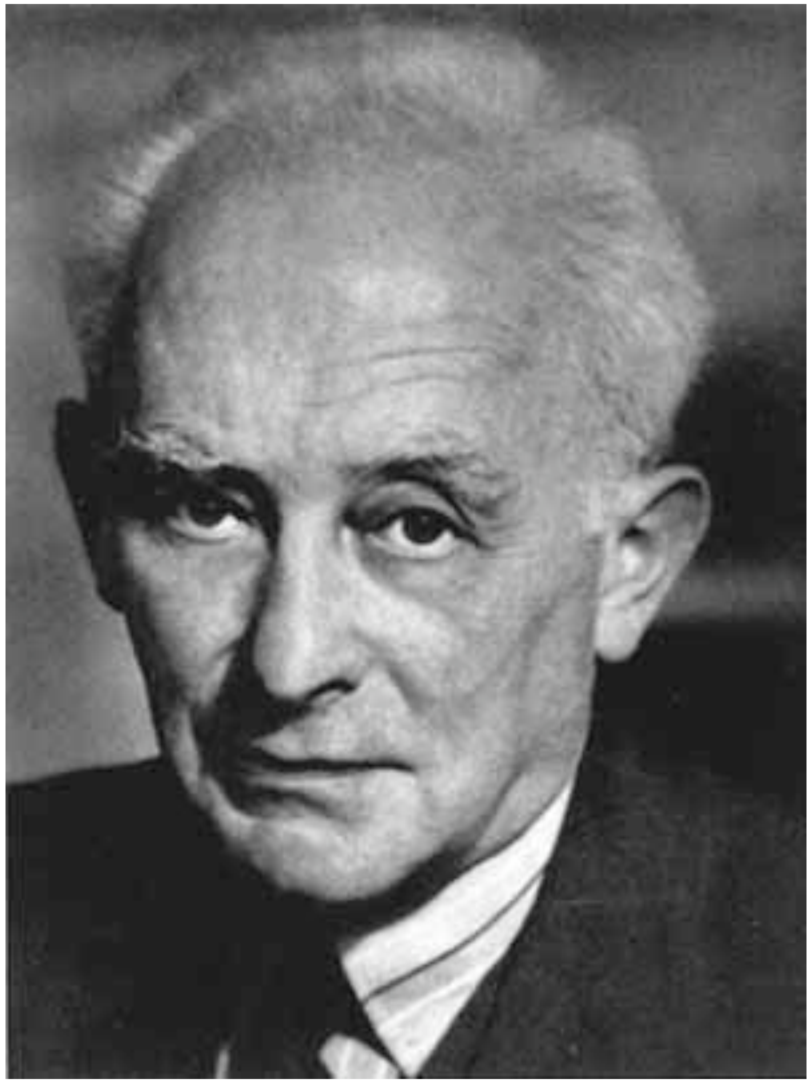
My two connections to Wrocław

My two connections to Wrocław

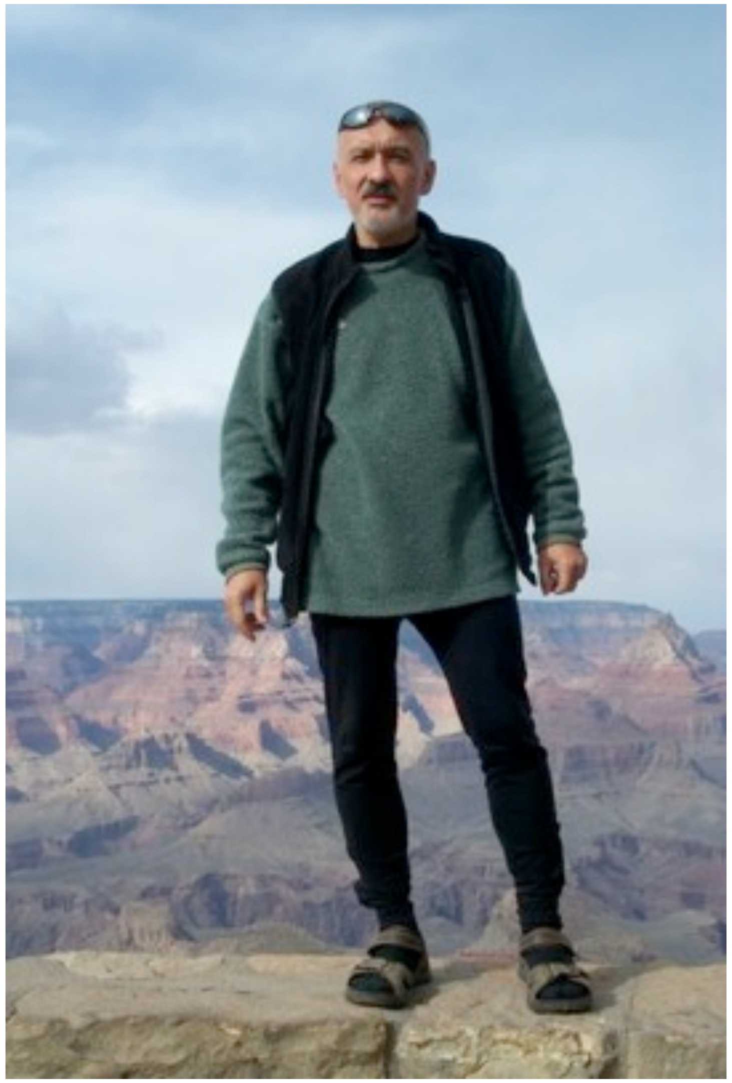


Max Born
(1882-1970)

My two connections to Wrocław



Max Born
(1882-1970)



Krzysztof Galicki
(1958-2007)

Introduction

Plan of the talk

Plan of the talk

- M2-branes and AdS/CFT

Plan of the talk

- M2-branes and AdS/CFT
- Superconformal Chern-Simons+matter

Plan of the talk

- M2-branes and AdS/CFT
- Superconformal Chern-Simons+matter
- Construction in terms of 3-algebras

Plan of the talk

- M2-branes and AdS/CFT
- Superconformal Chern-Simons+matter
- Construction in terms of 3-algebras
- Some open questions

Based on:

arXiv:0809.1086 [hep-th]

with Paul de Medeiros + Elena Méndez-Escobar
+ Patricia Ritter

and preprint in preparation

with Paul de Medeiros + Elena Méndez-Escobar

M2-branes and AdS/CFT

M2-branes

M2-branes

Eleven dimensional supergravity admits a two-parameter family of half-supersymmetric backgrounds:

M2-branes

Eleven dimensional supergravity admits a two-parameter family of half-supersymmetric backgrounds:

