Extremal Dependence in International Output Growth: Tales from the Tails[*]

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Abstract

This paper explores the comovement of the economic activity of several OECD countries during periods of large negative and positive growth. Extremal dependence measures are here applied to assess the degree of cross-country tail dependence of output growth rates. Our main empirical findings are: (i) cross-country tail dependence is much stronger during periods of large negative growth, than during the ones of large positive growth; (ii) cross-country growth is asymptotically independent; (iii) cross-country tail dependence is considerably stronger than the one arising from a Gaussian dependence model. In addition, our results suggest that, among the typical determinants for explaining international output growth synchronization, only economic specialization similarity seems to play a role during such extreme periods.

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I.   Introduction

The fallout of the recent 2009 Great Recession has been widely felt. Although such extreme events are infrequent, but recurrent in our profession, many of our data modeling tools have been devised for (regular) central events, and tend to perform poorly on (rare) tail events—where it is becomes challenging to extrapolate beyond the observed data. The huge losses linked to the occurrence of such rare but catastrophic events, have led to an increasing interest in the topic of statistics of extremes (Coles, 2001; Beirlant *et al.*, 2004; de Haan and Ferreira, 2006); for some recent applications in economics, see Straetmans *et al.* (2008), Mohtadi and Murshid (2009), Ren and Giles (2010), Bollerslev *et al.* (2013), and de Carvalho *et al.* (2013).

Here we will examine what insights can methods of statistics of extremes provide on the comovement of the economic activity of several countries, during periods of large negative and positive growth—left and right tails, respectively. Many statistical tools often applied in the analysis of central events, are inappropriate for the analysis of extremes, and in particular Pearson correlation—the most used measure of synchronization of economic activity in the growth cycle literature—fails to be a reliable measure for assessing the level of agreement of two variables at their extreme levels.[1] Despite its broad use, the price of its simplicity comes at the cost of some important limitations. First, Pearson correlation makes no distinction between large negative and positive values; specifically, in the context of the growth cycle literature, this implies that it places the same weight on negative and positive growth rates. Second, Pearson correlation is defined through an average of departures from the mean, so that its unsuitableness for quantifying dependence at tail events is self evident; hence, it is inappropriate for evaluating the strength of the comovement of output growth rates for periods which are far from average levels, such as during moments for which there is an extremely sharp decline in economic activity.

In this paper we explore the comovement of the economic activity of several OECD countries during periods of large negative and positive growth. Extremal dependence measures (Coles *et al.*, 1999; Poon *et al.*, 2003, 2004) are here used to evaluate the degree of cross-country tail dependence of output growth rates, over the past 50 years. Although there has been an increasing interest in studying the distribution of output growth rates (Canning *et al.*, 1998; Lee *et al.*, 1998; Fagiolo *et al.*, 2008; Castaldi and Dosi, 2009; Fagiolo *et al.*, 2009), our analysis allows us to study the comovements of international output growth from a completely novel standpoint, and to our knowledge this is the first paper modeling business cycle data with statistics of multivariate extremes. This allows us to collect some new stylized facts for cross-country output dynamics. *First*, the application of extremal dependence measures, allows us to observe that the comovement of output growth rates is much stronger in left tails than in right tails. Hence, during acute recession periods the strength of comovement of growth cycles is much stronger than during the utmost expansionary periods. *Second*, cross-country growth is asymptotically independent. This is in line with Poon *et al.* (2004), who also find evidence of asymptotic independence in stock markets returns, and who note that this has important implications for understanding what models can be compatible with the data. *Third*, dependence in the tails is shown to be much

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[1]Correlation-based techniques are of course not the unique way to address the study of comovements; see Gligor and Ausloos (2008) for a clustering-based approach, and Diebold and Yilmaz (2011) for a connectedness-based method.

stronger than the one arising from a Gaussian dependence model. Thus, if we use Pearson correlation for measuring synchronization of output growth rates during extreme scenarios, we will tend to underestimate dependence in the tails. The above-mentioned caveats of the most employed measure of synchronization of economic activity motivates the question: are the factors driving the mechanics of propagation of shocks the same over junctures of sharp variations in output? Put differently, are the typical determinants of synchronization tenable throughout moments of exceptionally large negative (and positive) growth? As a byproduct of our analysis puts forward, among some of the most standard determinants for explaining international output synchronization (Baxter and Kouparitsas, 2005; Inklaar *et al.*, 2008), only economic specialization similarity seems to have an impact during such extreme periods.

The plan of this paper is as follows. The next section introduces extremal dependence measures along with guidelines for estimation and inference. In section III we explore the insights offered by these measures, to the analysis of the comovement of the economic activity of several OECD countries, during periods of extreme negative (and positive) growth. Concluding remarks are given in section IV.

II.   Measuring Extremal Dependence

Dual measures of tail dependence

The link between the joint distribution function and its corresponding marginals can provide helpful information regarding the dependence of two random variables. In statistical parlance, the function C establishing such connection is defined as a copula (Nelsen, 2006). A key result in copula modeling is Sklar's theorem which, in its simplest form, establishes the existence and unicity of *a* copula C, for any given set of continuous marginals assigned to a certain joint distribution (Embrechts, 2009, Theorem 1). The most straightforward example of copula arises when the variables of interest are independent, so that the joint distribution function can be written as the product of the marginals, and so the corresponding copula is simply given by $C(u, v) = uv$, for $(u, v) \in [0, 1]^2$; other examples of copulas can be found, for instance, in Granger *et al.* (2006), and references therein. As we shall see below, copulas also play a role in joint tail dependence modeling.

