Revision

(1) **Rings** Multiple choice | One answer only

True or false? You could write down the definition of the quotient ring (factor ring) R/I, for an ideal I of a commutative ring R.

a. True
b. False

(2) **Rings** Multiple choice | One answer only

True or false? You could write down the definition of an *ideal* in a commutative ring.

a. False
b. True

(3) **Linear algebra** Multiple choice | One answer only

True or false? Every surjective endomorphism of a finite-dimensional vector space is injective.

a. True
b. False

(4) **Linear algebra** Multiple choice | One answer only

True or false? Every injective endomorphism of a finite-dimensional vector space is surjective.

a. False
b. True

(5) **Linear algebra** Multiple choice | One answer only

True or false? Every injective endomorphism of a vector space is surjective.

a. True
b. False

(6) **Rings** Multiple choice | One answer only

Let I be an ideal of a commutative ring R, and let S be another commutative ring. Which of the following statements is true?

a. The homomorphisms $R/I \to S$ correspond naturally to the homomorphisms $\phi : R \to S$ such that $\ker \phi \supseteq I$.
b. The homomorphisms $R/I \to S$ correspond naturally to the homomorphisms $\phi : R \to S$ such that $\ker \phi \subseteq I$.
c. The homomorphisms $R/I \to S$ correspond naturally to the homomorphisms $\phi : R \to S$ such that $\ker \phi = I$.
d. None of the other statements is true.

(7) **Groups** [MULTIPLE CHOICE] [One answer only]

Let N be a normal subgroup of a group G, and let H be another group. Which of the following statements is true?

a. The homomorphisms $G/N \to H$ correspond naturally to the homomorphisms $\phi : G \to H$ such that $\ker \phi \supseteq N$.
b. None of the other statements is true.
c. The homomorphisms $G/N \to H$ correspond naturally to the homomorphisms $\phi : G \to H$ such that $\ker \phi \subseteq N$.
d. The homomorphisms $G/N \to H$ correspond naturally to the homomorphisms $\phi : G \to H$ such that $\ker \phi = N$.

(8) **Groups** [MULTIPLE CHOICE] [One answer only]

True or false? You could write down the definition of solvable group.

a. True
b. False

(9) **Groups** [MULTIPLE CHOICE] [One answer only]

True or false? Let G be a finite group and let H be a subgroup of G. Then the order of H divides the order of G.

a. False
b. True

(10) **Groups** [MULTIPLE CHOICE] [One answer only]

True or false? Let G be a finite group and let k be a positive integer that divides the order of G. Then G has a subgroup of order k.

a. True
b. False

(11) **Groups** Multiple choice | One answer only

True or false? Let G be a finite group of order 20. Then G has a subgroup of order 5.

a. False
b. True

(12) **Groups** Multiple choice | One answer only

Up to isomorphism, how many groups are there of order < 8?

a. 7
b. 8
c. 9
d. 6
e. 10
f. None of the other answers is correct.

(13) **Rings** Multiple choice | One answer only

True or false? Every ideal of a commutative ring R is a subring of R.

a. False
b. True

(14) **Rings** Multiple choice | One answer only

True or false? The set of units in a ring is closed under multiplication.

a. True
b. False

(15) **Polynomials** Multiple choice | One answer only

True or false? Let f and g be polynomials over a field K. If $f(a) = g(a)$ for all $a \in K$ then $f = g$.

a. True
b. False

(16) **Groups** Multiple choice | One answer only

What is the order of the smallest simple nonabelian group?

a. 12
b. 24
c. None of the other answers is correct.
d. 48
e. 6

(17) **Groups** MULTIPLE CHOICE One answer only

What is the order of the smallest non-solvable group?

a. 3
b. 60
c. 24
d. 120
e. None of the other answers is correct.

(18) **Rings** MULTIPLE CHOICE One answer only

True or false? Every integral domain is a field.

a. False
b. True

(19) **Rings** MULTIPLE CHOICE One answer only

True or false? Every field is an integral domain.

a. True
b. False

(20) **Rings** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Z}/n\mathbb{Z}$ is an integral domain if and only if it is a field, for integers $n \neq 0$.

a. False
b. True

(21) **Groups** MULTIPLE CHOICE One answer only

True or false? Every solvable group is abelian.

a. False
b. True

(22) **Groups** Multiple choice | One answer only

True or false? Every abelian group is solvable.

a. True
b. False

(23) **Groups** Multiple choice | One answer only

True or false? The group A_5 is solvable.

a. True
b. False

(24) **Groups** Multiple choice | One answer only

True or false? The group S_5 is solvable.

a. False
b. True

(25) **Groups** Multiple choice | One answer only

True or false? The group A_6 is solvable.

a. False
b. True

(26) **Groups** Multiple choice | One answer only

True or false? The group S_6 is solvable.

a. False
b. True

(27) **Groups** Multiple choice | One answer only

True or false? The group A_4 is solvable.

a. False
b. True

(28) **Groups** Multiple choice | One answer only

True or false? The group S_4 is solvable.

a. False

b. True

(29) **Groups** MULTIPLE CHOICE One answer only

True or false? The dihedral group of order 56 is solvable.

