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Jordan Richards, Myung Won Lee, Viviana Carcaiso, and Miguel de Carvalho's contribution to the Discussion of 'Inference for extreme spatial temperature events in a changing climate with application to Ireland' by Healy et al.

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We congratulate the authors for a thought-provoking case study, and for their novel treatment of nonstationary spatial extreme value analysis. In what follows, we focus on aspects of the analysis related to missing data. In Section 4.3, the authors note that:

> *'The issue of missing data in spatial extremes applications seems to be rarely discussed'.*

In fact, the term 'missing data', and any discussion of its treatment, is almost *completely missing* in leading monographs on extreme value theory (e.g. Beirlant et al., 2004; Coles, 2001; de Haan & Ferreira, 2006; Resnick, 2007); this omission is not an issue with the monographs themselves, but rather a consequence of the level of attention the issue has received within the field. To complicate matters even further, many analyses of extremes disregard the missing values; this poor practice is called, in statistical jargon, a 'complete case analysis'—an approach that produces valid, yet ineffective, results under only specific conditions [Missing Completely at Random (MCAR); Little & Rubin, 2002]. Ignoring the uncertainty surrounding the missing values is well-known to impact the inferences leading, for example, to larger confidence intervals.

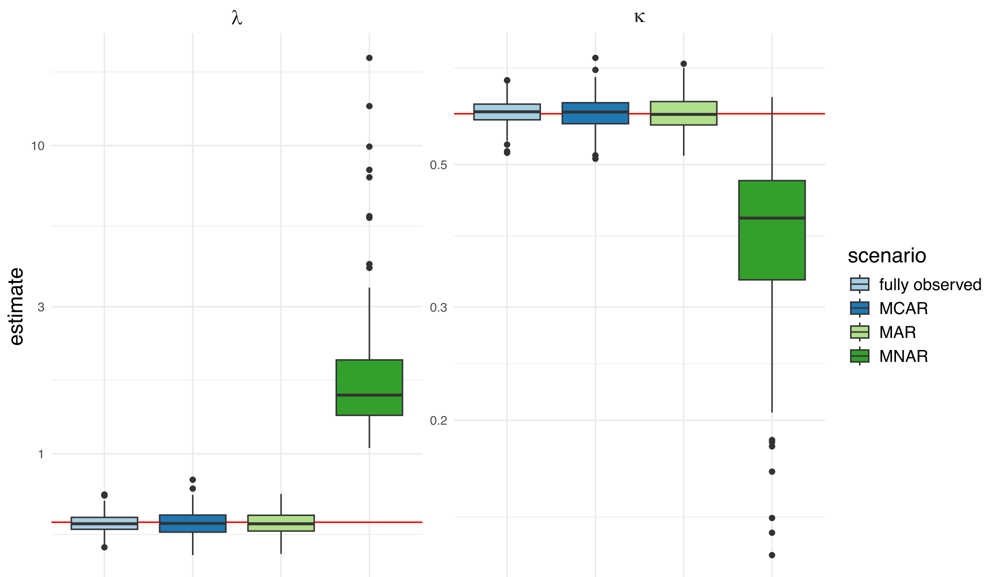

Figure 1. Boxplots of estimated (λ, κ) when data are (from left to right) (i) fully observed, (ii) Missing Completely at Random, (iii) Missing at Random, and (iv) Missing Not at Random. The true values, $(\lambda, \kappa) = (0.6, 0.6)$, are denoted by the horizontal lines.

To illustrate the strengths and weaknesses of the author's approach to handling missing data, we consider likelihood inference for the Husler–Reiss r-Pareto process; we use the risk functional in Eq. (20) with variogram $\gamma(h) = (h/\lambda)^\kappa$, for $\lambda > 0$ and $\kappa \in (0, 2)$. Four scenarios are considered: (i) one with fully observed data, and three where $\approx 50\%$ of observations are missing. In the latter three scenarios, we consider (ii) MCAR, with values missing uniformly-at-random; (iii) Missing at Random (MAR), with observations at sites with smaller inter-site distances more likely to be missing; and (iv) Missing Not at Random (MNAR), where higher values are more likely to be missing. We use the same approach as this article to estimate (λ, κ) for 200 datasets, which each contain 1,000 independent replicates of the standardized process observed at 25 sites (in $[0, 1] \times [0, 1]$); Figure 1 displays the results from this experiment. The inference framework presented by this article provides rigorous results for the case of MCAR and MAR, with a small increase in uncertainty relative to the case of fully observed data. However, Figure 1 shows significant bias in parameter estimates under MNAR, suggesting that any inferences conducted in this data setting should be highly scrutinized.

Despite the rigorous modelling of missing values in an r-Pareto process framework by the authors, we add that other standard missing data approaches—such as the Expectation–Maximization algorithm—could be viable alternatives for some applications, particularly in cases where the MAR assumption is tenable. Although computationally intensive, these approaches could in principle be adapted for the censored likelihood of the r-Pareto process. Moreover, exploring their viability for score matching, as discussed by de Fondeville and Davison (2018), present an intriguing possibility.

Finally, we wonder about the authors' perspective on integrating calibration methods into their framework. As Turkman et al. (2021) suggest, calibrating artificial data may be beneficial for some applications, especially if their tails are lighter. This calibration can be achieved using the transformation:

$$x^*(t, \mathrm{s}) = F^{-1}_{X_{\mathrm{o}}(t,\mathrm{s})}[F_{X_{\mathrm{c}}(t,\mathrm{s})}\{x(t, \mathrm{s})\}]. \tag{1}$$

Incidentally, Eq. (1) can be shown to be an optimal transport between the distribution functions of observed and climate model temperatures (Santambrogio, 2015, Theorem 2.2).

Conflicts of interest: None declared.

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Data availability

Data are available upon request from the corresponding author.

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