Before we are able to measure dependence in the extreme levels of the variables G_1 and G_2, here representing the output growth rates of two countries of interest, we first need to convert the data into an appropriate common scale. Only if the data are transformed into a unified scale fair comparisons can be made. Output growth rates are known to possess fat tails (Fagiolo *et al.*, 2008), so that transforming the data into the unit Fréchet scale becomes the natural choice.[2] This can be accomplished by turning the original pair (G_1, G_2) into

$$(Z_1, Z_2) = \left(-(\log F_{G_1})^{-1}, -(\log F_{G_2})^{-1} \right). \tag{1}$$

The marginal distribution functions G_1 and G_2 are typically unknown so that in practice the empirical distribution functions $\widehat{F}_{G_1}$ and $\widehat{F}_{G_2}$ are plugged in (1). After such relocation

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[2]Although, we are restricting the exposition to the unit Fréchet scale, the conceptual framework underlying all measures presented here remains unchanged for cases wherein the variables are transformed into unit Pareto margins as, for instance, in Straetmans *et al.* (2008, p. 23). In such case, instead of using (1), we would convert the pair (G_1, G_2) into $(\tilde{Z}_1, \tilde{Z}_2) = \left((1 - F_{G_1})^{-1}, (1 - F_{G_2})^{-1} \right)$.

has been performed, the order of magnitude of the high quantiles of G_1 becomes comparable with those of G_2, so that all differences in the distributions that may persist are simply due to the dependence between the variables. A natural measure for assessing the degree of dependence at an arbitrary high level z, is given by the bivariate tail dependence index χ (Coles *et al.*, 1999; Poon *et al.*, 2003, 2004), defined as

$$\chi = \lim_{z \to \infty} \Pr\{Z_1 > z \mid Z_2 > z\}. \tag{2}$$

Roughly speaking, χ measures the degree of dependence which may eventually prevail in the limit. As it is clear from (2), χ is constrained to live in the interval $[0, 1]$. If dependence persists as $z \to \infty$, so that $0 < \chi \leq 1$, then we say that G_1 and G_2 are *asymptotically dependent*. If the degree of dependence vanishes in the limit, then $\chi = 0$, and in this case we say that the variables are *asymptotically independent*.

As discussed by Poon *et al.* (2003, 2004), this extremal dependence characterization is not only consequential for a more fine understanding of the comovement of the variables during extreme events as it also brings deep implications for statistically modeling the data. In particular, if the variables are asymptotically independent then any naive application of multivariate extreme value distributions leads to an overrepresentation of the occurrence of simultaneous extreme events. It can be shown (Coles *et al.*, 1999), that χ can also be rewritten in terms of a limit of a function of the copula C, i.e.

$$\chi = \lim_{u \to 1} 2 - \frac{\log \mathrm{C}(u, u)}{\log u}. \tag{3}$$

Hence, the function C not only 'couples' the joint distribution function and its corresponding marginals, as it also provides helpful information for modeling joint tail dependence.

Tail dependence should be measured according to the dependence structure underlying the variables under analysis. If the variables are asymptotically dependent, the measure χ is appropriate for assessing what is the strength of dependence which links the variables at the extremes. If the variables are asymptotically independent then $\chi = 0$, so that χ unfairly pools the cases wherein although dependence may not prevail in the limit, it may still persist for relatively large levels of the variables. To measure extremal dependence under asymptotic independence, Coles *et al.* (1999) introduced the following measure

$$\overline{\chi} = \lim_{z \to \infty} \frac{2 \log \Pr\{Z_1 > z\}}{\log \Pr\{Z_1 > z, Z_2 > z\}} - 1, \tag{4}$$

which takes values on the interval $(-1; 1]$. The interpretation of $\overline{\chi}$ is to a certain extent analogous to Pearson correlation, namely: values of $\overline{\chi} > 0$, $\overline{\chi} = 0$, and $\overline{\chi} < 0$, respectively correspond to positive association, exact independence, and negative association in the extremes. It follows that if the dependence structure is Gaussian then $\overline{\chi} = \rho$ (Poon *et al.*, 2003, 2004). This benchmark case is particularly helpful for guiding how does the dependence in the tails—as measured by $\overline{\chi}$—compares with the one arising from fitting a Gaussian dependence model; for a comprehensive inventory for the functional forms of the extremal measure(s) $\overline{\chi}$ (and χ), over a broad variety of dependence models, see Heffernan (2000).

The concepts of asymptotic dependence and asymptotic independence can also be characterized through $\overline{\chi}$. More specifically, for asymptotically dependent variables, it holds that $\overline{\chi} = 1$, while for asymptotically independent variables $\overline{\chi}$ takes values in $(-1, 1)$. Hence χ

and $\overline{\chi}$ can be seen as dual measures of joint tail dependence: if $\overline{\chi} = 1$ and $0 < \chi \leq 1$, the variables are asymptotically dependent, and χ assesses the size of dependence within the class of asymptotically dependent distributions; if $-1 \leq \overline{\chi} < 1$ and $\chi = 0$, the variables are asymptotically independent, and $\overline{\chi}$ evaluates the extent of dependence within the class of asymptotically independent distributions.

In a similar way to (3), the extremal measure $\overline{\chi}$ can also be written using copulas, viz.

$$\overline{\chi} = \lim_{u \to 1} \frac{2 \log(1 - u)}{\log\{1 - 2u + \mathrm{C}(u, u)\}}. \tag{5}$$

Hence, the function C can provide helpful information for assessing dependence in the extremes, both under asymptotic dependence and asymptotic independence; further details on these connections can be found in de Carvalho and Ramos (2012).

In the next subsection we focus on estimation features of the dual measures of joint tail dependence introduced above.

