Chapter 2: Group actions, rings and fields

(1) **Actions** MULTIPLE CHOICE One answer only

True or false? For every group G and set X, there is some action of G on X.

a. True
b. False

(2) **Actions** MULTIPLE CHOICE One answer only

True or false? For every nontrivial group G and set X with at least two elements, there exists a nontrivial action of G on X.

a. False
b. True

(3) **Actions** MULTIPLE CHOICE One answer only

True or false? If a finite group G acts faithfully on a finite set X with n elements, then $|G| \leq n!$.

a. True
b. False

(4) **Actions** MULTIPLE CHOICE One answer only

True or false? The actions of a group G on a set X correspond naturally to the homomorphisms $\mathrm{Sym}(X) \to G$.

a. False
b. True

(5) **Actions** MULTIPLE CHOICE One answer only

True or false? A finite group cannot act transitively on an infinite set.

a. True
b. False

(6) **Rings** MULTIPLE CHOICE One answer only

Which of the following statements is true?

a. There are no rings in which 0 is irreducible.

b. There are some rings in which 0 is irreducible, and some in which it is not.
c. In every ring, 0 is irreducible.

(7) **Rings** [Multiple choice] [One answer only]

Which of the following statements is true?

a. There are no rings in which 0 is reducible.
b. In every ring, 0 is reducible.
c. There are some rings in which 0 is reducible, and some in which it is not.

(8) **Rings** [Multiple choice] [One answer only]

Which of the following statements is true?

a. There are no rings in which 1 is irreducible.
b. In every ring, 1 is irreducible.
c. There are some rings in which 1 is irreducible, and some in which it is not.

(9) **Rings** [Multiple choice] [One answer only]

Which of the following statements is true?

a. There are some rings in which 1 is reducible, and some in which it is not.
b. There are no rings in which 1 is reducible.
c. In every ring, 1 is reducible.

(10) **Rings** [Multiple choice] [One answer only]

For rings R and S, let ψ_{RS} denote the function $R \to S$ defined by $\psi_{RS}(r) = 0$ for all $r \in R$. Which of the following statements is true?

a. ψ_{RS} is a homomorphism for all rings R and S.
b. There are no rings R and S for which ψ_{RS} is a homomorphism.
c. ψ_{RS} is a homomorphism for some rings R and S but not others.

(11) **Rings** [Multiple choice] [One answer only]

True or false? The subset $2\mathbb{Z}$ of $\mathbb{Z}$ is a subring.

a. True
b. False

(12) **Rings** MULTIPLE CHOICE One answer only

True or false? The subset $2\mathbb{Z}$ of $\mathbb{Z}$ is an ideal.

a. False
b. True

(13) **Rings** MULTIPLE CHOICE One answer only

True or false? The union of two subrings of a ring is a subring.

a. False
b. True

(14) **Rings** MULTIPLE CHOICE One answer only

True or false? Let R be a ring. Any ideal of R containing a unit must be equal to R.

a. True
b. False

(15) **Rings** MULTIPLE CHOICE One answer only

True or false? For every ring R, there is exactly one homomorphism $R \to \mathbb{Z}$.

a. False
b. True

(16) **Rings** MULTIPLE CHOICE One answer only

True or false? Every ideal of a ring is a subring.

a. False
b. True

(17) **Rings** MULTIPLE CHOICE One answer only

True or false? For a ring R and elements $r, u \in R$, if u is a unit then $\langle r \rangle = \langle ur \rangle$.

a. False

b. True

(18) **Rings** [Multiple choice] [One answer only]

True or false? In the ring $\mathbb{Z}$, every integer divides 0.

a. False
b. True

(19) **Rings** [Multiple choice] [One answer only]

True or false? If r is an element of a ring such that 0 divides r, then $r = 0$.

a. False
b. True

(20) **Rings** [Multiple choice] [One answer only]

True or false? In any ring, the product of two units is a unit.

a. False
b. True

(21) **Rings** [Multiple choice] [One answer only]

True or false? In any ring, 0 and 1 are coprime.

a. False
b. True

(22) **Fields** [Multiple choice] [One answer only]

How many ring homomorphisms $\mathbb{Q} \to \mathbb{Q}$ are there?

a. infinitely many
b. none of the other answers is correct
c. 0
d. 1
e. 2

(23) **Fields** [Multiple choice] [One answer only]

True or false? There exists a field of characteristic 11.

a. False

b. True

(24) **Fields** MULTIPLE CHOICE One answer only

True or false? There exists a field of characteristic 12.

a. False
b. True

(25) **Fields** MULTIPLE CHOICE One answer only

True or false? There exists a ring of characteristic 12.

a. True
b. False

(26) **Fields** MULTIPLE CHOICE One answer only

True or false? There exists an integral domain of characteristic 12.

a. True
b. False

(27) **Fields** MULTIPLE CHOICE One answer only

True or false? There exists a homomorphism of fields $\mathbb{Q} \to \mathbb{F}_2$.

a. False
b. True

(28) **Fields** MULTIPLE CHOICE One answer only

How many ring homomorphisms $\mathbb{F}_2 \to \mathbb{F}_3$ are there?

a. 9
b. 1
c. 3
d. 8
e. none of the other answers is correct

(29) **Fields** MULTIPLE CHOICE One answer only

Call a field K 'special' if the prime subfield of K is isomorphic to K. Up to isomorphism, how many infinite fields are special?

a. none of the other answers is correct

b. 0
c. infinitely many
d. 1

(30) **Fields** MULTIPLE CHOICE One answer only

True or false? Let K and L be fields. If there exists a homomorphism $K \to L$ then K and L have isomorphic prime subfields.

a. True
b. False

(31) **Fields** MULTIPLE CHOICE One answer only

True or false? Let K and L be fields. If there exist a field M and homomorphisms $K \to M \leftarrow L$ then K and L have isomorphic prime subfields.

a. True
b. False

(32) **Fields** MULTIPLE CHOICE One answer only

True or false? Every field of positive characteristic is finite.

a. False
b. True

(33) **Fields** MULTIPLE CHOICE One answer only

True or false? Every field of characteristic 0 is infinite.

a. True
b. False

(34) **Fields** MULTIPLE CHOICE One answer only

True or false? Every infinite field has characteristic 0.

a. True
b. False

(35) **Fields** MULTIPLE CHOICE One answer only

True or false? Every finite field has positive characteristic.

a. False
b. True

(36) **Fields** MULTIPLE CHOICE One answer only

True or false? Let R be a principal ideal domain. If r is an element of R such that $R/\langle r\rangle$ is a field then r is irreducible.

a. True
b. False

(37) **Fields** MULTIPLE CHOICE One answer only

True or false? Let R be a principal ideal domain. If r is an irreducible element of R then $R/\langle r\rangle$ is an integral domain.

a. True
b. False

Total of marks: 37