Chapter 10: Finite fields

(1) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every $q \geq 2$ that is an integer power of a prime number, there is a field of order q.

a. True
b. False

(2) **Solvability by radicals** MULTIPLE CHOICE One answer only

Up to isomorphism, how many fields are there of order ≤ 20?

a. none of the other answers is correct
b. 13
c. 8
d. 10
e. 20

(3) **Solvability by radicals** MULTIPLE CHOICE One answer only

Up to isomorphism, how many fields are there whose order is ≥ 80 and ≤ 90?

a. 1
b. 4
c. 2
d. 3
e. none of the other answers is correct

(4) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $M : K$ is separable.

a. True
b. False

(5) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $M : K$ is normal.

a. True

b. False

(6) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension, with M finite. Then M is the splitting field of some polynomial over K.

a. True
b. False

(7) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension, with M finite. Then $\mathrm{Gal}(M : K)$ is abelian.

a. True
b. False

(8) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension, with M finite. Then $\mathrm{Gal}(M : K)$ is simple.

a. False
b. True

(9) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let p be a prime and let $q \geq 1$ be an integer. Then $(\alpha + \beta)^q = \alpha^q + \beta^q$ for all elements α, β of any field of characteristic p.

a. True
b. False

(10) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let p be a prime and let $q \geq 1$ be an integer power of p. Then $(\alpha + \beta)^q = \alpha^q + \beta^q$ for all elements α, β of any field of characteristic p.

a. False
b. True

(11) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every field K of characteristic $p > 0$, the function $K \to K$ defined by $\alpha \mapsto \alpha^p$ is an isomorphism.

a. False
b. True

(12) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every finite field K of characteristic $p > 0$, the function $K \to K$ defined by $\alpha \mapsto \alpha^p$ is an isomorphism.

a. False
b. True

(13) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every field K of characteristic $p > 0$, the Frobenius map $K \to K$ has trivial kernel.

a. True
b. False

(14) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every prime p, the Frobenius map $\mathbb{F}_p(t) \to \mathbb{F}_p(t)$ is bijective.

a. False
b. True

(15) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The field of four elements is $\mathbb{Z}/\langle 4 \rangle$.

a. True
b. False

(16) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In a finite field of characteristic 5, every element has a unique 5th root.

a. True

b. False

(17) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In a field of characteristic 5, every element has a unique 5th root.

a. False
b. True

(18) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In a finite field of characteristic 5, every element has a unique 25th root.

a. True
b. False

(19) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every prime p and integer $n \geq 0$, there is precisely one field of order p^n (up to isomorphism).

a. True
b. False

(20) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The splitting field of $t^{p^n} - t$ over $\mathbb{F}_p$ has order n.

a. False
b. True

(21) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The splitting field of $t^{p^n} - t$ over $\mathbb{F}_p$ has degree p^n over $\mathbb{F}_p$.

a. True
b. False

(22) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The splitting field of $t^{p^n} - t$ over $\mathbb{F}_p$ has degree n over $\mathbb{F}_p$.

a. False
b. True

(23) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The field extension $\mathrm{SF}_{\mathbb{F}_p}(t^{p^n} - t) : \mathbb{F}_p$ is simple.

a. True
b. False

(24) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let M be a finite field of order p^n, where p is prime and $n \geq 1$. Then $\alpha^p = \alpha$ for all $\alpha \in M$.

a. False
b. True

(25) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Up to isomorphism, the number of fields of order 27 is equal to the number of monic irreducible cubic polynomials over $\mathbb{F}_3$.

a. False
b. True

(26) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let K be a field. Then every subgroup of the multiplicative group $K^\times$ is cyclic.

a. True
b. False

(27) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let K be a finite field. Then every subgroup of the multiplicative group $K^\times$ is cyclic.

a. True
b. False

(28) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let K be a field and let H be a finite subgroup of $K^\times$. Then every element of H has finite order in the group $K^\times$.

a. False
b. True

(29) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let p be a prime. Every element of $\mathbb{F}_p$ apart from 0 and ± 1 is a generator (in the group theory sense) of the multiplicative group $\mathbb{F}_p^\times$.

a. False
b. True

(30) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? The field extension $\mathbb{F}_{343} : \mathbb{F}_7$ is simple.

a. False
b. True

(31) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let p be a prime and $1 \leq m \leq n$. There are exactly m subfields of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(32) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let p be a prime and $1 \leq m \leq n$. There is exactly one subfield of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(33) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let p be a prime and $m, n \geq 1$ with $m|n$. There is exactly one subfield of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(34) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $m, n \geq 1$ with $m|n$. There are exactly m subfields of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(35) **Solvability by radicals** MULTIPLE CHOICE One answer only

How many subfields does $\mathbb{F}_{16}$ have?

a. 3
b. none of the other answers is correct
c. 2
d. 4
e. 5

(36) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime. Then every polynomial in t^p over $\mathbb{F}_p$ has a pth root in $\mathbb{F}_p[t]$.

a. True
b. False

(37) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every irreducible polynomial over a finite field is separable.

a. False
b. True

(38) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $\mathrm{Gal}(M : K)$ is cyclic of order $[M : K]$.

a. True
b. False

(39) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? $\mathbb{F}_{125}$ has a subfield with 25 elements.

a. True
b. False

(40) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? $\mathbb{F}_{625}$ has a subfield with 25 elements.

a. True
b. False

(41) **Solvability by radicals** MULTIPLE CHOICE One answer only

How many subgroups does C_{45} have?

a. 5
b. 3
c. 2
d. 45
e. none of the other answers is correct

(42) **Solvability by radicals** MULTIPLE CHOICE One answer only

How many subgroups of order 9 does C_{45} have?

a. 1
b. none of the other answers is correct
c. 2
d. 3
e. 0

(43) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $m, n \geq 1$, with $m|n$. Then the unique subfield of $\mathbb{F}_{p^n}$ of order p^m consists of the roots of $t^{p^m} - t$.

a. False
b. True

(44) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $m, n \geq 1$, with $m|n$. Then the unique subfield of $\mathbb{F}_{p^n}$ of order p^m consists of the roots of $t^{p^{n/m}} - t$.

a. False
b. True

(45) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The unique subfield of $\mathbb{F}_{32}$ of order 4 consists of the elements x such that $x^4 = x$.

a. False
b. True

(46) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The unique subfield of $\mathbb{F}_{32}$ of order 4 consists of the elements x such that $x^8 = x$.

a. True
b. False

(47) **Solvability by radicals** MULTIPLE CHOICE One answer only

Let p be a prime and $m, n \geq 1$, with $m < n$. How many homomorphisms $\mathbb{F}_{p^m} \to \mathbb{F}_{p^n}$ are there?

a. more than 1
b. depends on m and n
c. 1
d. none
e. none of the other answers is correct

(48) **Solvability by radicals** MULTIPLE CHOICE One answer only

Let p be a prime and $m, n \geq 1$, with $m|n$. How many homomorphisms $\mathbb{F}_{p^m} \to \mathbb{F}_{p^n}$ are there?

a. none of the other answers is correct
b. depends on m and n
c. 1
d. more than 1
e. none

(49) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There are exactly 6 automorphisms of the field with 64 elements.

a. True
b. False

(50) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There are exactly 6 homomorphisms $\mathbb{F}_{64} \to \mathbb{F}_{64}$

a. False
b. True

(51) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There are exactly 6 surjective homomorphisms $\mathbb{F}_{64} \to \mathbb{F}_{64}$

a. False
b. True

Total of marks: 51