Estimation and inference

Although the copula-based representations provided above are conceptually enlightening, they are not the most appropriate for estimation purposes. These can however be suitably reparametrized by using a result due to Ledford and Tawn (1996, 1998), which establishes that, under fairly mild assumptions, the univariate variable $\mathcal{Z} = \min\{Z_1, Z_2\}$ has a regularly varying tail with index $-1/\eta$. Formally

$$\Pr\{\mathcal{Z} > z\} \sim \frac{\mathcal{L}(z)}{z^{1/\eta}}, \qquad z \to \infty, \tag{6}$$

where $\mathcal{L}(z)$ is used to denote a slowly varying function, i.e., $\lim_{x \to \infty} \mathcal{L}(xz)/\mathcal{L}(x) = 1$, for every $z > 0$; the constant η, which is constrained to the interval $(0,1]$, is the so-called coefficient of tail dependence. The result reported in (6) can be restated as

$$\Pr\{Z_1 > z, Z_2 > z\} \sim \frac{\mathcal{L}(z)}{z^{1/\eta}}, \qquad z \to \infty. \tag{7}$$

Hence, if we plug in (7) in equation (4), the following reparametrization (Coles *et al.*, 1999) of $\overline{\chi}$, in terms of the coefficient of tail dependence η, arises

$$\overline{\chi} = 2\eta - 1. \tag{8}$$

From the practical stance this representation is quite appealing since it only depends on η, which can be estimated through the well-known Hill estimator (Hill, 1975), defined as

$$\widehat{\eta}_{\mathrm{H}} = \frac{1}{k} \sum_{i=1}^{k} \left\{ \log \mathcal{Z}_{(n-k+i)} - \log \mathcal{Z}_{(n-k)} \right\}, \tag{9}$$

We use $\mathcal{Z}_{(1)} \leq \cdots \leq \mathcal{Z}_{(n)}$, to denote the order statistics of a random sample $\{\mathcal{Z}_i\}_{i=1}^{n}$ from $\mathcal{Z} = \min\{Z_1, Z_2\}$. Hence, from the discussion above, estimation of $\overline{\chi}$ (Poon *et al.*, 2003, 2004) follows naturally as

$$\widehat{\overline{\chi}} = 2\widehat{\eta}_{\mathrm{H}} - 1, \tag{10}$$

with corresponding variance

$$\text{var}\{\widehat{\overline{\chi}}\} = \frac{(\widehat{\overline{\chi}} + 1)^2}{\mathcal{Z}_{(n-k)}}. \tag{11}$$

A remark regarding practicalities. Note that eq. (9) depends on k, which represents the number of observations used to conduct the tail index estimation, and its selection entails a bias–variance tradeoff: if too few observations are used, the produced estimate is subject to a large variance, whereas if too many observations are used, a bias will arise. Here, we select k using the iterative subsample bootstrap method discussed in Daníelsson and de Vries (1997), so that a compromise in this tradeoff can be achieved. The inference is based on the asymptotic normality of the Hill estimator (de Haan and Ferreira, 2006, section III); hence, if $\widehat{\overline{\chi}}$ is significantly less than 1, at the α-level, so that

$$\widehat{\overline{\chi}} < 1 - z_\alpha \sqrt{\text{var}\{\widehat{\overline{\chi}}\}}\;,$$

then we infer that the variables are asymptotically independent and take $\chi = 0$. It is important to underscore that only if there is no significant evidence to reject $\overline{\chi} = 1$, we prosecute with the estimation of χ, which is done under the assumption $\overline{\chi} = \eta = 1$.

An estimator for χ, can be constructed using the maximum likelihood estimator of the slowly varying function

$$\widehat{\mathcal{L}(z)} = (1 - k/n)\{\mathcal{Z}_{(n-k)}\}^{1/\eta}, \tag{12}$$

(Poon *et al.*, 2003, 2004). Thus, if we plug in (12) in equation (7), under the constraint $\widehat{\overline{\chi}} = 1$, and use the definition of the extremal measure χ, the following estimator arises

$$\widehat{\chi} = (k/n)\mathcal{Z}_{(n-k)}, \qquad \text{var}\{\widehat{\chi}\} = k(n-k)/n^3(\mathcal{Z}_{(n-k)})^2.$$

III. Synchronization at the extremes

Extremal dependence in international output growth

Our empirical analysis entails 15 OECD countries: Austria, Belgium, Canada, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, UK, and US. The main criteria for selecting these countries was the period for which the first observation was available, because methods introduced in section II are based on large sample results, and hence we need to confine the breadth of the study to countries for which a longer span of data is available. We use the first differences of the logarithm of the (seasonally adjusted) Industrial Production (IP) index, with the time horizon ranging from 1960:1 to 2009:12, gathered from Thompson Financial Datastream.[3] As mentioned above the presented methods are asymptotically motivated, so that other economic activity measures such as the GDP—which is only available on a quarterly basis and, for most countries, over shorter periods of time—are not considered; although we are aware that the index used here is a proxy for measuring economic activity evolution, it is widely known that the IP is strongly correlated with the aggregate activity as measured by GDP (see, for instance Fagiolo *et al.*, 2008).

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[3]There are two exceptions to be noted. Namely for Canada and Spain, the data was only available starting from 1961:1 and 1964:1, respectively.

[Insert Table 1 about here]

We start the analysis with Pearson correlation ρ which is reported in Table 1. This table summarizes correlation between all possible pairs of economies and thus supplies an important benchmark for comparison with tail dependence measures in the following sense: if we believed that a Gaussian dependence model was ruling the mechanics of the comovement of international output growth then dependence in the left and right tails should coincide with Pearson correlation coefficient.[4] In particular, this would imply that the degree of association should be alike in periods of extreme declines and increases in economic activity. As we shall see below, this happens not to be the case.

[Insert Table 2 about here]

A short comment on notation: below, we use the notations $\overline{\chi}_L$ and $\overline{\chi}_R$ to distinguish left-tail dependence from right-tail dependence. In Table 2 we outline the results from the examination of left-tail dependence in the comovement of economic output. Some brief remarks regarding the construction of this table are in order. First, the optimal $k^\star$ was estimated by dint of the iterative subsample bootstrap procedure of Daníelsson and de Vries (1997), for each possible pair of countries. Second, the corresponding estimates of the coefficient of tail dependence η are obtained through (9). Finally, to work out the estimates of $\overline{\chi}_L$, the Hill estimates obtained in the latter step are introduced in (10).

