Chapter 9: Solvability by radicals

(1) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? $\sqrt{i}$ is a respectable mathematical expression.

a. True
b. False

(2) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every root of a quadratic with rational coefficients is radical.

a. False
b. True

(3) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $\alpha \in \mathbb{C}$ and $n \geq 1$. If α is radical then so is α^n.

a. True
b. False

(4) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $\alpha \in \mathbb{C}$ and $n \geq 1$. If α^n is radical then so is α.

a. False
b. True

(5) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every polynomial over $\mathbb{Q}$ that splits in $\mathbb{Q}$ is solvable by radicals.

a. True
b. False

(6) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For all $n \geq 1$, the Galois group of $t^n - 1$ over $\mathbb{Q}$ is cyclic.

a. True
b. False

(7) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha)$ be a simple extension of a field K, let $L : K$ be any extension of K, and let $\phi, \psi \colon K(\alpha) \to L$ be homomorphisms over K. Then $\phi = \psi$ if and only if $\phi(\alpha) = \psi(\alpha)$.

a. True
b. False

(8) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For all $n \geq 1$ and $a \in \mathbb{Q}$, the Galois group of $t^n - a$ over $\mathbb{Q}$ is abelian.

a. True
b. False

(9) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension. Then every element of $\mathrm{Gal}(M : K)$ is a linear operator on the vector space M over K.

a. True
b. False

(10) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every finite normal extension of $\mathbb{Q}$ is solvable.

a. True
b. False

(11) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There is a subfield M of $\mathbb{C}$ that is solvable over $\mathbb{Q}$ and contains every other subfield of $\mathbb{C}$ solvable over $\mathbb{Q}$.

a. False
b. True

(12) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let M be a subfield of $\mathbb{C}$ such that $M : \mathbb{Q}$ is finite, normal and solvable. Let $a \in M$ and $n \geq 1$. Then $\mathrm{SF}_M(t^n - a) : \mathbb{Q}$ is also solvable.

a. True
b. False

(13) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The symmetric group S_n is generated by σ and τ, for any n-cycle σ and transposition τ.

a. False
b. True

(14) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The symmetric group S_n is generated by σ and τ, for any n-cycle σ and transposition $\tau = (i\ j)$ such that $\sigma(i) = j$.

a. False
b. True

(15) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every $f \in \mathbb{Q}[t]$, the degree of f divides the order of $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. False
b. True

(16) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every irreducible $f \in \mathbb{Q}[t]$, the degree of f divides the order of $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. True
b. False

(17) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In the symmetric group S_n, every power of an n-cycle is either an n-cycle or the identity.

a. False
b. True

(18) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In the symmetric group S_n, every power of a p-cycle (where p is prime) is either a p-cycle or the identity.

a. True
b. False

(19) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In the symmetric group S_n, every power of a cycle is a cycle.

a. True
b. False

(20) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If f and g are solvable by radicals then so is fg.

a. False
b. True

(21) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If fg is solvable by radicals then so are f and g.

a. True
b. False

(22) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If f is solvable by radicals then so is fg.

a. True
b. False

(23) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If the group $\mathrm{Gal}_{\mathbb{Q}}(fg)$ is solvable then so are $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(g)$.

a. False

b. True

(24) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. Then $\mathrm{Gal}_{\mathbb{Q}}(fg)$ is isomorphic to the direct product $\mathrm{Gal}_{\mathbb{Q}}(f) \times \mathrm{Gal}_{\mathbb{Q}}(g)$.

a. True
b. False

(25) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every reducible polynomial over $\mathbb{Q}$ of degree 5 is solvable by radicals.

a. False
b. True

(26) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every reducible polynomial over $\mathbb{Q}$ of degree 6 is solvable by radicals.

a. True
b. False

(27) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every radical number is algebraic.

a. False
b. True

(28) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There is a field extension with Galois group A_5.

a. False
b. True

(29) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$ that is solvable by radicals. Then the splitting field of f is a solvable extension of $\mathbb{Q}$.

a. True

b. False

(30) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let $f \in \mathbb{Q}[t]$. If $\mathrm{Gal}_{\mathbb{Q}}(f)$ is not solvable then none of the complex roots of f are radical.

a. True
b. False

(31) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let $\alpha, \beta \in \mathbb{C}$. If α and β are radical then so is $\alpha\beta$.

a. False
b. True

(32) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? For $\alpha \in \mathbb{C}$, if α is algebraic then α is radical.

a. False
b. True

(33) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Every splitting field extension of a polynomial over $\mathbb{Q}$ is solvable.

a. False
b. True

(34) **Solvability by radicals** [Multiple choice] [One answer only]

What is the smallest number k with the following property: for every $n \geq 3$, there is some k-element subset of S_n that generates S_n?

a. 2
b. 1
c. none of the other answers is correct.
d. 3

(35) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? No irreducible polynomial over $\mathbb{Q}$ of degree ≥ 5 is solvable by radicals.

a. False
b. True

(36) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? An irreducible cubic over $\mathbb{Q}$ with exactly one real root has Galois group of order 6.

a. False
b. True

(37) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? $(-2)^{1/4}$ is a respectable mathematical expression.

a. True
b. False

(38) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Every root of a cubic over $\mathbb{Q}$ is a radical number.

a. False
b. True

(39) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let K be a subfield of $\mathbb{C}$, let $a \in K$, and let $n \geq 1$. Let $M \subseteq \mathbb{C}$ be a splitting field of $t^n - a$ over K. If every element of K is radical then so is every element of M.

a. False
b. True

(40) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? For all $n \geq 1$, the Galois group of $t^n - 1$ over $\mathbb{Q}$ is abelian.

a. True
b. False

(41) **Solvability by radicals** [Multiple choice] [One answer only]

True or false? Let $L : K$ be a field extension and $Y \subseteq L$ with $L = K(Y)$. Let $M : K$ be another field extension, and let $\phi, \psi \colon L \to M$ be homomorphisms over K. If $\phi(y) = \psi(y)$ for all $y \in Y$ then $\phi = \psi$.

a. True
b. False

(42) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let K be a field, $0 \neq a \in K$, and $n \geq 1$. Then $t^n - a$ has n distinct roots in its splitting field over K.

a. True
b. False

(43) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For all $n \geq 1$, the Galois group of $t^n - 1$ over $\mathbb{Q}$ has order n.

a. True
b. False

(44) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every finite normal extension of $\mathbb{Q}$ is separable.

a. False
b. True

(45) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? A polynomial over $\mathbb{Q}$ with exactly 4 distinct roots in $\mathbb{C}$ is solvable by radicals.

a. False
b. True

(46) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? A polynomial over $\mathbb{Q}$ with Galois group A_5 is solvable by radicals.

a. True
b. False

(47) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? A polynomial over $\mathbb{Q}$ with Galois group of order 60 is unsolvable by radicals.

a. True
b. False

(48) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If every complex root of f is a root of g and the group $\mathrm{Gal}_{\mathbb{Q}}(g)$ is solvable then so is the group $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. False
b. True

(49) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every extension of $\mathbb{Q}$ of degree 2 is solvable.

a. False
b. True

(50) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every normal extension of $\mathbb{Q}$ of prime degree is solvable.

a. True
b. False

Total of marks: 50