a. False
b. True

(30) **Groups** MULTIPLE CHOICE One answer only

True or false? Let G be a group, let H be a normal subgroup of G, and let K be a normal subgroup of H. Then K is a normal subgroup of G.

a. False
b. True

(31) **Groups** MULTIPLE CHOICE One answer only

True or false? Let G be a group, let H be a subgroup of G, and let K be a subgroup of H. Then K is a subgroup of G.

a. True
b. False

(32) **Groups** MULTIPLE CHOICE One answer only

True or false? Let G be a group, let H be a subgroup of G, and let K be a subgroup of H. If K is normal in G then K is normal in H.

a. False
b. True

(33) **Groups** MULTIPLE CHOICE One answer only

True or false? Let G be a group, let H be a subgroup of G, and let K be a subgroup of H. If K is normal in G then H is normal in G.

a. False
b. True

(34) **Groups** MULTIPLE CHOICE One answer only

True or false? You know what it means for one element of a group to be *conjugate* to another.

a. True
b. False

(35) **Groups** Multiple choice One answer only

True or false? You know what it means for one subgroup of a group to be *conjugate* to another.

a. True
b. False

(36) **Groups** Multiple choice One answer only

True or false? Let H and H' be subgroups of a group G. If H and H' are conjugate subgroups then they are isomorphic as groups.

a. True
b. False

(37) **Groups** Multiple choice One answer only

True or false? You could state the first isomorphism theorem for groups.

a. True
b. False

(38) **Rings** Multiple choice One answer only

True or false? You could state the first isomorphism theorem for rings.

a. False
b. True

(39) **Rings** Multiple choice One answer only

True or false? You could state the universal property of quotient rings (factor rings), as stated in Honours Algebra.

a. False
b. True

(40) **Linear algebra** Multiple choice One answer only

In the context of vector spaces, what is an *automorphism*?

a. A linear map from a vector space to itself.

b. A linear map from a vector space to itself that has an inverse.
c. A linear map that has an inverse.
d. None of the other answers is correct.
e. A linear map.

Total of marks: 40

Chapter 1: Overview of Galois theory

(1) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers $3 + 4i$ and $3 - 4i$ are conjugate over $\mathbb{R}$.

a. True
b. False

(2) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers 1, $2i$ and $1 + 2i$ are conjugate over $\mathbb{R}$.

a. False
b. True

(3) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers $7 + \sqrt{7}$ and $7 - \sqrt{7}$ are conjugate over $\mathbb{Q}$.

a. True
b. False

(4) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The numbers $7 + \sqrt{7}$ and $-7 - \sqrt{7}$ are conjugate over $\mathbb{Q}$.

a. False
b. True

(5) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The 5th roots of unity are all conjugate over $\mathbb{Q}$.

a. True
b. False

(6) **Conjugacy** MULTIPLE CHOICE One answer only

True or false? The 9th roots of unity, excluding 1, are all conjugate over $\mathbb{Q}$.

a. True
b. False

(7) **Conjugacy** Multiple choice | One answer only

True or false? The 19th roots of unity, excluding 1, are all conjugate over $\mathbb{Q}$.

a. True
b. False

(8) **Conjugacy** Multiple choice | One answer only

True or false? The triples $(1, 2i, 1+2i)$ and $(1, -2i, 1-2i)$ are conjugate over $\mathbb{Q}$.

a. True
b. False

(9) **Conjugacy** Multiple choice | One answer only

True or false? The triples $(1, 2i, 1+2i)$ and $(2i, 1+2i, 1)$ are conjugate over $\mathbb{Q}$.

a. True
b. False

(10) **Conjugacy** Multiple choice | One answer only

True or false? Let $z, z' \in \mathbb{C}$. If z and z' are conjugate over $\mathbb{R}$ then they are conjugate over $\mathbb{Q}$.

a. True
b. False

(11) **Conjugacy** Multiple choice | One answer only

True or false? Let $z, z' \in \mathbb{C}$. If z and z' are conjugate over $\mathbb{Q}$ then they are conjugate over $\mathbb{R}$.

a. False
b. True

(12) **Galois groups** Multiple choice | One answer only

What is the Galois group of the polynomial $t^2 - 3t + 2$?

a. the trivial group 1

b. C_3
c. S_3
d. C_2
e. none of the other answers

(13) **Galois groups** MULTIPLE CHOICE One answer only

What is the Galois group of the polynomial $t^2 - 2t + 3$?

a. C_3
b. C_2
c. S_3
d. the trivial group 1
e. none of the other answers

(14) **Galois groups** MULTIPLE CHOICE One answer only

What is the Galois group of the polynomial $(t + 2)(t + 1/2)(t - 2)(t - 1/2)$?

a. $C_2 \times C_2$
b. none of the other answers
c. the trivial group 1
d. C_4
e. S_4

(15) **Galois groups** MULTIPLE CHOICE One answer only

What is the Galois group of the polynomial $(t^5 - 1)/(t - 1)$?

a. S_4
b. none of the other answers
c. S_5
d. C_4
e. C_5

(16) **Galois groups** MULTIPLE CHOICE One answer only

True or false? The Galois group of a polynomial of degree n has order at most $n!$.

a. True
b. False

(17) **Galois groups** MULTIPLE CHOICE One answer only

True or false? The Galois group of a polynomial of degree n has order dividing $n!$.