From the inspection of Table 2 we can ascertain that overall, the reported results are considerably higher than the corresponding counterparts reported in Table 1. To be more precise, in 90.48% of the cases it is verified that the estimated value of $\overline{\chi}_L$ lies above ρ. The lesson here is the following: the strength of economic activity comovement is much stronger during sharp declines than a Pearson correlation would foretell. Additionally, there is strong evidence to support the hypothesis of asymptotic independence in left tails. In 96 pairs, out of a total of $105 = \binom{15}{2}$, we are unable to reject the null of asymptotic independence at the α-level of 5%. Moreover, the percentage of non-rejections increases into 97.1%, with only 3 pairs suggesting asymptotic dependence, if we consider an α-level of 10%. Such pairs are (Japan, Germany), (Canada, Spain) and (UK, Canada), with the corresponding χ values given by 0.3090, 0.3160, and 0.3173, respectively.

[Insert Table 3 about here]

Table 3 sums up an analogous exercise to the one reported in Table 2, but now focusing on right tails. Likewise, there is also a general evidence for the estimated values of $\overline{\chi}_R$ to be larger than their corresponding correlations, as measured by ρ, although the strength of the dominance is here markedly lower. Still, in 71.90% of the cases the computed values of $\overline{\chi}_R$ remain above Pearson correlation. Particularly, this implies that the extent of the synchronization is manifestly larger during periods of sharp increases in the economic activity growth than a naive estimate of ρ would predict. Furthermore, the statistical evidence in favor of the hypothesis of asymptotic independence is also here remarkably clear with all pairs supporting the null at the α-level of 10%. The comparison of Tables 2 and 3 also brings an enlightening point into the discussion: overall, left-tail dependence is markedly stronger than right-tail dependence. To be more specific, in 78.57% of the cases the estimated value of $\overline{\chi}_L$ dominates $\overline{\chi}_R$. The message here is the following: dependence is more pronounced in periods of sharp declines in output, than during epochs of steep increases.

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[4]As discussed above, in a Gaussian dependence model it holds that $\overline{\chi} = \rho$.

[**Insert Figure 1 about here**]

To make a long story short, we depict in Figure 1 the average values per country for $\overline{\chi}_{\text{L}}$, $\overline{\chi}_{\text{R}}$ and for ρ. This figure wraps up the discussion given above concerning the relative ordering between these measures. On one hand, Figure 1 highlights that in average $\overline{\chi}_{\text{L}}$ dominates $\overline{\chi}_{\text{R}}$, which is consistent with the observations made above vis-à-vis the dominance of left tails over right tails. On the other hand, it is also clear from the inspection of this figure that, on average, $\overline{\chi}_{\text{L}}$ and $\overline{\chi}_{\text{R}}$ lie above ρ. This complies with the aforementioned discussion regarding the supremacy of the dependence in the tails in comparison with the one which would arise from a Gaussian dependence model.

Do typical determinants of comovement hold in the tails?

We now assess if the determinants typically found as important in explaining international output synchronization are tenable when one focuses on tail dependence. Among the several variables deemed to influence output synchronization, the foremost candidates are trade variables. Although it has long been acknowledged that trade is an important linkage between economies, theory is ambiguous whether intensified trade relations result in more or in less output comovement. From one point of view, comparative advantage trade theories postulate that increasing trade leads to a higher degree of production specialization and consequently to a lower comovement (see, for example, Krugman 1992). From another point of view, according to a wide range of theoretical models of international trade, with either technology or monetary shocks, increasing trade often results in higher comovement. For instance, Frankel and Rose (1998) assert that closer trade links lead to higher output synchronization as an outcome. The underlying issue is whether bilateral trade is mainly intra-industry or inter-industry driven. In the former case one would expect higher comovement whereas in the latter lower comovement would be predicted. Hence, along with the role of bilateral trade, one should also take into account the relative trade specialization.

Another potential determinant often considered in the literature is the similarity of the production composition. The intuition here is that countries with similar economic structure should be in like manner affected by sector-specific shocks which may induce an higher output comovement (see, for example, Imbs 2004). The existence of other similarity mechanisms paralleling in the economies is also reckoned among the conceivable determinants of synchronization. For example, the implementation of coordinated policies may also have an effect in synchronization. If two countries adopt similar policies, either monetary or fiscal, an higher synchronization may be induced (see, for example, Inklaar *et al.* 2008).

As in theory, many factors may potentially underlie output synchronization, identifying the determinants of comovement becomes an empirical matter. Among the variables that have been pointed out in the literature as possible explanatory determinants of international output comovement (for a comprehensive overview see, for example, Inklaar *et al.* 2008, and references therein), we concern ourselves with the variables that have been found robust in related work.[5] Two influential papers in this respect are Baxter and Kouparitsas (2005)

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[5]Recent work makes use of extreme bounds analysis, suggested by Leamer (1983) and developed by Levine and Renelt (1992) and Sala-i-Martin (1997), to ascertain the 'robustness' of the determinants. Here the word 'robust' should be understood in Leamer's terminology, and hence it applies to variables whose statistical significance does not depend on the information set.

and Inklaar *et al.* (2008). On one hand, Baxter and Kouparitsas (2005) consider over one hundred countries and the variables under analysis are: bilateral trade between countries; total trade in each country; sectoral structure; similarity in export and import baskets; factor endowments; and gravity variables. On the other hand, Inklaar *et al.* (2008) considered an even larger assortment of potential variables for 21 OECD countries. The results of the latter suggest that besides bilateral trade between countries (as in Baxter and Kouparitsas 2005), variables capturing similarity of monetary and fiscal policies, as well as specialization measures are robust determinants of international output comovement.