$$\begin{aligned} g &= H^{-2/3} g\left(\mathbb{R}^{1,2}\right) + H^{1/3}\left(dr^2 + r^2 g\left(S^7\right)\right) \\ F &= \mathrm{dvol}\left(\mathbb{R}^{1,2}\right) \wedge dH^{-1} \end{aligned}$$

M2-branes

Eleven dimensional supergravity admits a two-parameter family of half-supersymmetric backgrounds:

$$
\begin{aligned}
g &= H^{-2/3} g\left(\mathbb{R}^{1,2}\right) + H^{1/3}\left(dr^2 + r^2 g\left(S^7\right)\right) \\
F &= \mathrm{dvol}\left(\mathbb{R}^{1,2}\right) \wedge dH^{-1}
\end{aligned}
$$

where

$$H = \alpha + \frac{\beta}{r^6}$$

Duff+Stelle (1991)

For generic α and β, this describes a "stack" of $n \propto \beta$ coincident **M2**-branes.

For generic α and β, this describes a "stack" of $n \propto \beta$ coincident **M2**-branes.

For $\beta = 0$, the background becomes (*11*-dimensional) Minkowski spacetime, whereas for $\alpha = 0$, it becomes

$$\mathrm{AdS}_4 \times S^7 \quad \text{with} \quad 2R_{\mathrm{AdS}} = R_S = \beta^{1/6}$$

which is the near-horizon geometry of the n coincident **M2**-branes.

