Chapter 4: Field extensions

(1) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every monic irreducible polynomial $f \in \mathbb{Q}[t]$, there is some element of $\mathbb{C}$ whose minimal polynomial over $\mathbb{Q}$ is f.

a. True
b. False

(2) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The complex conjugation map $\mathbb{C} \to \mathbb{C}$, given by $z \mapsto \overline{z}$, defines a field extension of $\mathbb{C}$ over itself.

a. False
b. True

(3) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The set $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a subfield of $\mathbb{C}$.

a. True
b. False

(4) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The set $\{a + b\sqrt{4} : a, b \in \mathbb{Q}\}$ is a subfield of $\mathbb{C}$.

a. False
b. True

(5) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The set $\{a + b\sqrt[3]{2} : a, b \in \mathbb{Q}\}$ is a subfield of $\mathbb{C}$.

a. True
b. False

(6) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists a field containing $K[t]$ as a subring.

a. True

b. False

(7) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $X \subseteq K$. If X is finite then the subfield of K generated by X is finite.

a. True
b. False

(8) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $X \subseteq K$. If X is finite then the subfield of K generated by X is countable.

a. True
b. False

(9) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field. The union of any family of subfields of K is a subfield.

a. True
b. False

(10) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The subfield of $\mathbb{C}$ generated by $\{i\}$ is $\mathbb{C}$ itself.

a. True
b. False

(11) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $Y \subseteq M$. Then $K(Y)$ is the largest subfield of M containing $K \cup Y$.

a. False
b. True

(12) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $Y \subseteq M$. Then $K(Y)$ is the smallest subfield of M containing $K \cup Y$.

a. True
b. False

(13) **Field extensions** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension and $Y \subseteq M$. Then $K(Y)$ is the smallest subfield of M containing Y.

a. False
b. True

(14) **Field extensions** [Multiple choice] [One answer only]

True or false? Let $M : \mathbb{Q}$ be a field extension and $Y \subseteq M$. Then $\mathbb{Q}(Y)$ is the smallest subfield of M containing Y.

a. False
b. True

(15) **Field extensions** [Multiple choice] [One answer only]

True or false? Let $M : \mathbb{F}_p$ be a field extension and $Y \subseteq M$. Then $\mathbb{F}_p(Y)$ is the smallest subfield of M containing Y.

a. False
b. True

(16) **Field extensions** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension and $X \subseteq Y \subseteq M$. Then $K(X) \subseteq K(Y)$.

a. True
b. False

(17) **Field extensions** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions and $\alpha \in M$. If α is algebraic over K then α is algebraic over L.

a. False
b. True

(18) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions and $\alpha \in M$. If α is algebraic over L then α is algebraic over K.

a. True
b. False

(19) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For a field K, every element of the complement $K(t) \setminus K$ is transcendental over K.

a. True
b. False

(20) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. If α and β have the same sets of annihilating polynomials then either both are algebraic over K or both are transcendental over K.

a. False
b. True

(21) **Field extensions** MULTIPLE CHOICE One answer only

True or false? There is an element of $\mathbb{C}$ whose minimal polynomial over $\mathbb{Q}$ is $1 + 2t + 3t^2 + 4t^3 + 5t^4$.

a. True
b. False

(22) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. If α and β are both algebraic and have the same minimal polynomial, then they have the same sets of annihilating polynomials.

a. True
b. False

(23) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let α and β be complex numbers algebraic over $\mathbb{Q}$. Then α and β are conjugate over $\mathbb{Q}$ if and only if they have the same minimal polynomial.

a. False
b. True

(24) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every monic polynomial $f \in \mathbb{Q}[t]$, there is an element of $\mathbb{C}$ whose minimal polynomial over $\mathbb{Q}$ is f.

a. True
b. False

(25) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, let α be an element of the complement $M \setminus K$, and let $f \in K[t]$ be a monic quadratic that annihilates α. Then f is the minimal polynomial of α over K.

a. False
b. True

(26) **Field extensions** MULTIPLE CHOICE One answer only

Let $M : K$ be a field extension and let α be an element of M algebraic over K. What is the smallest possible degree of the minimal polynomial of α?

a. 2
b. 1
c. None of the other answers is correct.
d. 0

(27) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let $\alpha \in K$. Then the minimal polynomial of α over K has degree 0.

a. False
b. True

(28) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For a prime p, the minimal polynomial of $e^{2\pi i/p}$ over $\mathbb{Q}$ is $t^p - 1$.

a. True
b. False

(29) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. If $K(\alpha) \cong K(\beta)$ then $\alpha = \beta$.

a. True
b. False

(30) **Field extensions** MULTIPLE CHOICE One answer only

Let $M : K$ be a field extension and let α and β be elements of M algebraic over K. Suppose that α and β have the same minimal polynomial. What is the relationship between $K(\alpha)$ and $K(\beta)$?

a. They are equal as subfields of M.
b. They are isomorphic as abstract fields, but not necessarily isomorphic over K.
c. They are isomorphic over K, but not necessarily equal as subfields of M.
d. None of the other answers is correct.
e. They are not necessarily isomorphic as abstract fields.

(31) **Field extensions** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Q}(i) = \mathbb{Q}(1 + i)$ as subfields of $\mathbb{C}$.

a. True
b. False

(32) **Field extensions** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Q}(\sqrt{5} + 5) = \mathbb{Q}(\sqrt{5} - 5)$ as subfields of $\mathbb{C}$.

a. False
b. True

(33) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The extension $K(t) : K$ is simple for all fields K, where $K(t)$ is the field of rational expressions over K.

a. False
b. True

(34) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let α, β be distinct elements of M such that $M = K(\alpha, \beta)$. Then $M : K$ is not simple.

a. True
b. False

(35) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $m \in K[t]$. Then $K[t]/\langle m \rangle$ is a field.

a. True
b. False

(36) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field and let $M : K$ and $M' : K$ be extensions of K. If M and M' each contain an element transcendental over K, then $M \cong M'$.

a. False
b. True

(37) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists an extension $M : K$ such that 2 has a square root in M.

a. True
b. False

(38) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists an extension $M : K$ such that the equation $t^2 + t + 1 = 0$ has a solution in M.

a. True
b. False

(39) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists an extension $M : K$ such that the polynomial $t^5 - 6t + 3$ has a root in M.

a. False
b. True

(40) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K and nonconstant polynomial f over K, there exists an extension $M : K$ such that f has at least one root in M.

a. True
b. False

Total of marks: 40