## Galois Theory Assignment 1

Work is to be submitted on Gradescope by 12:00 on Thursday 2 February. This is a hard deadline.

Please report any mistakes on this sheet to Tom.Leinster@ed.ac.uk.

Take care over communication and presentation. Your answers should be coherent, logical arguments written in full sentences. Marks will be awarded for this.

- 1. This question leads you through Exercise 1.2.2 (a harder exercise than almost all the others).
  - (i) Let  $k \ge 0$ . For  $\sigma \in S_k$  and  $p \in \mathbb{Q}[t_1, \ldots, t_k]$ , define  $\sigma p \in \mathbb{Q}[t_1, \ldots, t_k]$  by

 $(\sigma p)(t_1,\ldots,t_k) = p(t_{\sigma(1)},\ldots,t_{\sigma(k)}).$ 

Prove that this defines a left action of  $S_k$  on  $\mathbb{Q}[t_1, \ldots, t_k]$ .

(ii) Let f be a polynomial over  $\mathbb{Q}$  with k distinct roots in  $\mathbb{C}$ . Show that  $\operatorname{Gal}(f)$ , as defined in Definition 1.2.1 of the notes, is a subgroup of  $S_k$ . (For this question, do not use any other definition of Galois group.)