Gibbons+Townsend (1993), Duff+Gibbons+Townsend (1994)

Generalisation

Generalisation

$$g = H^{-2/3} g(\mathbb{R}^{1,2}) + H^{1/3}(dr^2 + r^2 g(X^7))$$

$$F = \mathrm{dvol}(\mathbb{R}^{1,2}) \wedge dH^{-1}$$

Generalisation

$$g = H^{-2/3} g(\mathbb{R}^{1,2}) + H^{1/3}(dr^2 + r^2 g(X^7))$$

$$F = \mathrm{dvol}(\mathbb{R}^{1,2}) \wedge dH^{-1}$$

Any Einstein 7-manifold,
admitting real Killing spinors:

$$\nabla_V \phi = \tfrac{1}{2} V \cdot \phi$$

Generalisation

$$g = H^{-2/3} g(\mathbb{R}^{1,2}) + H^{1/3}(dr^2 + r^2 g(X^7))$$

$$F = \mathrm{dvol}(\mathbb{R}^{1,2}) \wedge dH^{-1}$$

Any Einstein 7-manifold, admitting real Killing spinors:

$$\nabla_V \phi = \tfrac{1}{2} V \cdot \phi$$

Interpretation: M2-branes at a conical singularity in a special holonomy 8-manifold. Of course, this breaks some supersymmetry.

Acharya+FO+Hull+Spence (1998)

The cone construction

The cone construction

The cone construction solves the problem of which riemannian manifolds admit real Killing spinors:

The cone construction

The cone construction solves the problem of which riemannian manifolds admit real Killing spinors:

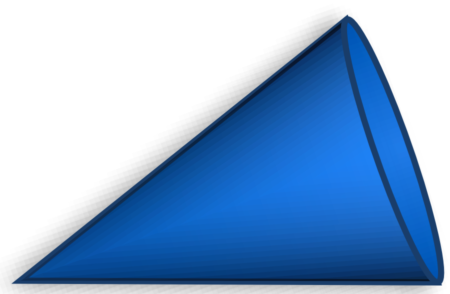
(X, g) admits real Killing spinors

The cone construction

The cone construction solves the problem of which riemannian manifolds admit real Killing spinors:

(X, g) admits real Killing spinors

$$\Updownarrow$$



$(\mathbb{R}^+ \times X, dr^2 + r^2 g)$ admits parallel spinors

Bär (1993)

If X is complete, then the cone is either flat or irreducible.

Gallot (1979)

If X is complete, then the cone is either flat or irreducible.

Gallot (1979)

Holonomy	Parallel spinors
Spin(7)	(1,0)
SU(4)	(2,0)
Sp(2)	(3,0)
{1}	(8,8)

Wang (1989)

7-manifolds with real Killing spinors

7-manifolds with real Killing spinors

7-dimensional geometry	Holonomy of cone	Killing spinors
Weak G_2 holonomy	Spin(7)	1
Sasaki-Einstein	SU(4)	2
3-Sasaki	Sp(2)	3
Sphere	$\{1\}$	8

Manifolds with $N = 4, 5, 6$ may be obtained by quotienting the sphere.

Manifolds with $N=4,5,6$ may be obtained by quotienting the sphere.

Every smooth quotient of the round 7-sphere by a subgroup of isometries admits a spin structure and some quotients also admit real Killing spinors.

Manifolds with $N=4,5,6$ may be obtained by quotienting the sphere.

Every smooth quotient of the round 7-sphere by a subgroup of isometries admits a spin structure and some quotients also admit real Killing spinors.

In summary, there exist finite subgroups $\Gamma \subset SO(8)$ such that

$$S^7/\Gamma$$

admits $N \leq 6$ Killing spinors. (Also $N=8$ for $\Gamma = \mathbb{Z}/2$.)

FO+Gadhia (2006,?)

AdS/CFT predicts the existence of a three-dimensional superconformal field theory dual to each of these M2-brane configurations.