a. False
b. True

(18) **Galois groups** MULTIPLE CHOICE One answer only

True or false? The Galois group of a polynomial of degree n has order at most n.

a. True
b. False

(19) **Solvability** MULTIPLE CHOICE One answer only

True or false? The Galois group of the polynomial $t^5 + 5$ is solvable.

a. False
b. True

(20) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the polynomial $(t^2-1)(t^6+2)$ solvable by radicals?

a. No
b. Yes
c. Not enough information in Chapter 1 to say

(21) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the polynomial

$$(5t^4 + 4t^3 - 3t^2 + 2t - 1)(t^3 + 8t - 14)$$

solvable by radicals?

a. No
b. Not enough information in Chapter 1 to say
c. Yes

(22) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the polynomial $t^5 - 6t + 3$ solvable by radicals?

a. No
b. Not enough information in Chapter 1 to say
c. Yes

(23) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the Galois group of the polynomial
$$(t^4 + 2t^2 - 2t - 1)(t^3 - 5t^2 - 3t + 15)$$
solvable?

a. No
b. Not enough information in Chapter 1 to say
c. Yes

(24) **Solvability** MULTIPLE CHOICE One answer only

From what you've been told in Chapter 1, is the Galois group of the polynomial $t^5 + 3t^4 - 2t^3 + 6t + 1$ solvable?

a. Yes
b. Not enough information in Chapter 1 to say
c. No

(25) **Solvability** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial of degree 4 with nontrivial Galois group G. Then the commutators $ghg^{-1}h^{-1}$ $(g, h \in G)$ generate G.

a. True
b. False

(26) **Solvability** MULTIPLE CHOICE One answer only

True or false? The number π is radical.

a. False
b. True

(27) **Solvability** MULTIPLE CHOICE One answer only

True or false? The number $(5+\sqrt{2})^{1/4} - 3^{1/6}$ is radical.

a. False
b. True

Total of marks: 27

Chapter 2: Group actions, rings and fields

(1) **Actions** MULTIPLE CHOICE | One answer only

True or false? For every group G and set X, there is some action of G on X.

a. True
b. False

(2) **Actions** MULTIPLE CHOICE | One answer only

True or false? For every nontrivial group G and set X with at least two elements, there exists a nontrivial action of G on X.

a. False
b. True

(3) **Actions** MULTIPLE CHOICE | One answer only

True or false? If a finite group G acts faithfully on a finite set X with n elements, then $|G| \leq n!$.

a. True
b. False

(4) **Actions** MULTIPLE CHOICE | One answer only

True or false? The actions of a group G on a set X correspond naturally to the homomorphisms $\mathrm{Sym}(X) \to G$.

a. False
b. True

(5) **Actions** MULTIPLE CHOICE | One answer only

True or false? A finite group cannot act transitively on an infinite set.

a. True
b. False

(6) **Rings** MULTIPLE CHOICE | One answer only

Which of the following statements is true?

a. There are no rings in which 0 is irreducible.

b. There are some rings in which 0 is irreducible, and some in which it is not.
c. In every ring, 0 is irreducible.

(7) **Rings** MULTIPLE CHOICE One answer only

Which of the following statements is true?

a. There are no rings in which 0 is reducible.
b. In every ring, 0 is reducible.
c. There are some rings in which 0 is reducible, and some in which it is not.

(8) **Rings** MULTIPLE CHOICE One answer only

Which of the following statements is true?

a. There are no rings in which 1 is irreducible.
b. In every ring, 1 is irreducible.
c. There are some rings in which 1 is irreducible, and some in which it is not.

(9) **Rings** MULTIPLE CHOICE One answer only

Which of the following statements is true?

a. There are some rings in which 1 is reducible, and some in which it is not.
b. There are no rings in which 1 is reducible.
c. In every ring, 1 is reducible.

(10) **Rings** MULTIPLE CHOICE One answer only

For rings R and S, let ψ_{RS} denote the function $R \to S$ defined by $\psi_{RS}(r) = 0$ for all $r \in R$. Which of the following statements is true?

a. ψ_{RS} is a homomorphism for all rings R and S.
b. There are no rings R and S for which ψ_{RS} is a homomorphism.
c. ψ_{RS} is a homomorphism for some rings R and S but not others.

(11) **Rings** MULTIPLE CHOICE One answer only

True or false? The subset $2\mathbb{Z}$ of $\mathbb{Z}$ is a subring.

a. True
b. False

(12) **Rings** MULTIPLE CHOICE One answer only

True or false? The subset $2\mathbb{Z}$ of $\mathbb{Z}$ is an ideal.

a. False
b. True

(13) **Rings** MULTIPLE CHOICE One answer only

True or false? The union of two subrings of a ring is a subring.

a. False
b. True

(14) **Rings** MULTIPLE CHOICE One answer only

True or false? Let R be a ring. Any ideal of R containing a unit must be equal to R.

a. True
b. False

(15) **Rings** MULTIPLE CHOICE One answer only

True or false? For every ring R, there is exactly one homomorphism $R \to \mathbb{Z}$.

a. False
b. True

(16) **Rings** MULTIPLE CHOICE One answer only

True or false? Every ideal of a ring is a subring.