As Inklaar *et al.* (2008) also consider the monthly IP as a measure of economic activity and the set of countries is closer to our case, we will draw heavily on their findings vis-à-vis the selection of the variables to be examined in the remaining analysis. Thus, we consider as possible determinants of output comovement the following variables: (i) bilateral trade between countries; (ii) three specialization indicators; (iii) a similarity measure of monetary policy stance; and (iv) a similarity measure of fiscal policy stance. Some specific comments, about the meaning and computation of each of these yardsticks, will be provided below. For the ease of exposition in the following we use some simplifying conventions regarding notation. The indices i and j are reserved to represent countries, whereas t is taken to denote time. Hence in cases where the respective meaning of these indices is clear from the context they may be omitted. In addition, capital letters are intended to represent 'totals' of the corresponding indices (for instance, T should be understood as the total number of time t periods).

Starting with the first variable mentioned above, here we use *bilateral trade intensity*, for the pair of countries (i, j), which is given by

$$\frac{1}{T} \sum_{t=1}^{T} \frac{x_{ijt} + m_{ijt} + x_{jit} + m_{jit}}{x_{it} + m_{it} + x_{jt} + m_{jt}}. \tag{13}$$

Here x_{ijt} and m_{ijt} respectively denote exports and imports from country i to country j, while x_{it} and m_{it} respectively represent total exports and imports of country i; this essentially corresponds to the preferred measure of Baxter and Kouparitsas (2005). All data regarding trade flows are taken from the CHELEM International Trade Database and covers the period from 1967 up to 2008.

As mentioned earlier three indicators of specialization measure are here calculated, viz.: industrial similarity; export similarity; and intra-industry trade. The *industrial similarity*, proposed by Imbs (2004), can be written as

$$\frac{1}{T} \sum_{t=1}^{T} \left(1 - \frac{1}{L} \sum_{l=1}^{L} |s_{ilt} - s_{jlt}| \right), \tag{14}$$

where s_{ilt} denotes the production share of industry l in country i. As in Inklaar *et al.* (2008), we resort to the 60-Industry Database of the Groningen Growth and Development Centre, which has data mainly at the 2-digit ISIC detail level and the sample period ranges from 1979 up to 2003. By its turn, *export similarity*, suggested by Baxter and Kouparitsas (2005), is computed as

$$\frac{1}{T} \sum_{t=1}^{T} \left(1 - \frac{1}{P} \sum_{p=1}^{P} |s_{ipt} - s_{jpt}| \right), \tag{15}$$

where s_{ipt} is product p's share of country i's total exports. Likewise Baxter and Kouparitsas (2005), export shares are obtained using trade data by commodity at the 2-digit ISIC detail level for all country pairs. Finally, the measure of *intra-industry trade* is given by

$$\frac{1}{T} \sum_{t=1}^{T} \left(1 - \frac{\sum_p |x_{ijpt} - m_{ijpt}|}{\sum_p (x_{ijpt} + m_{ijpt})} \right), \tag{16}$$

where x_{ijpt} and m_{ijpt} respectively denote the exports and imports of product p from country i to country j. Again, trade data by commodity at the 2-digit ISIC detail level is used.

Concerning the similarity measure of monetary policy stance, we follow Inklaar *et al.* (2008) and compute the correlation for all country pairs of the monthly short-term interest rates taken from the OECD Main Economic Indicators Database, using the available data up to December 2009. Regarding the measure of fiscal policy stance, we compute the correlation for all country pairs of the cyclically adjusted government primary balance, as a percentage of potential GDP, available at the OECD Economic Outlook Database, with the sample period ranging in most cases from 1970 up to 2009.

[Insert Table 4 about here]

In Table 4, we present the regression results using as dependent variable a measure of the degree of association (namely, the Pearson correlation coefficient, the left and right tail dependence, respectively measured by $\overline{\chi}_{\mathrm{L}}$ and $\overline{\chi}_{\mathrm{R}}$) and as covariates the above described factors, to wit: bilateral trade intensity; a specialization measure; and two policy stance similarity indicators. For the Pearson correlation coefficient, the results are broadly similar to those obtained by Inklaar *et al.* (2008). We also find evidence supporting the importance of bilateral trade intensity, specialization measure and monetary policy stance similarity for explaining comovement. In contrast, the fiscal policy stance indicator is not statistically significant in our case. Besides the fact that both the set of countries and the sample period are not the same, we use the cyclically adjusted government primary balance whereas Inklaar *et al.* (2008) use the cyclically adjusted government total balance. As it is widely acknowledged, the government primary balance is a more adequate measure of the current fiscal policy stance since it is not affected by interest rate payments on the government debt which reflects an accumulated governmental deficit over previous years.

The question that now arises is the following. Are the standard determinants of synchronization tenable over periods of exceptional positive and negative growth? An answer to this question is given by examining in Table 4 the regression outputs for the cases wherein $\overline{\chi}_{\mathrm{L}}$ and $\overline{\chi}_{\mathrm{R}}$ are taken as dependent variables. From this exercise, a major conclusion can be readily gathered. With the exception of the specialization measure, all the above determinants are not statistically significant. This means that for the comovement in extreme events what really seems to matter is the specialization similarity between economies. On the face of it, the vehicle of propagation of shocks over scenarios of sharp variations in output appears to be the specialization similarity across economies. Among the specialization indicators considered, the evidence for the export similarity measure, proposed by Baxter and Kouparitsas (2005), is the strongest as it is statistically significant in the regression for both tails. By its turn, the industrial similarity measure, as suggested by Imbs (2004), is clearly important for explaining left-tail dependence whereas the intra-industry trade, used by Inklaar *et al.* (2008), seems to be more relevant for right-tail dependence.