AdS/CFT predicts the existence of a three-dimensional superconformal field theory dual to each of these M2-brane configurations.

The **superconformal algebra** of the field theory is isomorphic to the **Killing superalgebra** of the near-horizon limit.

AdS/CFT predicts the existence of a three-dimensional superconformal field theory dual to each of these M2-brane configurations.

The **superconformal algebra** of the field theory is isomorphic to the **Killing superalgebra** of the near-horizon limit.

The Killing superalgebra of the near-horizon limit of the M2-branes is isomorphic to $\mathfrak{osp}(N|4)$, in agreement with Nahm's classification of 3-dimensional superconformal algebras.

Acharya+FO+Hull+Spence (1998), FO (1999)

The even Lie subalgebra is isomorphic to

The even Lie subalgebra is isomorphic to

$$\mathfrak{so}(N) \oplus \mathfrak{sp}(4, \mathbb{R})$$

The even Lie subalgebra is isomorphic to

SUGRA

isometries of 7-manifold

isometries of $AdS4$

$$\mathfrak{so}(N) \oplus \mathfrak{sp}(4, \mathbb{R})$$

The even Lie subalgebra is isomorphic to

SUGRA

isometries of 7-manifold

isometries of AdS_4

$$\mathfrak{so}(N) \oplus \mathfrak{sp}(4, \mathbb{R})$$

CFT

R-symmetry

conformal algebra

Chern-Simons theories

Chern-Simons theories

It took a decade to construct candidate theories realising these superconformal algebras.

Chern-Simons theories

It took a decade to construct candidate theories realising these superconformal algebras.

They are constructed by coupling supersymmetric Chern-Simons theory to matter hypermultiplets ***not*** (necessarily) in the adjoint representation.

Chern-Simons theories

It took a decade to construct candidate theories realising these superconformal algebras.

They are constructed by coupling supersymmetric Chern-Simons theory to matter hypermultiplets ***not*** (necessarily) in the adjoint representation.

They can be formulated succinctly in terms of certain **3-algebras**.

Superconformal Chern-Simons + matter theories

Some references

Some references

N≤3 theories Gaiotto+Yin (2007)

Some references

$N \leq 3$ theories — Gaiotto+Yin (2007)

$N=4$ theories — Gaiotto+Witten, Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)

Some references

$\mathcal{N}\leq3$ theories — Gaiotto+Yin (2007)

$\mathcal{N}=4$ theories — Gaiotto+Witten, Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)

$\mathcal{N}=5$ theories — Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)

Some references

N≤3 theories — Gaiotto+Yin (2007)

N=4 theories — Gaiotto+Witten, Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)

N=5 theories — Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)

N=6 theories — Aharony+Bergman+Jafferis+Maldacena, Bagger+Lambert, Schnabl+Tachikawa (2008)

Some references

$N \leq 3$ theories	Gaiotto+Yin (2007)
N=4 theories	Gaiotto+Witten, Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)
N=5 theories	Hosomichi+Lee+Lee+Lee+Park, Bergshoeff+Hohm+Roest+Samtleben+Sezgin (2008)
N=6 theories	Aharony+Bergman+Jafferis+Maldacena, Bagger+Lambert, Schnabl+Tachikawa (2008)
N=8 theories	Bagger+Lambert (2007), Gustavsson (2007)

The field content consists of a $\mathfrak{g}$-valued gauge field $\boldsymbol{A}$ and its superpartner χ. The Chern-Simons lagrangian is

$$\frac{1}{2}\mathrm{Tr}(A \wedge dA) + \frac{1}{3}\mathrm{Tr}(A \wedge A \wedge A) - \mathrm{Tr}(\overline{\chi}\chi)$$

The field content consists of a $\mathfrak{g}$-valued gauge field $\boldsymbol{A}$ and its superpartner χ. The Chern-Simons lagrangian is

$$\tfrac{1}{2}\mathrm{Tr}(A \wedge dA) + \tfrac{1}{3}\mathrm{Tr}(A \wedge A \wedge A) - \mathrm{Tr}(\overline{\chi}\chi)$$

where $\boldsymbol{Tr}$ stands for an ad-invariant inner product on $\mathfrak{g}$. The theory is uniquely defined by specifying $\mathfrak{g}$ and $\boldsymbol{Tr}$. For $\mathfrak{g}$ simple, $\boldsymbol{Tr}$ is a multiple of the Killing form. This multiple is quantised: the **level** of the Chern-Simons theory.

