Chapter 5: Degree

(1) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha) : K$ be a simple field extension. If α satisfies a quadratic equation over K then $[K(\alpha) : K] \in \{1, 2\}$.

a. True
b. False

(2) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha) : K$ be a simple field extension. If α satisfies a quadratic equation over K then $[K(\alpha) : K] = 2$.

a. False
b. True

(3) **Degree** MULTIPLE CHOICE One answer only

True or false? $[\mathbb{C} : \mathbb{Q}] = 2$.

a. True
b. False

(4) **Degree** MULTIPLE CHOICE One answer only

True or false? $[\mathbb{Q} : \mathbb{Q}] = 0$.

a. False
b. True

(5) **Degree** MULTIPLE CHOICE One answer only

True or false? $\deg_{\mathbb{Q}}(e^{2\pi i/p}) = p$ for all primes p.

a. True
b. False

(6) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha) : K$ be a simple field extension and $n \geq 1$. If α is a root of some polynomial over K of degree n then $[K(\alpha) : K] \leq n$.

a. True
b. False

(7) **Degree** MULTIPLE CHOICE One answer only

True or false? $M : K$ is finite whenever M is a simple extension of K.

a. True
b. False

(8) **Degree** MULTIPLE CHOICE One answer only

For $n \geq 1$, what is $[\mathbb{Q}(e^{2\pi i/n}) : \mathbb{Q}]$?

a. 1
b. n
c. none of the other answers is correct
d. $n - 1$
e. ∞

(9) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $\alpha, \beta \in \mathbb{Q}$ with $[\mathbb{Q}(\alpha) : \mathbb{Q}] = [\mathbb{Q}(\beta) : \mathbb{Q}] = 2$. Then $[\mathbb{Q}(\alpha, \beta) : \mathbb{Q}] = 4$.

a. True
b. False

(10) **Degree** MULTIPLE CHOICE One answer only

True or false? Let p and q be distinct primes. Then $[\mathbb{Q}(\sqrt{p}, \sqrt{q}) : \mathbb{Q}] = 4$.

a. True
b. False

(11) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha, \beta) : K(\alpha)] = [K(\beta) : K]$.

a. False
b. True

(12) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. Then $[L : K] \leq [M : K]$.

a. True
b. False

(13) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. Then $[M : L] \leq [M : K]$.

a. False
b. True

(14) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha + \beta) : K] \leq [K(\alpha, \beta) : K]$.

a. False
b. True

(15) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha^2\beta - \alpha\beta^2) : K] \leq [K(\alpha, \beta) : K]$.

a. True
b. False

(16) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha, \beta) : K(\alpha)] \geq [K(\beta) : K]$.

a. False
b. True

(17) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha, \beta) : K(\alpha)] = [K(\beta) : K]$.

a. False
b. True

(18) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension such that $M = K(\alpha_1, \ldots, \alpha_n)$ for some $\alpha_1, \ldots, \alpha_n \in M$. Then every element of M can be expressed as a polynomial over K in $\alpha_1, \ldots, \alpha_n$.

a. False
b. True

(19) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions such that $[M : K]$ is a prime number. Then $L = M$ or $L = K$.

a. False
b. True

(20) **Degree** MULTIPLE CHOICE One answer only

True or false? There exist field extensions $M : L : K$ such that $M : K$ is infinite but $M : L$ and $L : K$ are finite.

a. False
b. True

(21) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : K$ is finite then so is $M : L$.

a. False
b. True

(22) **Degree** MULTIPLE CHOICE One answer only

True or false? There exist field extensions $M : L : K$ such that $[M : K] = 6$ and $[M : L] = 4$.

a. False
b. True

(23) **Degree** MULTIPLE CHOICE One answer only

True or false? There exist field extensions $M : L : K : F$, all nontrivial, such that $[M : F] = 6$.

a. False
b. True

(24) **Degree** [Multiple choice] [One answer only]