a. False
b. True

(17) **Rings** MULTIPLE CHOICE One answer only

True or false? For a ring R and elements $r, u \in R$, if u is a unit then $\langle r \rangle = \langle ur \rangle$.

a. False

b. True

(18) **Rings** Multiple choice One answer only

True or false? In the ring $\mathbb{Z}$, every integer divides 0.

a. False
b. True

(19) **Rings** Multiple choice One answer only

True or false? If r is an element of a ring such that 0 divides r, then $r = 0$.

a. False
b. True

(20) **Rings** Multiple choice One answer only

True or false? In any ring, the product of two units is a unit.

a. False
b. True

(21) **Rings** Multiple choice One answer only

True or false? In any ring, 0 and 1 are coprime.

a. False
b. True

(22) **Fields** Multiple choice One answer only

How many ring homomorphisms $\mathbb{Q} \to \mathbb{Q}$ are there?

a. infinitely many
b. none of the other answers is correct
c. 0
d. 1
e. 2

(23) **Fields** Multiple choice One answer only

True or false? There exists a field of characteristic 11.

a. False

b. True

(24) **Fields** Multiple choice | One answer only

True or false? There exists a field of characteristic 12.

a. False
b. True

(25) **Fields** Multiple choice | One answer only

True or false? There exists a ring of characteristic 12.

a. True
b. False

(26) **Fields** Multiple choice | One answer only

True or false? There exists an integral domain of characteristic 12.

a. True
b. False

(27) **Fields** Multiple choice | One answer only

True or false? There exists a homomorphism of fields $\mathbb{Q} \to \mathbb{F}_2$.

a. False
b. True

(28) **Fields** Multiple choice | One answer only

How many ring homomorphisms $\mathbb{F}_2 \to \mathbb{F}_3$ are there?

a. 9
b. 1
c. 3
d. 8
e. none of the other answers is correct

(29) **Fields** Multiple choice | One answer only

Call a field K 'special' if the prime subfield of K is isomorphic to K. Up to isomorphism, how many infinite fields are special?

a. none of the other answers is correct

b. 0
c. infinitely many
d. 1

(30) **Fields** Multiple choice One answer only

True or false? Let K and L be fields. If there exists a homomorphism $K \to L$ then K and L have isomorphic prime subfields.

a. True
b. False

(31) **Fields** Multiple choice One answer only

True or false? Let K and L be fields. If there exist a field M and homomorphisms $K \to M \leftarrow L$ then K and L have isomorphic prime subfields.

a. True
b. False

(32) **Fields** Multiple choice One answer only

True or false? Every field of positive characteristic is finite.

a. False
b. True

(33) **Fields** Multiple choice One answer only

True or false? Every field of characteristic 0 is infinite.

a. True
b. False

(34) **Fields** Multiple choice One answer only

True or false? Every infinite field has characteristic 0.

a. True
b. False

(35) **Fields** Multiple choice One answer only

True or false? Every finite field has positive characteristic.

a. False
b. True

(36) **Fields** MULTIPLE CHOICE One answer only

True or false? Let R be a principal ideal domain. If r is an element of R such that $R/\langle r\rangle$ is a field then r is irreducible.

a. True
b. False

(37) **Fields** MULTIPLE CHOICE One answer only

True or false? Let R be a principal ideal domain. If r is an irreducible element of R then $R/\langle r\rangle$ is an integral domain.

a. True
b. False

Total of marks: 37

Chapter 3: Polynomials

(1) **Polynomials** Multiple choice One answer only

Let K be a field. Consider the statement 'if $f, g \in K[t]$ with $f(a) = g(a)$ for all $a \in K$, then $f = g$'. Which of the following holds?

a. None of the other answers is correct.
b. The statement is true for all fields K.
c. The statement is false when $K = \mathbb{F}_2$ and true for all other fields.
d. The statement is false when K is finite and true when K is infinite.
e. The statement is false for all fields K.

(2) **Polynomials** Multiple choice One answer only

True or false? n divides $\binom{n}{i}$ whenever $0 < i < n$ and n is prime.

a. False
b. True

(3) **Polynomials** Multiple choice One answer only

True or false? n divides $\binom{n}{i}$ whenever $0 < i < n$.

a. True
b. False

(4) **Polynomials** Multiple choice One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is irreducible over $\mathbb{Q}$ then f is irreducible over $\mathbb{Z}$.

a. False
b. True

(5) **Polynomials** Multiple choice One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is primitive and irreducible over $\mathbb{Q}$ then f is irreducible over $\mathbb{Z}$.

a. False
b. True

(6) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is irreducible over $\mathbb{Z}$ then f is irreducible over $\mathbb{Q}$.

a. False
b. True

(7) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is reducible over $\mathbb{Q}$ then f is reducible over $\mathbb{Z}$.

a. True
b. False

(8) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Z}[t]$. If f is irreducible over $\mathbb{Z}$ then f is primitive.

a. True
b. False

(9) **Polynomials** MULTIPLE CHOICE One answer only

True or false? For every nonzero polynomial f, the codegree of f is less than or equal to the degree of f.

a. True
b. False

(10) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in K[t]$. If $f(a) = g(a)$ for all $a \in K$, then the multiplicity of any root of f and g is the same for f and g.