The field content consists of a $\mathfrak{g}$-valued gauge field $\boldsymbol{A}$ and its superpartner χ . The Chern-Simons lagrangian is

$$\tfrac{1}{2}\mathrm{Tr}(A \wedge dA) + \tfrac{1}{3}\mathrm{Tr}(A \wedge A \wedge A) - \mathrm{Tr}(\overline{\chi}\chi)$$

where $\boldsymbol{Tr}$ stands for an ad-invariant inner product on $\mathfrak{g}$. The theory is uniquely defined by specifying $\mathfrak{g}$ and $\boldsymbol{Tr}$. For $\mathfrak{g}$ simple, $\boldsymbol{Tr}$ is a multiple of the Killing form. This multiple is quantised: the **level** of the Chern-Simons theory.

For the M2 theories, however, $\mathfrak{g}$ is **not** simple.

... plus matter

... plus matter

The matter content consists of a scalar field $\boldsymbol{X}$ in a representation

$$B \otimes M$$

of $\mathfrak{so}(N) \times \mathfrak{g}$ and a fermionic spinor ψ in a representation

$$F \otimes M$$

... plus matter

The matter content consists of a scalar field X in a representation

$$B \otimes M$$

of $\mathfrak{so}(N) \times \mathfrak{g}$ and a fermionic spinor ψ in a representation

$$F \otimes M$$

The supercharges, and hence the supersymmetry parameters, are spinors with values in the vector representation V of $\mathfrak{so}(N)$.

The supersymmetry transformations take the form

$$\delta_\varepsilon X = \overline{\varepsilon}\psi$$

$$\delta_\varepsilon \psi = dX \cdot \varepsilon + \cdots$$

The supersymmetry transformations take the form

$$\delta_\varepsilon X = \bar{\varepsilon}\psi$$

$$\delta_\varepsilon \psi = dX \cdot \varepsilon + \cdots$$

whence

$$V \otimes F \supset B \qquad\qquad V \otimes B \supset F$$

The supersymmetry transformations take the form

$$\delta_\varepsilon X = \overline{\varepsilon}\psi$$
$$\delta_\varepsilon \psi = dX \cdot \varepsilon + \cdots$$

whence

$$V \otimes F \supset B \qquad\qquad V \otimes B \supset F$$

which suggests taking $\boldsymbol{B}$ and $\boldsymbol{F}$ to be spinor representations with the above relations induced by Clifford multiplication.

Spinor representations

N	so(N)	spinor reps
2	$\mathfrak{u}(1)$	$\mathbb{C}$
3	$\mathfrak{sp}(1)$	$\mathbb{H}$
4	$\mathfrak{sp}(1) \oplus \mathfrak{sp}(1)$	$\mathbb{H} \oplus \mathbb{H}$
5	$\mathfrak{sp}(2)$	$\mathbb{H}^2$
6	$\mathfrak{su}(4)$	$\mathbb{C}^4$
7	$\mathfrak{so}(7)$	$\mathbb{R}^8$
8	$\mathfrak{so}(8)$	$\mathbb{R}^8 \oplus \mathbb{R}^8$

Matter is always in a real representation, so this dictates the type of the representation M of $\mathfrak{g}$:

Matter is always in a real representation, so this dictates the type of the representation M of $\mathfrak{g}$:

$\mathbb{R}$ for N=1,7,8

Matter is always in a real representation, so this dictates the type of the representation M of $\mathfrak{g}$:

$\mathbb{R}$ for N=1,7,8

$\mathbb{C}$ for N=2,6

Matter is always in a real representation, so this dictates the type of the representation M of $\mathfrak{g}$:

$\mathbb{R}$ for N=1,7,8

$\mathbb{C}$ for N=2,6

$\mathbb{H}$ for N=3,4,5

Matter is always in a real representation, so this dictates the type of the representation M of $\mathfrak{g}$:

$\mathbb{R}$ for $N=1,7,8$

$\mathbb{C}$ for $N=2,6$

$\mathbb{H}$ for $N=3,4,5$

For $N=1,2,3$ any (unitary) representation is allowed. For $N\geq 4$ the representation must give rise to a particular kind of 3-algebra.

