# Solutions to exercises from Lecture 4

**4.13** Take an isomorphism $\alpha : H_A \xrightarrow{\sim} H_B$. Put

$$\begin{aligned} f &= \alpha_A(1_A) \in H_B(A) = \mathcal{A}(A, B), \\ g &= \alpha_B^{-1}(1_B) \in H_A(B) = \mathcal{A}(B, A). \end{aligned}$$

By naturality of $\alpha$, the square

$$\begin{array}{ccc} H_A(A) & \xrightarrow{\alpha_A} & H_B(A) \\ {\scriptstyle H_A(g)}\downarrow & & \downarrow{\scriptstyle H_B(g)} \\ H_A(B) & \xrightarrow[\alpha_B]{} & H_B(B) \end{array}$$

commutes. This square is

$$\begin{array}{ccc} \mathcal{A}(A, A) & \xrightarrow{\alpha_A} & \mathcal{A}(A, B) \\ {\scriptstyle g^*}\downarrow & & \downarrow{\scriptstyle g^*} \\ \mathcal{A}(B, A) & \xrightarrow[\alpha_B]{} & \mathcal{A}(B, B), \end{array}$$

and we have

$$\begin{array}{ccc} 1_A & \longmapsto & f \\ \big\downarrow & & \big\downarrow \\ & & f \circ g \\ \big\downarrow & & \\ g & \longmapsto & 1_B, \end{array}$$

so $f \circ g = 1_B$. Dually, $g \circ f = 1_A$. Hence $A \cong B$.

**4.14** Remain calm.