Chapter 3: Polynomials

(1) **Polynomials** [Multiple choice] [One answer only]

Let K be a field. Consider the statement 'if $f, g \in K[t]$ with $f(a) = g(a)$ for all $a \in K$, then $f = g$'. Which of the following holds?

a. None of the other answers is correct.
b. The statement is true for all fields K.
c. The statement is false when $K = \mathbb{F}_2$ and true for all other fields.
d. The statement is false when K is finite and true when K is infinite.
e. The statement is false for all fields K.

(2) **Polynomials** [Multiple choice] [One answer only]

True or false? n divides $\binom{n}{i}$ whenever $0 < i < n$ and n is prime.

a. False
b. True

(3) **Polynomials** [Multiple choice] [One answer only]

True or false? n divides $\binom{n}{i}$ whenever $0 < i < n$.

a. True
b. False

(4) **Polynomials** [Multiple choice] [One answer only]

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is irreducible over $\mathbb{Q}$ then f is irreducible over $\mathbb{Z}$.

a. False
b. True

(5) **Polynomials** [Multiple choice] [One answer only]

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is primitive and irreducible over $\mathbb{Q}$ then f is irreducible over $\mathbb{Z}$.

a. False
b. True

(6) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is irreducible over $\mathbb{Z}$ then f is irreducible over $\mathbb{Q}$.

a. False
b. True

(7) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is reducible over $\mathbb{Q}$ then f is reducible over $\mathbb{Z}$.

a. True
b. False

(8) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is irreducible over $\mathbb{Z}$ then f is primitive.

a. True
b. False

(9) **Polynomials** MULTIPLE CHOICE One answer only

True or false? For every nonzero polynomial f, the codegree of f is less than or equal to the degree of f.

a. True
b. False

(10) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in K[t]$. If $f(a) = g(a)$ for all $a \in K$, then the multiplicity of any root of f and g is the same for f and g.

a. True
b. False

(11) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $n \in \mathbb{Z}$ and suppose that $\sqrt{n} \in \mathbb{Q}$. Then $\sqrt{n} \in \mathbb{Z}$.

a. False

b. True

(12) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and write $\phi_* : R[t] \to S[t]$ for the induced homomorphism. Then $\deg(\phi_*(f)) = \deg(f)$ for all $f \in R[t]$.

a. True
b. False

(13) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and write $\phi_* : R[t] \to S[t]$ for the induced homomorphism. Then $\deg(\phi_*(f)) \leq \deg(f)$ for all $f \in R[t]$.

a. False
b. True

(14) **Polynomials** MULTIPLE CHOICE One answer only

True or false? For every ring R and element $r \in R$, there is a unique homomorphism $\phi : \mathbb{Z}[t] \to R$ such that $\phi(t) = r$.

a. False
b. True

(15) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f \in \mathbb{Z}[t]$ and $g, h \in \mathbb{Q}[t]$. If $f = gh$ then $g, h \in \mathbb{Z}[t]$.

a. True
b. False

(16) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If p is prime then $1 + t + \cdots + t^{p-1}$ is irreducible over $\mathbb{Q}$.

a. False
b. True

(17) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If p is prime then $1 + t + \cdots + t^{p-1}$ is irreducible over $\mathbb{F}_p$.

a. False
b. True

(18) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If $n \geq 1$ then $1 + t + \cdots + t^{n-1}$ is irreducible over $\mathbb{Q}$.

a. False
b. True

(19) **Polynomials** MULTIPLE CHOICE One answer only

True or false? There is an irreducible polynomial over $\mathbb{Q}$ of degree 1000.

a. False
b. True

(20) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If R is a principal ideal domain then $R[t]$ is a principal ideal domain.

a. False
b. True

(21) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If K is a field then $K[t]$ is a field.

a. False
b. True

(22) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If R is an integral domain then $R[t]$ is an integral domain.

a. False
b. True

(23) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If K is a field then $K[t]$ is a principal ideal domain.

a. False
b. True

(24) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f(t) \in K[t]$. If f has no roots in K then f is irreducible.

a. True
b. False

(25) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f(t) \in K[t]$. If f is irreducible then f has no roots in K.

a. True
b. False

(26) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f(t) \in K[t]$ with $\deg(f) \geq 2$. If f is irreducible then f has no roots in K.

a. False
b. True

(27) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and let f be a polynomial over K of degree 3. If f has no roots in K then f is irreducible.

a. True
b. False

(28) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let R be a ring and $f, g \in R[t]$. If f and g are equal as polynomials then f and g induce the same function $K \to K$.

a. False
b. True

(29) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let R be a ring and $f \in R[t]$. If $f(t)$ is reducible then $f(t+4)$ is reducible.

a. True
b. False

(30) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{R}$ is algebraically closed.

a. False
b. True

(31) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{F}_2$ is algebraically closed.

a. False
b. True

(32) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Z}[t]$ is a principal ideal domain.

a. True
b. False

(33) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Q}[t, u]$ is a principal ideal domain.

a. False
b. True

(34) **Polynomials** MULTIPLE CHOICE One answer only

True or false? There is a homomorphism $\phi : \mathbb{Q}[t] \to \mathbb{Q}[t]$ such that $\phi(t) = t^2$.

a. False
b. True

(35) **Polynomials** MULTIPLE CHOICE One answer only

True or false? There is an isomorphism $\phi : \mathbb{Q}[t] \to \mathbb{Q}[t]$ such that $\phi(t) = t^2$.

a. True
b. False

(36) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and $r \in R$. If r is a unit then $\phi(r)$ is a unit.

a. True
b. False

(37) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and $r \in R$. If $\phi(r)$ is a unit then r is a unit.

a. False
b. True

(38) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over $\mathbb{Z}$, let p be a prime, and suppose that f reduced mod p is irreducible over $\mathbb{F}_p$. Then f is irreducible over $\mathbb{Q}$.

a. True
b. False

(39) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let f be a monic polynomial over $\mathbb{Z}$, let p be a prime, and suppose that f reduced mod p is irreducible over $\mathbb{F}_p$. Then f is irreducible over $\mathbb{Q}$.

a. True
b. False

Total of marks: 39