3-algebras

Metric 3-algebras

The data of the Chern-Simons theory

- a metric Lie algebra $\mathfrak{g}$, Tr
- a unitary representation, M

defines a **metric 3-Leibniz algebra**.

$T : M \times M \to \mathfrak{g}$ is the transpose of the $\mathfrak{g}$-action on M

$$\mathrm{Tr}(T(x,y)X) = \langle X \cdot x, y \rangle$$

$T : M \times M \to \mathfrak{g}$ is the transpose of the $\mathfrak{g}$-action on M

$$\mathrm{Tr}(T(x,y)X) = \langle X \cdot x, y \rangle$$

The 3-bracket $M \times M \times M \to M$ is given by

$$[x,y,z] := T(x,y) \cdot z$$

Faulkner (1973)

$T : M \times M \to \mathfrak{g}$ is the transpose of the $\mathfrak{g}$-action on M

$$\operatorname{Tr}(T(x,y)X) = \langle X \cdot x, y \rangle$$

The 3-bracket $M \times M \times M \to M$ is given by

$$[x,y,z] := T(x,y) \cdot z$$

Faulkner (1973)

Unitary representations come in three types: **real**, **complex** and **quaternionic** — each one giving rise to a different class of 3-algebra.

It obeys three main properties:

It obeys three main properties:

the **fundamental identity**:

$$[x, y, [z_1, z_2, z_3]] = [[x, y, z_1], z_2, z_3] + [z_1, [x, y, z_2], z_3] + [z_1, z_2, [x, y, z_3]]$$

It obeys three main properties:

the **fundamental identity**:

$$[x,y,[z_1,z_2,z_3]] = [[x,y,z_1],z_2,z_3] + [z_1,[x,y,z_2],z_3] + [z_1,z_2,[x,y,z_3]]$$

the **metricity** condition:

$$\langle [x,y,z_1],z_2 \rangle = - \langle z_1,[x,y,z_2] \rangle$$

It obeys three main properties:

the **fundamental identity**:

$$[x, y, [z_1, z_2, z_3]] = [[x, y, z_1], z_2, z_3] + [z_1, [x, y, z_2], z_3] + [z_1, z_2, [x, y, z_3]]$$

the **metricity** condition:

$$\langle [x, y, z_1], z_2 \rangle = - \langle z_1, [x, y, z_2] \rangle$$

and also a **symmetry** condition:

$$\langle [x, y, z_1], z_2 \rangle = + \langle z_1, [z_2, x, y] \rangle$$

Real representations

Real representations

These include two extremal cases:

Real representations

These include two extremal cases:

- **3-Lie algebras**, where $[x, y, z]$ is totally skewsymmetric, and

Filippov (1985)

Real representations

These include two extremal cases:

- **3-Lie algebras**, where $[x,y,z]$ is totally skewsymmetric, and

Filippov (1985)

- **Lie triple systems**, where $[x,y,z]+[y,z,x]+[z,x,y]=0$, corresponding to **Riemannian symmetric spaces**.

