

Galois Theory Assignment 4

The fundamental theorem of Galois theory

The deadline for submitting this work is **12 noon on Tuesday 16 March**, through Gradescope. To avoid technical problems, please start uploading by 11.55am. Please report any mistakes to Tom.Leinster@ed.ac.uk.

Take care over communication and presentation. Your answers should be coherent, logical arguments written in full sentences. Marks will be awarded for this.

1. Work through the details of the Galois correspondence for the polynomial $t^3 - 2$ over \mathbb{Q} .

By ‘work through the details’, I mean that you should do all the things I did for $t^4 - 2$ in Section 8.3 of the notes. Your answer should include the following points (not necessarily in this order):

- a description of the splitting field;
- the isomorphism type of the Galois group;
- a diagram showing the subgroups of the Galois group and the inclusions between them;
- a diagram showing the intermediate fields of the splitting field extension and the inclusions between them;
- a statement of the orders of the subgroups and the degrees of the extensions;
- a description of which subgroups correspond to which intermediate fields;
- a statement of which subgroups of the Galois group are normal, and which intermediate fields are normal extensions of \mathbb{Q} ;
- for the normal subgroups, an analysis of the group isomorphism (involving a quotient) provided by the last part of the fundamental theorem of Galois theory.

Some aspects are already covered in Example 7.1.15, but you should include them anyway, making sure you understand them and expressing them in your own words.