a. True
b. False

(11) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let $n \in \mathbb{Z}$ and suppose that $\sqrt{n} \in \mathbb{Q}$. Then $\sqrt{n} \in \mathbb{Z}$.

a. False

b. True

(12) **Polynomials** Multiple choice | One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and write $\phi_* : R[t] \to S[t]$ for the induced homomorphism. Then $\deg(\phi_*(f)) = \deg(f)$ for all $f \in R[t]$.

a. True
b. False

(13) **Polynomials** Multiple choice | One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and write $\phi_* : R[t] \to S[t]$ for the induced homomorphism. Then $\deg(\phi_*(f)) \leq \deg(f)$ for all $f \in R[t]$.

a. False
b. True

(14) **Polynomials** Multiple choice | One answer only

True or false? For every ring R and element $r \in R$, there is a unique homomorphism $\phi : \mathbb{Z}[t] \to R$ such that $\phi(t) = r$.

a. False
b. True

(15) **Polynomials** Multiple choice | One answer only

True or false? Let $f \in \mathbb{Z}[t]$ and $g, h \in \mathbb{Q}[t]$. If $f = gh$ then $g, h \in \mathbb{Z}[t]$.

a. True
b. False

(16) **Polynomials** Multiple choice | One answer only

True or false? If p is prime then $1 + t + \cdots + t^{p-1}$ is irreducible over $\mathbb{Q}$.

a. False
b. True

(17) **Polynomials** Multiple choice | One answer only

True or false? If p is prime then $1 + t + \cdots + t^{p-1}$ is irreducible over $\mathbb{F}_p$.

a. False
b. True

(18) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If $n \geq 1$ then $1 + t + \cdots + t^{n-1}$ is irreducible over $\mathbb{Q}$.

a. False
b. True

(19) **Polynomials** MULTIPLE CHOICE One answer only

True or false? There is an irreducible polynomial over $\mathbb{Q}$ of degree 1000.

a. False
b. True

(20) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If R is a principal ideal domain then $R[t]$ is a principal ideal domain.

a. False
b. True

(21) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If K is a field then $K[t]$ is a field.

a. False
b. True

(22) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If R is an integral domain then $R[t]$ is an integral domain.

a. False
b. True

(23) **Polynomials** MULTIPLE CHOICE One answer only

True or false? If K is a field then $K[t]$ is a principal ideal domain.

a. False
b. True

(24) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f(t) \in K[t]$. If f has no roots in K then f is irreducible.

a. True
b. False

(25) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f(t) \in K[t]$. If f is irreducible then f has no roots in K.

a. True
b. False

(26) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $f(t) \in K[t]$ with $\deg(f) \geq 2$. If f is irreducible then f has no roots in K.

a. False
b. True

(27) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let K be a field and let f be a polynomial over K of degree 3. If f has no roots in K then f is irreducible.

a. True
b. False

(28) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let R be a ring and $f, g \in R[t]$. If f and g are equal as polynomials then f and g induce the same function $K \to K$.

a. False
b. True

(29) **Polynomials** MULTIPLE CHOICE One answer only

True or false? Let R be a ring and $f \in R[t]$. If $f(t)$ is reducible then $f(t+4)$ is reducible.

a. True
b. False

(30) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{R}$ is algebraically closed.

a. False
b. True

(31) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{F}_2$ is algebraically closed.

a. False
b. True

(32) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Z}[t]$ is a principal ideal domain.

a. True
b. False

(33) **Polynomials** MULTIPLE CHOICE One answer only

True or false? $\mathbb{Q}[t, u]$ is a principal ideal domain.

a. False
b. True

(34) **Polynomials** MULTIPLE CHOICE One answer only

True or false? There is a homomorphism $\phi : \mathbb{Q}[t] \to \mathbb{Q}[t]$ such that $\phi(t) = t^2$.

a. False
b. True

(35) **Polynomials** MULTIPLE CHOICE One answer only

True or false? There is an isomorphism $\phi : \mathbb{Q}[t] \to \mathbb{Q}[t]$ such that $\phi(t) = t^2$.

a. True
b. False

(36) **Polynomials** Multiple choice | One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and $r \in R$. If r is a unit then $\phi(r)$ is a unit.

a. True
b. False

(37) **Polynomials** Multiple choice | One answer only

True or false? Let $\phi : R \to S$ be a ring homomorphism, and $r \in R$. If $\phi(r)$ is a unit then r is a unit.

a. False
b. True

(38) **Polynomials** Multiple choice | One answer only

True or false? Let f be a polynomial over $\mathbb{Z}$, let p be a prime, and suppose that f reduced mod p is irreducible over $\mathbb{F}_p$. Then f is irreducible over $\mathbb{Q}$.

a. True
b. False

(39) **Polynomials** Multiple choice | One answer only

True or false? Let f be a monic polynomial over $\mathbb{Z}$, let p be a prime, and suppose that f reduced mod p is irreducible over $\mathbb{F}_p$. Then f is irreducible over $\mathbb{Q}$.

a. True
b. False

Total of marks: 39

Chapter 4: Field extensions

(1) **Field extensions** Multiple choice One answer only

True or false? For every monic irreducible polynomial $f \in \mathbb{Q}[t]$, there is some element of $\mathbb{C}$ whose minimal polynomial over $\mathbb{Q}$ is f.