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IV.   Discussion

This paper examines the synchronization of several OECD countries during periods of abrupt declines and sudden increases in international economic activity, over the last 50 years. From the conducted analysis some noteworthy empirical findings are here collected: synchronization is more intense during periods of sharp declines than during scenarios of large positive growth; our results pinpoint statistical evidence in favor of asymptotic independence, with a stronger tail dependence than the one suggested by a Gaussian dependence model. Thus, in particular, this implies that Pearson correlation considerably underestimates the level of synchronization in periods large negative and positive growth. Lastly, our results put forward that, among the standard determinants used for explaining international output growth synchronization, only specialization similarity seems to play a role during extreme events. As mentioned by a reviewer, the application of techniques of statistics of extremes to business cycle data is still in its infancy, and one needs to be aware of its limits—especially in terms of small sample biases. A reliable application of the techniques requires as much data as one can get, so that it may be important to understand how the extremes of macroeconomic variables available on a higher frequency, connect with the cycle. Our analysis merely provides a first step towards understanding the mechanisms of propagation of shocks during periods of large negative and positive growth. An open question that also remains is whether other key variables not considered as typical determinants for explaining international output growth synchronization, can actually have an effect during such extreme periods.

Appendix: Iterative Subsample Bootstrap

To select the optimal $k = k^\star$, we use the Daníelsson–de Vries iterative subsample bootstrap procedure (Daníelsson and de Vries, 1997). This procedure is based on a recursive application of the following stages. In a *first step* a Hall subsample bootstrap (Hall, 1990) is employed to subsamples of size n_1 to yield a starting value for $k^\star$ (*say* $k_1^\star$). In a *second step*, the Hill estimator (9) is routinely applied to the subsamples using the starting value $k_1^\star$ to consistently estimate a first order parameter α. Lastly, we estimate a second order parameter β using the estimator proposed in Daníelsson and de Vries (1997, p. 247). The optimal value $k^\star$ is then given by properly combining k_1 with the first and second order parameters, viz.: $k^\star = k_1^\star (n/n_1)^{2\beta/(2\beta+\alpha)}$.

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TABLE 1

Pearson correlation of the output growth rates for OECD countries

	AUS	BEL	CN	FIN	FR	GER	IT	JP	NL	NOR	POR	SP	SWE	UK	US
						Pearson correlation ρ									
AUS		0.3073	0.0820	0.0743	0.0515	0.1449	0.2760	-0.0489	0.1767	0.0181	-0.0325	-0.0018	0.0467	0.1881	0.0206
BEL	0.3073		0.0133	0.0414	0.0463	0.1603	0.2736	-0.0425	0.0382	0.0190	-0.0291	0.0400	0.0788	0.2181	0.0470
CN	0.0820	0.0133		0.1664	0.0939	0.0848	0.0737	0.1749	0.0693	0.0690	-0.0030	0.1162	0.0506	0.1569	0.3715
FIN	0.0743	0.0414	0.1664		0.0726	0.1285	0.0828	0.0673	0.0488	-0.0362	0.1048	0.0195	0.2594	0.0368	0.0704
FR	0.0515	0.0463	0.0939	0.0726		0.1093	0.0536	0.1110	0.0898	-0.0375	0.0178	0.0823	0.1213	0.0215	0.0443
GER	0.1449	0.1603	0.0848	0.1285	0.1093		0.0684	0.2262	0.1136	-0.0157	0.0969	0.0922	0.0350	0.1527	0.1430
IT	0.2760	0.2736	0.0737	0.0828	0.0536	0.0684		0.0511	0.1678	0.0829	0.0576	0.1196	0.1088	0.1715	0.1180
JP	-0.0489	-0.0425	0.1749	0.0673	0.1110	0.2262	0.0511		0.0237	0.0073	0.1232	0.1434	0.0401	0.0831	0.2068
NL	0.1767	0.0382	0.0693	0.0488	0.0898	0.1136	0.1678	0.0237		-0.1022	0.0541	0.1139	0.0376	0.1817	0.0545
NOR	0.0181	0.0190	0.0690	-0.0362	-0.0375	-0.0157	0.0829	0.0073	-0.1022		0.0288	0.0352	0.0266	-0.0423	0.0270
POR	-0.0325	-0.0291	-0.0030	0.1048	0.0178	0.0969	0.0576	0.1232	0.0541	0.0288		0.1793	-0.0832	0.0426	-0.0391
SP	-0.0018	0.0400	0.1162	0.0195	0.0823	0.0922	0.1196	0.1434	0.1139	0.0352	0.1793		0.1336	0.0208	0.0776
SWE	0.0467	0.0788	0.0506	0.2594	0.1213	0.0350	0.1088	0.0401	0.0376	0.0266	-0.0832	0.1336		0.1265	0.0936
UK	0.1881	0.2181	0.1569	0.0368	0.0215	0.1527	0.1715	0.0831	0.1817	-0.0423	0.0426	0.0208	0.1265		0.1493
US	0.0206	0.0470	0.3715	0.0704	0.0443	0.1430	0.1180	0.2068	0.0545	0.0270	-0.0391	0.0776	0.0936	0.1493	

Notes: AUS = Austria ; BEL = Belgium ; CN = Canada ; DK = Denmark ; FIN = Finland ; FR = France ; GER = Germany ; IT = Italy ; JP = Japan ;

NL = Netherlands ; NOR = Norway ; POR = Portugal ; SP = Spain ; SWE = Sweden ; UK = United Kingdom ; US = United States of America.

TABLE 2

Left-tail dependence of the output growth rates for OECD countries

$\overline{\chi}_{L}$ (*Left-tail dependence as measured by* $\overline{\chi}$)

