Galois Theory Assignment 2

Work is to be submitted on Gradescope by 12:00 on Thursday 9 March. This is a hard deadline.

Please report any mistakes on this sheet to Tom.Leinster@ed.ac.uk.

Take care over communication and presentation. Your answers should be coherent, logical arguments written in full sentences. Marks will be awarded for this.

1. Prove that $\cos(\pi/9)$ is algebraic over $\mathbb{Q}$, and find its minimal polynomial. (Hint: begin by finding a general formula for $\cos 3\theta$.)

2. Show that for every $n \geq 1$, there exists an extension of $\mathbb{Q}$ of degree n.

   (If you rely on the result of any exercise in the notes, you should prove it here.)