Chapter 8: The fundamental theorem of Galois theory

(1) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? The order of the Galois group of a polynomial of degree n is divisible by n.

a. True
b. False

(2) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? The order of the Galois group of an irreducible polynomial of degree n is divisible by n.

a. False
b. True

(3) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a field extension $M : K$, the Galois correspondence associates with each subgroup H of $\mathrm{Gal}(M : K)$ the intermediate field $\mathrm{Fix}(H)$.

a. True
b. False

(4) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a field extension $M : K$, the Galois correspondence associates with each intermediate field L the group $\mathrm{Gal}(M : L)$.

a. False
b. True

(5) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a field extension $M : K$, the Galois correspondence associates with each intermediate field L the group $\mathrm{Gal}(L : K)$.

a. True
b. False

(6) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a field extension $M : K$ and subgroups $H_1 \subseteq H_2 \subseteq \mathrm{Gal}(M : K)$, we have $\mathrm{Fix}(H_1) \subseteq \mathrm{Fix}(H_2)$.

a. True
b. False

(7) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a field extension $M : K$ and subgroups $H_1 \subseteq H_2 \subseteq \mathrm{Gal}(M : K)$, we have $\mathrm{Fix}(H_2) \subseteq \mathrm{Fix}(H_1)$.

a. True
b. False

(8) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(M : L_1) \subseteq \mathrm{Gal}(M : L_2)$.

a. True
b. False

(9) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(M : L_2) \subseteq \mathrm{Gal}(M : L_1)$.

a. True
b. False

(10) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(L_1 : K) \subseteq \mathrm{Gal}(L_2 : K)$.

a. True
b. False

(11) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(L_2 : K) \subseteq \mathrm{Gal}(L_1 : K)$.

a. False
b. True

(12) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For field extensions $M : L : K$, where $M : K$ is finite and normal, the group $\mathrm{Gal}(L : K)$ is a quotient of the group $\mathrm{Gal}(M : K)$.

a. True
b. False

(13) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For field extensions $M : L : K$, where $M : K$ and $L : K$ are finite and normal, the group $\mathrm{Gal}(L : K)$ is a quotient of the group $\mathrm{Gal}(M : K)$.

a. False
b. True

(14) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Every splitting field extension is separable.

a. False
b. True

(15) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Every splitting field extension is algebraic.

a. False
b. True

(16) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Every splitting field extension in characteristic 0 is separable.

a. True
b. False

(17) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Every splitting field extension in characteristic > 0 is separable.

a. False
b. True

(18) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions and let H be a subgroup of $\mathrm{Gal}(M : K)$. If $L \subseteq \mathrm{Fix}(H)$ then $H \subseteq \mathrm{Gal}(M : L)$.

a. True
b. False

(19) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions and let H be a subgroup of $\mathrm{Gal}(M : K)$. If $\mathrm{Fix}(H) \subseteq L$ then $\mathrm{Gal}(M : L) \subseteq H$.

a. True
b. False

(20) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions. Then $L \subseteq \mathrm{Fix}(\mathrm{Gal}(M : L))$.

a. False
b. True

(21) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. Then $\mathrm{Fix}(\mathrm{Gal}(M : L)) \subseteq L$.

a. True
b. False

(22) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let H be a subgroup of $\mathrm{Gal}(M : K)$. Then $H \subseteq \mathrm{Gal}(M : \mathrm{Fix}(H))$.

a. True
b. False

(23) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let H be a subgroup of $\mathrm{Gal}(M : K)$. Then $\mathrm{Gal}(M : \mathrm{Fix}(H)) \subseteq H$.

a. False
b. True

(24) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f \in \mathbb{Q}[t]$ be a reducible polynomial of degree 5. Then $|\mathrm{Gal}_{\mathbb{Q}}(f)| \leq 24$.

a. True
b. False

(25) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a finite separable field extension $M : K$, the number of automorphisms of M over K is greater than or equal to the number of intermediate fields of $M : K$.

a. False
b. True

(26) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. If $L : K$ is also normal then $\mathrm{Gal}(M : K)/\mathrm{Gal}(L : K) \cong \mathrm{Gal}(M : L)$.