...,Jacobson (1951), Lister (1952), Yamaguti (1957)

Complex representations

Complex representations

These include two extremal cases:

Complex representations

These include two extremal cases:

- **N=6 triple systems**, where $[x,y,z]=-[z,y,x]$, and

Bagger+Lambert (2008)

Complex representations

These include two extremal cases:

- **N=6 triple systems**, where $[x,y,z]=-[z,y,x]$, and

Bagger+Lambert (2008)

- **hermitian Lie triple systems**, corresponding to **hermitian symmetric spaces**

Quaternionic representations

Quaternionic representations

These include two extremal cases:

Quaternionic representations

These include two extremal cases:

- **anti-3-Lie algebras**, where $[x,y,z]$ is totally symmetric, corresponding to **quaternionic Kähler symmetric spaces**, and

Quaternionic representations

These include two extremal cases:

- **anti-3-Lie algebras**, where $[x,y,z]$ is totally symmetric, corresponding to **quaternionic Kähler symmetric spaces**, and

- **anti-Lie triple systems**, where $[x,y,z]+[y,z,x]+[z,x,y]=0$

An important characteristic of the extremal 3-algebras is that they admit embedding into Lie (super)algebras in such a way that the 3-bracket is a nested Lie bracket:

$$[x,y,z] = [[x,y],z]$$

An important characteristic of the extremal 3-algebras is that they admit embedding into Lie (super)algebras in such a way that the 3-bracket is a nested Lie bracket:

$$[x,y,z] = [[x,y],z]$$

This is a classical result for the symmetric spaces:

An important characteristic of the extremal 3-algebras is that they admit embedding into Lie (super)algebras in such a way that the 3-bracket is a nested Lie bracket:

$$[x,y,z] = [[x,y],z]$$

This is a classical result for the symmetric spaces:

- **RSS**: 2-graded Lie algebra

An important characteristic of the extremal 3-algebras is that they admit embedding into Lie (super)algebras in such a way that the 3-bracket is a nested Lie bracket:

$$[x,y,z] = [[x,y],z]$$

This is a classical result for the symmetric spaces:

- **RSS**: 2-graded Lie algebra
- **HSS**: 3-graded Lie algebra

An important characteristic of the extremal 3-algebras is that they admit embedding into Lie (super)algebras in such a way that the 3-bracket is a nested Lie bracket:

$$[x,y,z] = [[x,y],z]$$

This is a classical result for the symmetric spaces:

- **RSS**: 2-graded Lie algebra
- **HSS**: 3-graded Lie algebra
- **QKSS**: 5-graded Lie algebra

The same is true for the other extremal cases relevant to superconformal Chern-Simons theories:

The same is true for the other extremal cases relevant to superconformal Chern-Simons theories:

- **3LA**: Lie superalgebra (at least in pos-def case)

The same is true for the other extremal cases relevant to superconformal Chern-Simons theories:

- **3LA**: Lie superalgebra (at least in pos-def case)
- **N=6**: 3-graded Lie superalgebra

The same is true for the other extremal cases relevant to superconformal Chern-Simons theories:

- **3LA**: Lie superalgebra (at least in pos-def case)
- **N=6**: 3-graded Lie superalgebra
- **aLTS**: Lie superalgebra

The same is true for the other extremal cases relevant to superconformal Chern-Simons theories:

- **3LA**: Lie superalgebra (at least in pos-def case)
- **N=6**: 3-graded Lie superalgebra
- **aLTS**: Lie superalgebra

This reduces their classification to that of certain Lie superalgebras.

Results

N	bosonic rep	3-algebra
4	$(P \otimes V_1) \oplus (N \otimes V_2)$	V_1, V_2 $\mathbb{H}$ anti-LTS
5	$S \otimes V$	V $\mathbb{H}$ anti-LTS
6	$(P \otimes V) \oplus c.c.$	V $\mathbb{C}$ N=6
7	$S \otimes V$	V $\mathbb{R}$ 3-Lie
8	$P \otimes V$	V $\mathbb{R}$ 3-Lie

P = positive-chirality spinor of $\mathfrak{so}(N)$
N = negative-chirality spinor of $\mathfrak{so}(N)$
S = spinor of $\mathfrak{so}(N)$

Some enhancements

Some enhancements

$N{=}4$ to $N{=}5$: take $V_1{=}V_2$

Some enhancements

$N=4$ to $N=5$: take $V_1=V_2$

$N=5$ to $N=6$: take $V=W \oplus W^*$

Some enhancements

$N=4$ to $N=5$: take $V_1 = V_2$

$N=5$ to $N=6$: take $V = W \oplus W^*$ (V **aLTS** iff W is **N=6**)

