Mistakes

Tom Leinster

Department of Pure Mathematics, University of Cambridge
Email: leinster@dpmms.cam.ac.uk
Web: http://www.dpmms.cam.ac.uk/~leinster

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This is a compilation of known errors in the Part III Category Theory lecture notes and problem sheets. Please let me know about any more that you find. Each error is marked 'announced' or 'unannounced', according to whether a correction was issued in lectures. They are listed in order of discovery.

Problem sheet 2, q. 5(iii), unannounced It's quite possible to prove that the chain of adjunctions $C \dashv D \dashv O \dashv I$ extends no further without knowing anything about limits. However, C and I do preserve terminal and initial objects, so the hint isn't any help here; the most efficient way to do this part is to show that C and I fail to preserve certain other (co)limits.

Cor D3.6 (limits commute), announced The statement, remarks and proof should be deleted and moved to D4.7.

In its earlier location, none of D3.6 makes sense: we need to know that $[\mathbb{I}, \mathcal{C}]$ and $[\mathbb{J}, \mathcal{C}]$ possess limits of shape $\mathbb{J}$ and $\mathbb{I}$ respectively, which only follows from D4.5. Moreover, in the remark before the proof I gave the example that $(A_1+B_1)+(A_2+B_2) \cong (A_1+A_2)+(B_1+B_2)$ in a category $\mathcal{C}$ with coproducts, and whilst this is easily seen to be true by a direct argument, it only follows from the Corollary once one knows that coproducts in a functor category $[\mathbb{K}, \mathcal{C}]$ are computed pointwise (D4.5 again).

Lemma D4.1 (maps between diagrams), announced When I wrote this up at the end of the lecture on Wednesday 15 November, I omitted the condition in part (b) that (f, α) forms a map of cones. The full statement of (b) should be:

> Let $(A \xrightarrow{q_I} D(I))_{I \in \mathbb{I}}$ and $(A' \xrightarrow{q'_I} D'(I))_{I \in \mathbb{I}}$ be cones in $\mathcal{C}$, and let $f : A \longrightarrow A'$ be a map such that

$$\begin{tikzcd} A \arrow[r, "q_I"] \arrow[d, "f"'] & D(I) \arrow[d, "\alpha_I"] \\ A' \arrow[r, "q'_I"'] & D'(I) \end{tikzcd}$$

commutes for all $I \in \mathbb{I}$. Then

$$\begin{tikzcd} A \arrow[r, "\overline{q}"] \arrow[d, "f"'] & \varprojlim_{\mathbb{I}} D \arrow[d, "\varprojlim_{\mathbb{I}} \alpha"] \\ A' \arrow[r, "\overline{q'}"'] & \varprojlim_{\mathbb{I}} D' \end{tikzcd}$$

also commutes.

Cor D4.3 ($\varprojlim_{\mathbb{I}}$ forms a functor), unannounced I stated that if $\mathcal{C}$ is a category with all limits of shape $\mathbb{I}$, then there is a *canonical* functor

$$\varprojlim_{\mathbb{I}} : [\mathbb{I}, \mathcal{C}] \longrightarrow \mathcal{C}$$

(and this is right adjoint to the diagonal). The word 'canonical' is certainly misleading here. What I should have said is that once one has chosen a specific limit cone for each diagram of shape $\mathbb{I}$ in $\mathcal{C}$, *then* $\varprojlim_{\mathbb{I}}$ becomes a functor in a canonical way. To say what $\varprojlim_{\mathbb{I}}$ does to objects of $[\mathbb{I}, \mathcal{C}]$, one has to make an arbitrary choice of one thing out of an isomorphism class (since limits are only unique up to iso); once this is done, Lemma D4.1 determines exactly what $\varprojlim_{\mathbb{I}}$ does to morphisms of $[\mathbb{I}, \mathcal{C}]$.

Problem sheet 3, q. 5(i), announced by email In the adjunction of the second sentence, $\mathcal{C}$ and **Set** should be swapped: so the correct version is $\mathbf{Set} \underset{U}{\overset{F}{\rightleftarrows}} \mathcal{C}$ (with $\perp$). The rest of (i) then makes sense. This error is already corrected in the downloadable version, but the copies I distributed have the error in them.

Example D3.2(b) (van Kampen's Theorem), unannounced This should read: let $\Pi_1 : \mathbf{Top} \longrightarrow \mathbf{Cat}$ be the fundamental groupoid functor, assigning to each space S the fundamental groupoid of S (the category whose objects are the points of S, and whose arrows are homotopy classes of paths in S). Fix a space A. Then the composite functor

$$\mathcal{O}(A) \hookrightarrow \mathbf{Top} \longrightarrow \mathbf{Cat}$$

preserves finite colimits. This is justified as in lectures: that it preserves initial objects is trivial, and that it preserves pushouts is a version of van Kampen's Theorem; then use Sheet 3, q. 8(iii), which says that finite colimits can be built from an initial object and pushouts. (Compare also Example 10 on the 'Informal Introduction' handout.)

(The problem with the version of this example given in lectures is that pushouts in $\mathbf{Top_{emb}}$ don't work in the immediately obvious way: e.g. if U and V are open subsets of a space A then the diagram of inclusions

$$\begin{tikzcd} U \cap V \arrow[r, hook] \arrow[d, hook] & U \arrow[d, hook] \\ V \arrow[r, hook] & U \cup V \end{tikzcd}$$

is not usually a pushout in $\mathbf{Top_{emb}}$, although it is in **Top** and in $\mathcal{O}(A)$.)

Theorem E1.6 (General Adjoint Functor Theorem), unannounced Not a mistake, but an expansion of the proof in case it seems mysterious.

By local smallness of $\mathcal{D}$ and Lemma E1.5, $(A \Rightarrow G)$ is locally small for each A. By completeness of $\mathcal{D}$ and Lemma E1.4, $(A \Rightarrow G)$ is complete for each A. By the SSC (which says exactly that $(A \Rightarrow G)$ has a weakly initial set) and Lemma E1.3, $(A \Rightarrow G)$ has an initial object for each A. By B3.5, G has a left adjoint.

Example E2.4 (pullbacks of monics in Set), unannounced The second arrow should be $M' \overset{m'}{\rightarrowtail} B$. This is on the handout for E2 (Special Adjoint Functor Theorem), and is corrected in the downloadable version.

Example E2.2(b) (subobjects), unannounced Delete the phrase 'subobjects in **Top** are subspaces'. This is on the handout for E2 (Special Adjoint Functor Theorem), and is corrected in the downloadable version.

In fact, a subobject of a topological space is a subset equipped with a toplogy at least as large as the subspace topology (so that the inclusion function is continuous).

Yoneda Lemma explanatory handout, unannounced On page 4, in the section entitled 'Monoid actions', $H^A : M \longrightarrow \mathbf{Set}$ should be changed to $H_A : M^{\mathrm{op}} \longrightarrow \mathbf{Set}$. This is now corrected in the downloadable version.

Example F4.10(a) (Gp monadic over Set), unannounced In the last line, e should be d. This is on the handout for F4 (the Monadicity Theorem), and is now corrected in the downloadable version.