a. True
b. False

(27) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. If $L : K$ is also normal then $\mathrm{Gal}(M : K)/\mathrm{Gal}(M : L) \cong \mathrm{Gal}(L : K)$.

a. False
b. True

(28) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. Then L is a normal extension of K if and only if $\mathrm{Gal}(L : K)$ is a normal subgroup of $\mathrm{Gal}(M : K)$.

a. False
b. True

(29) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. Then L is a normal extension of K if and only if $\mathrm{Gal}(M : L)$ is a normal subgroup of $\mathrm{Gal}(M : K)$.

a. True
b. False

(30) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let $M : L_1 : L_2 : K$ be field extensions. If $[M : L_1] = [M : L_2] < \infty$ then $L_1 = L_2$.

a. True
b. False

(31) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let f be a polynomial over a field K, with splitting field M. Then any two distinct automorphisms of M over K must take different values on some root of f in M.

a. False
b. True

(32) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let f be a polynomial over a field K, with splitting field M. Let L be a subfield of M containing K, such that $L : K$ is normal. Then $|\mathrm{Gal}(L : K)| \leq |\mathrm{Gal}(M : K)|$.

a. False
b. True

(33) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? Let f be a polynomial over a field K, with splitting field M. Let L be a subfield of M containing K, such that $L : K$ is normal. Then every automorphism of L over K can be extended to an automorphism of M.

a. False
b. True

(34) **The fundamental theorem of Galois theory** [Multiple choice] [One answer only]

True or false? The intermediate fields of a finite normal separable extension $M : K$ correspond one-to-one with the elements of $\mathrm{Gal}(M : K)$.

a. False
b. True

(35) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? The Galois correspondence preserves inclusions.

a. False
b. True

(36) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f \in K[t]$ where K is a field of characteristic 0. Then $|\mathrm{Gal}_K(f)| = \deg(f)$.

a. False
b. True

(37) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f \in K[t]$. If $Df = 0$ then f is constant.

a. False
b. True

(38) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? The polynomial $t^3 - 2$ is separable over $\mathbb{F}_7$.

a. True
b. False

(39) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Every normal separable extension of prime degree has abelian Galois group.

a. False
b. True

(40) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For nonzero polynomials $f, g \in \mathbb{Q}[t]$, if $\mathrm{Gal}_{\mathbb{Q}}(f) \cong \mathrm{Gal}_{\mathbb{Q}}(g)$ then f is a constant multiple of g.

a. False
b. True

(41) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For $f, g \in \mathbb{Q}[t]$, if $\mathrm{Gal}_{\mathbb{Q}}(f) \cong \mathrm{Gal}_{\mathbb{Q}}(g)$ then both sides are isomorphic to $\mathrm{Gal}_{\mathbb{Q}}(fg)$.

a. True
b. False

(42) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For $f \in \mathbb{Q}[t]$, the groups $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(f^2)$ are isomorphic.

a. True
b. False

(43) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For $f \in \mathbb{Q}[t]$, the groups $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(tf)$ are isomorphic.

a. True
b. False

(44) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$ be irreducible polynomials. Then the order of $\mathrm{Gal}_{\mathbb{Q}}(fg)$ is the product of the orders of $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(g)$.

a. True
b. False

(45) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f \in \mathbb{Q}[t]$. If not all the complex roots of f are real then $|\mathrm{Gal}_{\mathbb{Q}}(f)|$ is even.

a. False
b. True

(46) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Rotation by $\pi/2$ about the origin defines an automorphism of the field $\mathbb{Q}(i)$.

a. False
b. True

(47) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f \in \mathbb{Q}[t]$ be a polynomial of degree 5 and M its splitting field. Then $M : \mathbb{Q}$ has at most 120 intermediate fields.

a. True
b. False

(48) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f \in \mathbb{Q}[t]$ be a polynomial and M its splitting field. Then $M : \mathbb{Q}$ has at most $\deg(f)$ intermediate fields.

a. True
b. False

(49) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : L$ and $L : K$ are finite, normal and separable, then so is $M : K$.

a. False
b. True

(50) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : K$ is finite, normal and separable, then so is $L : K$.

a. False
b. True

(51) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : K$ is finite, normal and separable, then so is $M : L$.

a. True
b. False

Total of marks: 51