a. True
b. False

(2) **Field extensions** Multiple choice One answer only

True or false? The complex conjugation map $\mathbb{C} \to \mathbb{C}$, given by $z \mapsto \overline{z}$, defines a field extension of $\mathbb{C}$ over itself.

a. False
b. True

(3) **Field extensions** Multiple choice One answer only

True or false? The set $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a subfield of $\mathbb{C}$.

a. True
b. False

(4) **Field extensions** Multiple choice One answer only

True or false? The set $\{a + b\sqrt{4} : a, b \in \mathbb{Q}\}$ is a subfield of $\mathbb{C}$.

a. False
b. True

(5) **Field extensions** Multiple choice One answer only

True or false? The set $\{a + b\sqrt[3]{2} : a, b \in \mathbb{Q}\}$ is a subfield of $\mathbb{C}$.

a. True
b. False

(6) **Field extensions** Multiple choice One answer only

True or false? For every field K, there exists a field containing $K[t]$ as a subring.

a. True

b. False

(7) **Field extensions** Multiple choice | One answer only

True or false? Let K be a field and $X \subseteq K$. If X is finite then the subfield of K generated by X is finite.

a. True
b. False

(8) **Field extensions** Multiple choice | One answer only

True or false? Let K be a field and $X \subseteq K$. If X is finite then the subfield of K generated by X is countable.

a. True
b. False

(9) **Field extensions** Multiple choice | One answer only

True or false? Let K be a field. The union of any family of subfields of K is a subfield.

a. True
b. False

(10) **Field extensions** Multiple choice | One answer only

True or false? The subfield of $\mathbb{C}$ generated by $\{i\}$ is $\mathbb{C}$ itself.

a. True
b. False

(11) **Field extensions** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and $Y \subseteq M$. Then $K(Y)$ is the largest subfield of M containing $K \cup Y$.

a. False
b. True

(12) **Field extensions** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and $Y \subseteq M$. Then $K(Y)$ is the smallest subfield of M containing $K \cup Y$.

a. True
b. False

(13) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $Y \subseteq M$. Then $K(Y)$ is the smallest subfield of M containing Y.

a. False
b. True

(14) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : \mathbb{Q}$ be a field extension and $Y \subseteq M$. Then $\mathbb{Q}(Y)$ is the smallest subfield of M containing Y.

a. False
b. True

(15) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : \mathbb{F}_p$ be a field extension and $Y \subseteq M$. Then $\mathbb{F}_p(Y)$ is the smallest subfield of M containing Y.

a. False
b. True

(16) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $X \subseteq Y \subseteq M$. Then $K(X) \subseteq K(Y)$.

a. True
b. False

(17) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions and $\alpha \in M$. If α is algebraic over K then α is algebraic over L.

a. False
b. True

(18) **Field extensions** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions and $\alpha \in M$. If α is algebraic over L then α is algebraic over K.

a. True
b. False

(19) **Field extensions** Multiple choice One answer only

True or false? For a field K, every element of the complement $K(t) \setminus K$ is transcendental over K.

a. True
b. False

(20) **Field extensions** Multiple choice One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. If α and β have the same sets of annihilating polynomials then either both are algebraic over K or both are transcendental over K.

a. False
b. True

(21) **Field extensions** Multiple choice One answer only

True or false? There is an element of $\mathbb{C}$ whose minimal polynomial over $\mathbb{Q}$ is $1 + 2t + 3t^2 + 4t^3 + 5t^4$.

a. True
b. False

(22) **Field extensions** Multiple choice One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. If α and β are both algebraic and have the same minimal polynomial, then they have the same sets of annihilating polynomials.

a. True
b. False

(23) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let α and β be complex numbers algebraic over $\mathbb{Q}$. Then α and β are conjugate over $\mathbb{Q}$ if and only if they have the same minimal polynomial.

a. False
b. True

(24) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every monic polynomial $f \in \mathbb{Q}[t]$, there is an element of $\mathbb{C}$ whose minimal polynomial over $\mathbb{Q}$ is f.

a. True
b. False

(25) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, let α be an element of the complement $M \setminus K$, and let $f \in K[t]$ be a monic quadratic that annihilates α. Then f is the minimal polynomial of α over K.

a. False
b. True

(26) **Field extensions** MULTIPLE CHOICE One answer only

Let $M : K$ be a field extension and let α be an element of M algebraic over K. What is the smallest possible degree of the minimal polynomial of α?

a. 2
b. 1
c. None of the other answers is correct.
d. 0

(27) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let $\alpha \in K$. Then the minimal polynomial of α over K has degree 0.

a. False
b. True

(28) **Field extensions** Multiple choice One answer only

True or false? For a prime p, the minimal polynomial of $e^{2\pi i/p}$ over $\mathbb{Q}$ is $t^p - 1$.

a. True
b. False

(29) **Field extensions** Multiple choice One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. If $K(\alpha) \cong K(\beta)$ then $\alpha = \beta$.

a. True
b. False

(30) **Field extensions** Multiple choice One answer only

Let $M : K$ be a field extension and let α and β be elements of M algebraic over K. Suppose that α and β have the same minimal polynomial. What is the relationship between $K(\alpha)$ and $K(\beta)$?

a. They are equal as subfields of M.
b. They are isomorphic as abstract fields, but not necessarily isomorphic over K.
c. They are isomorphic over K, but not necessarily equal as subfields of M.
d. None of the other answers is correct.
e. They are not necessarily isomorphic as abstract fields.