	AUS	BEL	CN	FIN	FR	GER	IT	JP	NL	NOR	POR	SP	SWE	UK	US
AUS		0.3306	0.2233	0.2082	0.3355	0.3089	0.2815	0.4649	0.1783	0.2174	-0.0602	0.1045	0.2144	0.4512	0.1616
BEL	0.3306		0.1193	-0.0221	0.2472	0.2042	0.4695	0.2222	0.1455	0.2685	-0.0286	0.4878	0.2374	0.6945	0.0304
CN	0.2233	0.1193		0.6593	0.5195	0.3835	0.3426	0.7964	0.4253	0.1821	0.1495	0.8673	0.1820	0.5378	0.7113
FIN	0.2082	-0.0221	0.6593		0.4634	0.7133	0.3876	0.5085	0.1248	0.2421	0.3009	0.2620	0.4097	0.2709	0.3070
FR	0.3355	0.2472	0.5195	0.4634		0.3134	0.2743	0.4468	0.2970	0.1311	0.1985	0.3537	0.4243	0.4265	0.1549
GER	0.3089	0.2042	0.3835	0.7133	0.3134		0.4390	0.6803	0.2650	-0.0718	-0.0177	0.0292	0.5709	0.5559	0.5489
IT	0.2815	0.4695	0.3426	0.3876	0.2743	0.4390		0.2807	0.1579	0.0760	0.4741	0.2466	0.6350	0.6165	0.3513
JP	0.4649	0.2222	0.7964	0.5085	0.4468	0.6803	0.2807		-0.0920	0.3153	0.0926	0.1648	0.3136	0.4895	0.3582
NL	0.1783	0.1455	0.4253	0.1248	0.2970	0.2650	0.1579	-0.0920		-0.1104	0.2051	0.2198	0.2431	0.2890	0.3893
NOR	0.2174	0.2685	0.1821	0.2421	0.1311	-0.0718	0.0760	0.3153	-0.1104		0.2435	0.5762	0.3724	0.0752	0.2126
POR	-0.0602	-0.0286	0.1495	0.3009	0.1985	-0.0177	0.4741	0.0926	0.2051	0.2435		0.1882	0.0904	0.1701	0.2392
SP	0.1045	0.4878	0.8673	0.2620	0.3537	0.0292	0.2466	0.1648	0.2198	0.5762	0.1882		0.1485	0.3036	0.6660
SWE	0.2144	0.2374	0.1820	0.4097	0.4243	0.5709	0.6350	0.3136	0.2431	0.3724	0.0904	0.1485		0.4895	0.5396
UK	0.4512	0.6945	0.5378	0.2709	0.4265	0.5559	0.6165	0.4895	0.2890	0.0752	0.1701	0.3036	0.4895		0.5002
US	0.1616	0.0304	0.7113	0.3070	0.1549	0.5489	0.3513	0.3582	0.3893	0.2126	0.2392	0.6660	0.5396	0.5002	

Notes: AUS = Austria ; BEL = Belgium ; CN = Canada ; DK = Denmark ; FIN = Finland ; FR = France ; GER = Germany ; IT = Italy ; JP = Japan
NL = Netherlands ; NOR = Norway ; POR = Portugal ; SP = Spain ; SWE = Sweden ; UK = United Kingdom ; US = United States of America.

TABLE 3

Right-tail dependence of the output growth rates for OECD countries

$\overline{\chi}_R$ (*Right-tail dependence as measured by* $\overline{\chi}$)

	AUS	BEL	CN	FIN	FR	GER	IT	JP	NL	NOR	POR	SP	SWE	UK	US
AUS		0.6020	-0.1907	0.1072	0.2875	0.0959	0.4584	-0.0053	0.1407	0.2027	0.1304	-0.0555	-0.0856	0.3831	0.0998
BEL	0.6020		-0.1243	0.1239	0.1812	0.4930	0.2857	-0.0490	0.1587	-0.0184	0.0108	0.2388	0.1102	0.3831	-0.1974
CN	-0.1907	-0.1243		0.0735	-0.0836	0.1854	0.3215	0.1592	-0.2801	0.3181	0.0693	0.0359	0.0835	0.1513	0.0721
FIN	0.1072	0.1239	0.0735		0.1750	0.2358	0.0839	0.0895	0.1574	0.1701	0.0198	-0.1508	0.5396	0.1720	0.1569
FR	0.2875	0.1812	-0.0836	0.1750		0.1403	0.1693	0.3287	0.3095	0.2648	0.0644	0.2075	0.3047	0.1609	0.3225
GER	0.0959	0.4930	0.1854	0.2358	0.1403		0.1651	0.2693	0.1188	0.0198	0.2646	0.4709	0.1236	0.2091	0.0873
IT	0.4584	0.2857	0.3215	0.0839	0.1693	0.1651		0.0043	0.0708	0.0794	0.4226	0.0937	0.1480	0.5779	0.4077
JP	-0.0053	-0.0490	0.1592	0.0895	0.3287	0.2693	0.0043		-0.0782	0.1348	0.4659	0.0242	0.0529	0.2261	0.2324
NL	0.1407	0.1587	-0.2801	0.1574	0.3095	0.1188	0.0708	-0.0782		0.1875	0.0252	0.1197	-0.0721	0.3355	0.1409
NOR	0.2027	-0.0184	0.3181	0.1701	0.2648	0.0198	0.0794	0.1348	0.1875		0.0292	0.3102	0.2181	0.1926	0.0765
POR	0.1304	0.0108	0.0693	0.0198	0.0644	0.2646	0.4226	0.4659	0.0252	0.0292		0.0945	-0.1106	0.1111	0.1880
SP	-0.0555	0.2388	0.0359	-0.1508	0.2075	0.4709	0.0937	0.0242	0.1197	0.3102	0.0945		0.2879	0.2222	0.0695
SWE	-0.0856	0.1102	0.0835	0.5396	0.3047	0.1236	0.1480	0.0529	-0.0721	0.2181	-0.1106	0.2879		0.0968	-0.0010
UK	0.3831	0.3831	0.1513	0.1720	0.1609	0.2091	0.5779	0.2261	0.3355	0.1926	0.1111	0.2222	0.0968		0.2691
US	0.0998	-0.1974	0.0721	0.1569	0.3225	0.0873	0.4077	0.2324	0.1409	0.0765	0.1880	0.0695	-0.0010	0.2691	

Notes: AUS = Austria ; BEL = Belgium ; CN = Canada ; DK = Denmark ; FIN = Finland ; FR = France ; GER = Germany ; IT = Italy ; JP = Japan
NL = Netherlands ; NOR = Norway ; POR = Portugal ; SP = Spain ; SWE = Sweden ; UK = United Kingdom ; US = United States of America.