True or false? There exist field extensions $M : L : K$ such that $[M : K] = 6$ and $[M : L] = 3$.

a. True
b. False

(25) **Degree** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension and let $\alpha_1, \ldots, \alpha_n$ be elements of M algebraic over K. Then $[K(\alpha_1, \ldots, \alpha_n) : K] = [K(\alpha_1) : K] \cdots [K(\alpha_n) : K]$.

a. True
b. False

(26) **Degree** [Multiple choice] [One answer only]

True or false? Every field extension that is finite is finitely generated.

a. False
b. True

(27) **Degree** [Multiple choice] [One answer only]

True or false? Every field extension that is finitely generated is finite.

a. True
b. False

(28) **Degree** [Multiple choice] [One answer only]

True or false? Every algebraic field extension is finite.

a. False
b. True

(29) **Degree** [Multiple choice] [One answer only]

True or false? Every field extension that is finite is algebraic.

a. True

b. False

(30) **Degree** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension where M is a finite field. Then the extension is finite.

a. False
b. True

(31) **Degree** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension. If $M = K(\alpha, \beta, \gamma)$ for some $\alpha, \beta, \gamma \in M$, then $M : K$ is a finitely generated extension.

a. False
b. True

(32) **Degree** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension. If $M = K(\alpha, \beta, \gamma)$ for some $\alpha, \beta, \gamma \in M$, then $M : K$ is a finite extension.

a. True
b. False

(33) **Degree** [Multiple choice] [One answer only]

True or false? Let $M : K$ be a field extension. Every K-linear subspace of M containing K is a subfield of M.

a. False
b. True

(34) **Degree** [Multiple choice] [One answer only]

True or false? If a simple extension is algebraic, it is finite.

a. False
b. True

(35) **Degree** [Multiple choice] [One answer only]

True or false? Given two points in the plane distance L apart, one can use ruler and compasses to construct two points distance $\sqrt{2}L$ apart.

a. True
b. False

(36) **Degree** [Multiple choice] [One answer only]

True or false? Given two points in the plane distance L apart, one can use ruler and compasses to construct two points distance $2^{1/5}L$ apart.

a. True
b. False

(37) **Degree** [Multiple choice] [One answer only]

True or false? Given the points $(0, 0)$ and $(1, 0)$ in the plane, one can use ruler and compasses to construct the point $(e, 1/e)$.

a. False
b. True

(38) **Degree** [Multiple choice] [One answer only]

True or false? The set of points constructible by ruler and compasses from $(0, 0)$ and $(1, 0)$ is countable.

a. True
b. False

(39) **Degree** [Multiple choice] [One answer only]

True or false? If $M : K$ is a field extension of degree 2 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. True
b. False

(40) **Degree** [Multiple choice] [One answer only]

True or false? If $M : K$ is a field extension of degree 3 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. False
b. True

(41) **Degree** MULTIPLE CHOICE One answer only

True or false? If $M : K$ is a field extension of degree 4 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. True
b. False

(42) **Degree** MULTIPLE CHOICE One answer only

True or false? If $M : K$ is a field extension of degree 23 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. True
b. False

(43) **Degree** MULTIPLE CHOICE One answer only

True or false? Every element of $\mathbb{C}$ algebraic over $\mathbb{Q}$ has a monic annihilating polynomial with coefficients in $\mathbb{Z}$.

a. True
b. False

(44) **Degree** MULTIPLE CHOICE One answer only

True or false? A regular 17-sided polygon can be constructed by ruler and compass (given two points in the plane).

a. False
b. True

(45) **Degree** MULTIPLE CHOICE One answer only

True or false? A regular 18-sided polygon can be constructed by ruler and compass (given two points in the plane).

a. True
b. False

(46) **Degree** MULTIPLE CHOICE One answer only

True or false? A regular 7-sided polygon can be constructed by ruler and compass (given two points in the plane).

a. False
b. True

Total of marks: 46