Galois Theory Assignment 3
Degree and splitting fields

Work is to be submitted on paper by 12:10 on Monday 28 March. I will collect it at the lecture. If you are unable to attend, you can put your work under my door (JCMB 5317). Please do not email it.

Please report any mistakes on this sheet to Tom.Leinster@ed.ac.uk.

Take care over communication and presentation. Your answers should be coherent, logical arguments written in full sentences. Marks will be awarded for this.

1. Let $M : K$ be a finite extension, let $\alpha \in M$, and let m be the minimal polynomial of α over K. Show that $\deg(m)$ divides $[M : K]$.

2. (i) Let K be a field and let f and g be nonzero polynomials over K. Put $L = \mathrm{SF}_K(g)$. Show that $\mathrm{SF}_L(f)$ and $\mathrm{SF}_K(fg)$ are isomorphic over K.

   (ii) Let f and g be nonzero polynomials over $\mathbb{Q}$. Prove that $\mathrm{SF}_\mathbb{Q}(fg)$ is the compositum of $\mathrm{SF}_\mathbb{Q}(f)$ and $\mathrm{SF}_\mathbb{Q}(g)$, where all three splitting fields are viewed as subfields of $\mathbb{C}$.