Chapter 6: Splitting fields

(1) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Given field extensions $\iota : K \to M$ and $\iota' : K' \to M'$, if ι and ι' are isomorphisms then every homomorphism $K \to K'$ extends to a homomorphism $M \to M'$.

a. True
b. False

(2) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $\psi : K \to K'$ be a homomorphism of fields. Then every polynomial over K' can be expressed as $\psi_*(f)$ for some $f \in K[t]$.

a. False
b. True

(3) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let K and K' be isomorphic fields. If K' is perfect then so is K.

a. True
b. False

(4) **Splitting fields** MULTIPLE CHOICE One answer only

How many automorphisms ϕ of $\mathbb{Q}(\sqrt{2})$ are there such that $\phi(\sqrt{2}) = -\sqrt{2}$?

a. 1
b. nothing in the notes so far gives us the answer
c. 0
d. 2

(5) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The splitting field of $f(t) \in K[t]$ over K is $K[t]/\langle f \rangle$.

a. True
b. False

(6) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The splitting field of $t^3 - 2$ over $\mathbb{Q}$ is $\mathbb{Q}(\sqrt[3]{2})$.

a. True
b. False

(7) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For a polynomial f over an algebraically closed field K, the splitting field of f over K is K.

a. True
b. False

(8) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $f(t) \in K[t]$. If f has a root in M then f splits in M.

a. True
b. False

(9) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If M is a splitting field of $f(t) \in K[t]$, and f splits in L, then $L = M$.

a. True
b. False

(10) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The splitting field of a cubic over $\mathbb{Q}$ has degree 3 over $\mathbb{Q}$.

a. False
b. True

(11) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every polynomial f over a field K, the splitting field of f over K has degree $\deg(f)!$.

a. True

b. False

(12) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every polynomial f over a field K, the splitting field of f over K has degree $\geq \deg(f)!$.

a. False
b. True

(13) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every polynomial f over a field K, the splitting field of f over K has degree $\leq \deg(f)!$.

a. False
b. True

(14) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Whenever M is the splitting field of some polynomial over K, there are at most $[M : K]$ automorphisms of M over K.

a. False
b. True

(15) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Whenever M is the splitting field of some polynomial over K, there are at most $[M : K]$ homomorphisms $M \to M$ over K.

a. True
b. False

(16) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The automorphism $a + b\sqrt{5} \mapsto a - b\sqrt{5}$ of $\mathbb{Q}(\sqrt{5})$ ($a, b \in \mathbb{Q}$) can be extended to an automorphism of $\mathbb{Q}(\sqrt{2}, \sqrt{5}, \sqrt{12})$.

a. False
b. True

(17) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For a polynomial f of degree n over K, there are at most n automorphisms of $\mathrm{SF}_K(f)$ over K.

a. True
b. False

(18) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For a polynomial f of degree n over K, there are at most $n!$ automorphisms of $\mathrm{SF}_K(f)$ over K.

a. True
b. False

(19) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K and let M and M' be splitting fields of f over K. Then there is a unique isomorphism $M \to M'$ over K.

a. False
b. True

(20) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K and let M be a splitting field of f over K. Then there is a unique automorphism of M over K.

a. True
b. False

(21) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K and let M and M' be splitting fields of f over K. Then there exists an isomorphism $M \to M'$ over K.

a. True
b. False

(22) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$ and let $\alpha \in \mathbb{C}$ be a root of f. Then $\mathrm{SF}_{\mathbb{Q}}(f)$ is a splitting field of f over $\mathbb{Q}(\alpha)$.

a. False

b. True

(23) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$. Let $\alpha \in \mathbb{C}$ be a complex number such that $f(\alpha) = 0$ but $f(-\alpha) \neq 0$. Then $\mathrm{SF}_{\mathbb{Q}}(f)$ is a splitting field of f over $\mathbb{Q}(-\alpha)$.

a. True
b. False

(24) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension. If θ and ϕ are automorphisms of M over K then $\theta \circ \phi$ is an automorphism of M over K.

a. False
b. True

(25) **Splitting fields** MULTIPLE CHOICE One answer only

Let $M : K$ and $M' : K$ be field extensions, and let $\theta : M \to M'$ be an isomorphism over K. Which of the following statements about $\theta^{-1} : M' \to M$ is true?

a. θ^{-1} is an isomorphism over K
b. θ^{-1} is an isomorphism of fields but not necessarily an isomorphism over K
c. θ^{-1} is a bijection but not necessarily a homomorphism of fields
d. none of the other answers is correct
e. θ^{-1} is a homomorphism of fields but not necessarily an isomorphism of fields

(26) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The field extension $\mathbb{C} : \mathbb{R}$ is finite.

a. True
b. False

(27) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The field extension $\mathbb{C} : \mathbb{R}$ has finite Galois group.

a. False
b. True

(28) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The field extension $\mathbb{R} : \mathbb{Q}$ is finite.

a. True
b. False

(29) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For a simple extension $K(\alpha) : K$ with $\alpha \notin K$, the Galois group of $K(\alpha) : K$ is nontrivial.

a. False
b. True

(30) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every field extension $M : K$, there exists a polynomial f over K such that M is the splitting field of f over K.

a. False
b. True

(31) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension. If $\alpha, \alpha' \in M$ are conjugate and α is algebraic then so is α'.

a. True
b. False

(32) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in K[t]$ be a polynomial with distinct roots $\alpha_1, \ldots, \alpha_k$ in its splitting field. Then the natural homomorphism $\mathrm{Gal}_K(f) \to S_k$ is injective.

a. False
b. True

(33) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in K[t]$ be a polynomial with distinct roots $\alpha_1, \ldots, \alpha_k$ in its splitting field. Then the natural homomorphism $\mathrm{Gal}_K(f) \to S_k$ is surjective.

a. False
b. True

(34) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in K[t]$ be a polynomial with distinct roots $\alpha_1, \ldots, \alpha_k$ in its splitting field. Then the natural homomorphism $\mathrm{Gal}_K(f) \to S_k$ is an isomorphism.

a. True
b. False

(35) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Q}[t]$. Then $\mathrm{Gal}_{\mathbb{Q}(i)}(f)$ embeds naturally as a subgroup of $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. True
b. False

(36) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Q}[t]$. Then $\mathrm{Gal}_{\mathbb{Q}}(f)$ embeds naturally as a subgroup of $\mathrm{Gal}_{\mathbb{Q}(i)}(f)$.

a. False
b. True

(37) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial of degree n over a field K. Then the order of the Galois group of f over K divides $n!$.

a. True
b. False

(38) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? There is a polynomial of degree 4 over $\mathbb{Q}$ whose Galois group is C_9.

a. True
b. False

(39) **Splitting fields** Multiple choice One answer only

True or false? There is a polynomial of degree 3 over $\mathbb{Q}$ whose Galois group is $C_2 \times C_2$.

a. False
b. True

(40) **Splitting fields** Multiple choice One answer only

True or false? Let $f(t) \in K[t]$, let $\alpha \in \mathrm{SF}_K(f)$ be a root of f, and let $\theta \in \mathrm{Gal}_K(f)$. Then $\theta(\alpha)$ is a root of f.

a. True
b. False

Total of marks: 40