Galois Theory Assignment 1

Overview of Galois theory; group actions, rings and fields

Work is to be submitted physically, on paper, by 12:10 on Monday 31 January. I will collect it at the lecture. If you are unable to attend, you can put your work under my office door (JCMB 5317). Please do not email it.

Please report any mistakes on this sheet to Tom.Leinster@ed.ac.uk.

Take care over communication and presentation. Your answers should be coherent, logical arguments written in full sentences. Marks will be awarded for this.

1. Let G be a group acting on a set X. Let $S \subseteq G$, and write $\langle S \rangle$ for the subgroup of G generated by S. Prove that $\mathrm{Fix}(S) = \mathrm{Fix}(\langle S \rangle)$.

2. Let K be a field such that for $\alpha, \beta \in K$,

$$\alpha \text{ is a square root of } \beta \iff \beta \text{ is a square root of } \alpha.$$

   How many elements does K have? Justify your answer fully.