Some enhancements

$N{=}4$ to $N{=}5$: take $V_1{=}V_2$

$N{=}5$ to $N{=}6$: take $V{=}W \oplus W^*$ (V **aLTS** iff W is **N=6**)

$N{=}6$ to $N{\geq}7$: take $V{=}\mathbb{C}{\otimes}U$

Some enhancements

$N{=}4$ to $N{=}5$: take $V_1{=}V_2$

$N{=}5$ to $N{=}6$: take $V{=}W \oplus W^*$ (V **aLTS** iff W is **N=6**)

$N{=}6$ to $N{\geq}7$: take $V{=}\mathbb{C}{\otimes}U$ (V **N=6** iff U is **3LA**)

Superpotentials

Superpotentials

The $\mathcal{N}=2$ superpotential associated to a complex unitary representation is given (in $\mathcal{N}=1$ superspace) by

$$W = \frac{1}{16}\int d^2\theta\, \mathrm{Tr} T(X,X)^2$$

Superpotentials

The $N\!=\!2$ superpotential associated to a complex unitary representation is given (in $N\!=\!1$ superspace) by

$$W = \frac{1}{16} \int d^2\theta \, \mathrm{Tr} T(X, X)^2$$

to which one can add an F-term superpotential without breaking any supersymmetry.

The $\mathcal{N}=3$ superpotential associated to a quaternionic representation is given (in $\mathcal{N}=1$ superspace) by

$$W = \frac{1}{16}\int d^2\theta \, \left(\mathrm{Tr}T(X,X)^2 + \mathrm{Re}\mathrm{Tr}T(X,JX)^2\right)$$

The $\mathcal{N}=3$ superpotential associated to a quaternionic representation is given (in $\mathcal{N}=1$ superspace) by

$$W = \frac{1}{16}\int d^2\theta \;\left(\mathrm{Tr}T(X,X)^2 + \mathrm{Re}\mathrm{Tr}T(X,JX)^2\right)$$

Invariant quaternionic structure map

The $N{=}3$ superpotential associated to a quaternionic representation is given (in $N{=}1$ superspace) by

$$W = \frac{1}{16} \int d^2\theta \ \left(\mathrm{Tr} T(X,X)^2 + \mathrm{Re}\mathrm{Tr} T(X,JX)^2\right)$$

Invariant quaternionic structure map

This superpotential is **rigid**, consistent with the infinitesimal rigidity of (complete) 3-Sasakian manifolds.

Pedersen+Poon (1999)

The superpotentials for the $N \geq 4$ theories are obtained by plugging the right representation-theoretic data into the $N = 3$ superpotential.

The superpotentials for the $\mathcal{N}\geq 4$ theories are obtained by plugging the right representation-theoretic data into the $\mathcal{N}=3$ superpotential.

Empirically we find that a theory has $\mathcal{N}\geq 4$ superconformal symmetry if and only if the $\mathcal{N}=3$ superpotential is $\mathfrak{so}(\mathcal{N}\text{-}1)$-invariant.

The superpotentials for the $\mathcal{N}\geq 4$ theories are obtained by plugging the right representation-theoretic data into the $\mathcal{N}=3$ superpotential.

Empirically we find that a theory has $\mathcal{N}\geq 4$ superconformal symmetry if and only if the $\mathcal{N}=3$ superpotential is $\mathfrak{so}(\mathcal{N}\text{-}1)$-invariant.

(All this is in the absence of flavour.)

Open questions

Open questions

Open questions

- How about flavour?

Open questions

- How about flavour?
- Are there new superconformal Chern-Simons theories associated to 3-algebras **not** obtained via the Faulkner construction?

Open questions

- How about flavour?
- Are there new superconformal Chern-Simons theories associated to 3-algebras **not** obtained via the Faulkner construction?
- How does the geometry transverse to the M2-branes manifest itself in the superconformal field theory?