(31) **Field extensions** Multiple choice One answer only

True or false? $\mathbb{Q}(i) = \mathbb{Q}(1 + i)$ as subfields of $\mathbb{C}$.

a. True
b. False

(32) **Field extensions** Multiple choice One answer only

True or false? $\mathbb{Q}(\sqrt{5} + 5) = \mathbb{Q}(\sqrt{5} - 5)$ as subfields of $\mathbb{C}$.

a. False
b. True

(33) **Field extensions** MULTIPLE CHOICE One answer only

True or false? The extension $K(t) : K$ is simple for all fields K, where $K(t)$ is the field of rational expressions over K.

a. False
b. True

(34) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let α, β be distinct elements of M such that $M = K(\alpha, \beta)$. Then $M : K$ is not simple.

a. True
b. False

(35) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field and $m \in K[t]$. Then $K[t]/\langle m \rangle$ is a field.

a. True
b. False

(36) **Field extensions** MULTIPLE CHOICE One answer only

True or false? Let K be a field and let $M : K$ and $M' : K$ be extensions of K. If M and M' each contain an element transcendental over K, then $M \cong M'$.

a. False
b. True

(37) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists an extension $M : K$ such that 2 has a square root in M.

a. True
b. False

(38) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists an extension $M : K$ such that the equation $t^2 + t + 1 = 0$ has a solution in M.

a. True
b. False

(39) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K, there exists an extension $M : K$ such that the polynomial $t^5 - 6t + 3$ has a root in M.

a. False
b. True

(40) **Field extensions** MULTIPLE CHOICE One answer only

True or false? For every field K and nonconstant polynomial f over K, there exists an extension $M : K$ such that f has at least one root in M.

a. True
b. False

Total of marks: 40

Chapter 5: Degree

(1) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha) : K$ be a simple field extension. If α satisfies a quadratic equation over K then $[K(\alpha) : K] \in \{1, 2\}$.

a. True
b. False

(2) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha) : K$ be a simple field extension. If α satisfies a quadratic equation over K then $[K(\alpha) : K] = 2$.

a. False
b. True

(3) **Degree** MULTIPLE CHOICE One answer only

True or false? $[\mathbb{C} : \mathbb{Q}] = 2$.

a. True
b. False

(4) **Degree** MULTIPLE CHOICE One answer only

True or false? $[\mathbb{Q} : \mathbb{Q}] = 0$.

a. False
b. True

(5) **Degree** MULTIPLE CHOICE One answer only

True or false? $\deg_{\mathbb{Q}}(e^{2\pi i/p}) = p$ for all primes p.

a. True
b. False

(6) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha) : K$ be a simple field extension and $n \geq 1$. If α is a root of some polynomial over K of degree n then $[K(\alpha) : K] \leq n$.

a. True
b. False

(7) **Degree** MULTIPLE CHOICE One answer only

True or false? $M : K$ is finite whenever M is a simple extension of K.

a. True
b. False

(8) **Degree** MULTIPLE CHOICE One answer only

For $n \geq 1$, what is $[\mathbb{Q}(e^{2\pi i/n}) : \mathbb{Q}]$?

a. 1
b. n
c. none of the other answers is correct
d. $n - 1$
e. ∞

(9) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $\alpha, \beta \in \mathbb{Q}$ with $[\mathbb{Q}(\alpha) : \mathbb{Q}] = [\mathbb{Q}(\beta) : \mathbb{Q}] = 2$. Then $[\mathbb{Q}(\alpha, \beta) : \mathbb{Q}] = 4$.

a. True
b. False

(10) **Degree** MULTIPLE CHOICE One answer only

True or false? Let p and q be distinct primes. Then $[\mathbb{Q}(\sqrt{p}, \sqrt{q}) : \mathbb{Q}] = 4$.

a. True
b. False

(11) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha, \beta) : K(\alpha)] = [K(\beta) : K]$.

a. False
b. True

(12) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. Then $[L : K] \leq [M : K]$.

a. True
b. False

(13) **Degree** Multiple choice | One answer only

True or false? Let $M : L : K$ be field extensions. Then $[M : L] \leq [M : K]$.

a. False
b. True

(14) **Degree** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha + \beta) : K] \leq [K(\alpha, \beta) : K]$.

a. False
b. True

(15) **Degree** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha^2\beta - \alpha\beta^2) : K] \leq [K(\alpha, \beta) : K]$.

a. True
b. False

(16) **Degree** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha, \beta) : K(\alpha)] \geq [K(\beta) : K]$.

a. False
b. True

(17) **Degree** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and $\alpha, \beta \in M$. Then $[K(\alpha, \beta) : K(\alpha)] = [K(\beta) : K]$.

a. False
b. True

(18) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension such that $M = K(\alpha_1, \ldots, \alpha_n)$ for some $\alpha_1, \ldots, \alpha_n \in M$. Then every element of M can be expressed as a polynomial over K in $\alpha_1, \ldots, \alpha_n$.

