Chapter 1: Overview of Galois theory

(1) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers $3 + 4i$ and $3 - 4i$ are conjugate over $\mathbb{R}$.

a. True
b. False

(2) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers 1, $2i$ and $1 + 2i$ are conjugate over $\mathbb{R}$.

a. False
b. True

(3) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers $7 + \sqrt{7}$ and $7 - \sqrt{7}$ are conjugate over $\mathbb{Q}$.

a. True
b. False

(4) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers $7 + \sqrt{7}$ and $-7 - \sqrt{7}$ are conjugate over $\mathbb{Q}$.

a. False
b. True

(5) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The 5th roots of unity are all conjugate over $\mathbb{Q}$.

a. True
b. False

(6) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The 9th roots of unity, excluding 1, are all conjugate over $\mathbb{Q}$.

a. True
b. False

(7) **Conjugacy** [Multiple choice] [One answer only]

True or false? The 19th roots of unity, excluding 1, are all conjugate over $\mathbb{Q}$.

a. True
b. False

(8) **Conjugacy** [Multiple choice] [One answer only]

True or false? The triples $(1, 2i, 1+2i)$ and $(1, -2i, 1-2i)$ are conjugate over $\mathbb{Q}$.

a. True
b. False

(9) **Conjugacy** [Multiple choice] [One answer only]

True or false? The triples $(1, 2i, 1+2i)$ and $(2i, 1+2i, 1)$ are conjugate over $\mathbb{Q}$.

a. True
b. False

(10) **Conjugacy** [Multiple choice] [One answer only]

True or false? Let $z, z' \in \mathbb{C}$. If z and z' are conjugate over $\mathbb{R}$ then they are conjugate over $\mathbb{Q}$.

a. True
b. False

(11) **Conjugacy** [Multiple choice] [One answer only]

True or false? Let $z, z' \in \mathbb{C}$. If z and z' are conjugate over $\mathbb{Q}$ then they are conjugate over $\mathbb{R}$.

a. False
b. True

(12) **Galois groups** [Multiple choice] [One answer only]

What is the Galois group of the polynomial $t^2 - 3t + 2$?

a. the trivial group 1

b. C_3
c. S_3
d. C_2
e. none of the other answers

(13) **Galois groups** MULTIPLE CHOICE One answer only

What is the Galois group of the polynomial $t^2 - 2t + 3$?

a. C_3
b. C_2
c. S_3
d. the trivial group 1
e. none of the other answers

(14) **Galois groups** MULTIPLE CHOICE One answer only

What is the Galois group of the polynomial $(t + 2)(t + 1/2)(t - 2)(t - 1/2)$?

a. $C_2 \times C_2$
b. none of the other answers
c. the trivial group 1
d. C_4
e. S_4

(15) **Galois groups** MULTIPLE CHOICE One answer only

What is the Galois group of the polynomial $(t^5 - 1)/(t - 1)$?

a. S_4
b. none of the other answers
c. S_5
d. C_4
e. C_5

(16) **Galois groups** MULTIPLE CHOICE One answer only

True or false? The Galois group of a polynomial of degree n has order at most $n!$.

a. True
b. False

(17) **Galois groups** MULTIPLE CHOICE One answer only

True or false? The Galois group of a polynomial of degree n has order dividing $n!$.

a. False
b. True

(18) **Galois groups** MULTIPLE CHOICE One answer only

True or false? The Galois group of a polynomial of degree n has order at most n.

a. True
b. False

(19) **Solvability** MULTIPLE CHOICE One answer only

True or false? The Galois group of the polynomial $t^5 + 5$ is solvable.

a. False
b. True

(20) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the polynomial $(t^2-1)(t^6+2)$ solvable by radicals?

a. No
b. Yes
c. Not enough information in Chapter 1 to say

(21) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the polynomial

$$(5t^4 + 4t^3 - 3t^2 + 2t - 1)(t^3 + 8t - 14)$$

solvable by radicals?

a. No
b. Not enough information in Chapter 1 to say
c. Yes

(22) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the polynomial $t^5 - 6t + 3$ solvable by radicals?

a. No
b. Not enough information in Chapter 1 to say
c. Yes

(23) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the Galois group of the polynomial

$$(t^4 + 2t^2 - 2t - 1)(t^3 - 5t^2 - 3t + 15)$$

solvable?

a. No
b. Not enough information in Chapter 1 to say
c. Yes

(24) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the Galois group of the polynomial $t^5 + 3t^4 - 2t^3 + 6t + 1$ solvable?

a. Yes
b. Not enough information in Chapter 1 to say
c. No

(25) **Solvability** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial of degree 4 with nontrivial Galois group G. Then the commutators $ghg^{-1}h^{-1}$ $(g, h \in G)$ generate G.

a. True
b. False

(26) **Solvability** MULTIPLE CHOICE One answer only

True or false? The number π is radical.

a. False
b. True

(27) **Solvability** [Multiple choice] [One answer only]

True or false? The number $(5+\sqrt{2})^{1/4} - 3^{1/6}$ is radical.

a. False
b. True

Total of marks: 27