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TABLE 4

Comovement Determinants over Pearson Correlation and Extremal Dependence Measures

	Specialization Measure					
	Industrial similarity		*Export similarity*		*Intra industry trade*	
Pearson Correlation						
	Coefficient	*t*-HCSE	Coefficient	*t*-HCSE	Coefficient	*t*-HCSE
Bilateral trade	0.571	2.89	0.533	2.20	0.526	2.07
Specialization measure	0.146	3.50	0.087	2.79	0.102	1.58
Short-term interest rate	0.091	2.06	0.102	2.31	0.107	2.34
Cyclically adjusted government primary balance	0.008	0.34	0.002	0.09	-0.005	-0.19
Left-tail Dependence						
	Coefficient	*t*-HCSE	Coefficient	*t*-HCSE	Coefficient	*t*-HCSE
Bilateral trade	0.528	1.25	0.385	0.71	0.578	1.22
Specialization measure	0.344	3.12	0.233	3.02	0.149	0.93
Short-term interest rate	-0.074	-0.57	-0.052	-0.41	-0.028	-0.21
Cyclically adjusted government primary balance	0.030	0.41	0.013	0.18	0.011	0.14
Right-tail Dependence						
	Coefficient	*t*-HCSE	Coefficient	*t*-HCSE	Coefficient	*t*-HCSE
Bilateral trade	0.439	1.02	0.293	0.77	0.059	0.14
Specialization measure	0.111	1.33	0.127	2.08	0.279	1.75
Short-term interest rate	-0.009	-0.08	-0.011	-0.10	-0.016	-0.15
Cyclically adjusted government primary balance	0.068	1.41	0.059	1.20	0.031	0.55

Notes: constant is included ; *t*-HCSE (Heteroscedasticity Consistent Standard Errors).
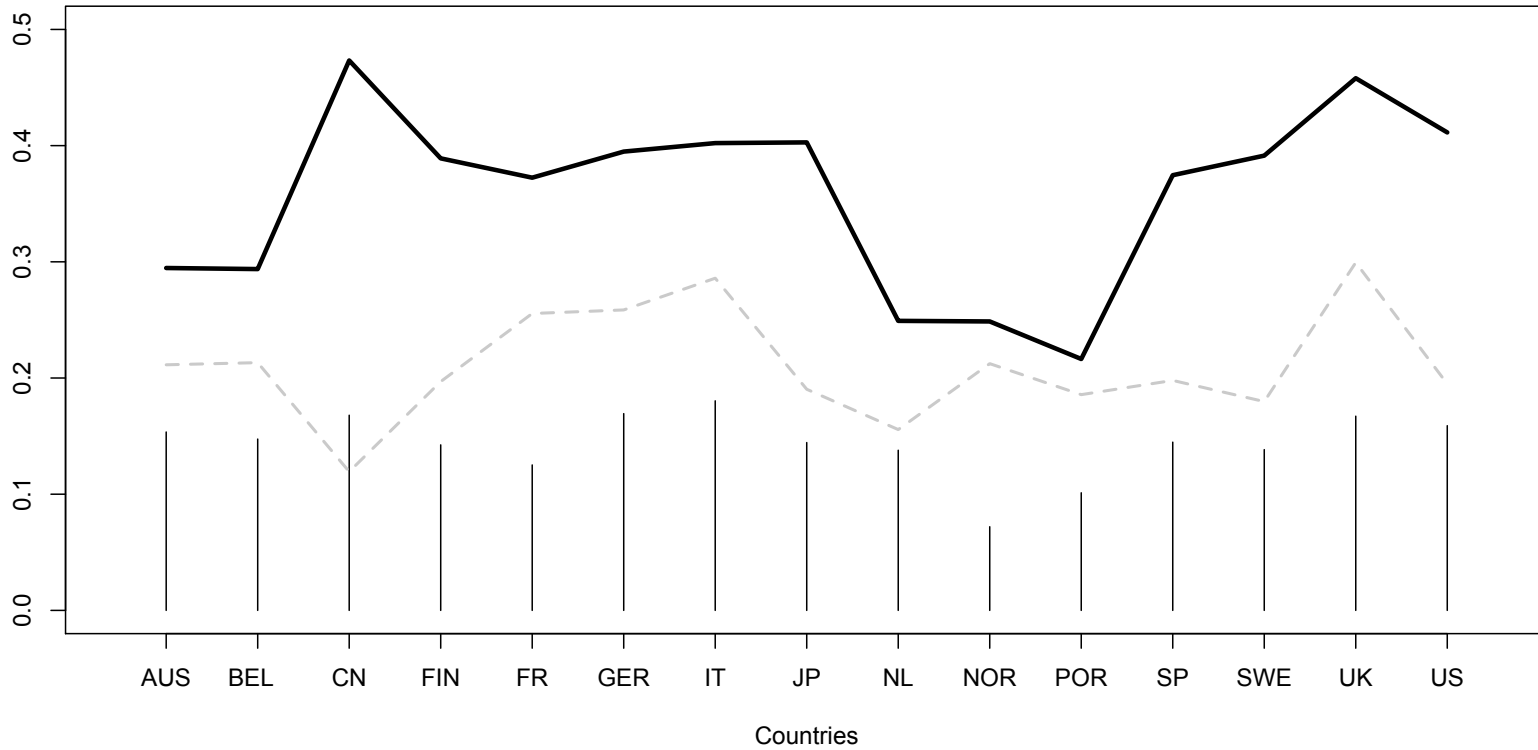
Figure 1. Average values per country for each of the dependence measures considered. The vertical bars correspond to Pearson correlation ρ, while the solid and dashed lines respectively correspond to the left-tail and right-tail dependence as measured by $\overline{\chi}_{\mathrm{L}}$ and $\overline{\chi}_{\mathrm{R}}$