a. False
b. True

(19) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions such that $[M : K]$ is a prime number. Then $L = M$ or $L = K$.

a. False
b. True

(20) **Degree** MULTIPLE CHOICE One answer only

True or false? There exist field extensions $M : L : K$ such that $M : K$ is infinite but $M : L$ and $L : K$ are finite.

a. False
b. True

(21) **Degree** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : K$ is finite then so is $M : L$.

a. False
b. True

(22) **Degree** MULTIPLE CHOICE One answer only

True or false? There exist field extensions $M : L : K$ such that $[M : K] = 6$ and $[M : L] = 4$.

a. False
b. True

(23) **Degree** MULTIPLE CHOICE One answer only

True or false? There exist field extensions $M : L : K : F$, all nontrivial, such that $[M : F] = 6$.

a. False
b. True

(24) **Degree** Multiple choice | One answer only

True or false? There exist field extensions $M : L : K$ such that $[M : K] = 6$ and $[M : L] = 3$.

a. True
b. False

(25) **Degree** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension and let $\alpha_1, \ldots, \alpha_n$ be elements of M algebraic over K. Then $[K(\alpha_1, \ldots, \alpha_n) : K] = [K(\alpha_1) : K] \cdots [K(\alpha_n) : K]$.

a. True
b. False

(26) **Degree** Multiple choice | One answer only

True or false? Every field extension that is finite is finitely generated.

a. False
b. True

(27) **Degree** Multiple choice | One answer only

True or false? Every field extension that is finitely generated is finite.

a. True
b. False

(28) **Degree** Multiple choice | One answer only

True or false? Every algebraic field extension is finite.

a. False
b. True

(29) **Degree** Multiple choice | One answer only

True or false? Every field extension that is finite is algebraic.

a. True

b. False

(30) **Degree** Multiple choice One answer only

True or false? Let $M : K$ be a field extension where M is a finite field. Then the extension is finite.

a. False
b. True

(31) **Degree** Multiple choice One answer only

True or false? Let $M : K$ be a field extension. If $M = K(\alpha, \beta, \gamma)$ for some $\alpha, \beta, \gamma \in M$, then $M : K$ is a finitely generated extension.

a. False
b. True

(32) **Degree** Multiple choice One answer only

True or false? Let $M : K$ be a field extension. If $M = K(\alpha, \beta, \gamma)$ for some $\alpha, \beta, \gamma \in M$, then $M : K$ is a finite extension.

a. True
b. False

(33) **Degree** Multiple choice One answer only

True or false? Let $M : K$ be a field extension. Every K-linear subspace of M containing K is a subfield of M.

a. False
b. True

(34) **Degree** Multiple choice One answer only

True or false? If a simple extension is algebraic, it is finite.

a. False
b. True

(35) **Degree** Multiple choice One answer only

True or false? Given two points in the plane distance L apart, one can use ruler and compasses to construct two points distance $\sqrt{2}L$ apart.

a. True
b. False

(36) **Degree** MULTIPLE CHOICE One answer only

True or false? Given two points in the plane distance L apart, one can use ruler and compasses to construct two points distance $2^{1/5}L$ apart.

a. True
b. False

(37) **Degree** MULTIPLE CHOICE One answer only

True or false? Given the points $(0, 0)$ and $(1, 0)$ in the plane, one can use ruler and compasses to construct the point $(e, 1/e)$.

a. False
b. True

(38) **Degree** MULTIPLE CHOICE One answer only

True or false? The set of points constructible by ruler and compasses from $(0, 0)$ and $(1, 0)$ is countable.

a. True
b. False

(39) **Degree** MULTIPLE CHOICE One answer only

True or false? If $M : K$ is a field extension of degree 2 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. True
b. False

(40) **Degree** MULTIPLE CHOICE One answer only

True or false? If $M : K$ is a field extension of degree 3 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. False
b. True

(41) **Degree** Multiple choice One answer only

True or false? If $M : K$ is a field extension of degree 4 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. True
b. False

(42) **Degree** Multiple choice One answer only

True or false? If $M : K$ is a field extension of degree 23 then $M = K(\alpha)$ for all $\alpha \in M \setminus K$.

a. True
b. False

(43) **Degree** Multiple choice One answer only

True or false? Every element of $\mathbb{C}$ algebraic over $\mathbb{Q}$ has a monic annihilating polynomial with coefficients in $\mathbb{Z}$.

a. True
b. False

(44) **Degree** Multiple choice One answer only

True or false? A regular 17-sided polygon can be constructed by ruler and compass (given two points in the plane).

a. False
b. True

(45) **Degree** Multiple choice One answer only

True or false? A regular 18-sided polygon can be constructed by ruler and compass (given two points in the plane).

a. True
b. False

(46) **Degree** Multiple choice One answer only

True or false? A regular 7-sided polygon can be constructed by ruler and compass (given two points in the plane).

a. False
b. True

Total of marks: 46

Chapter 6: Splitting fields

(1) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Given field extensions $\iota : K \to M$ and $\iota' : K' \to M'$, if ι and ι' are isomorphisms then every homomorphism $K \to K'$ extends to a homomorphism $M \to M'$.

a. True
b. False

(2) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $\psi : K \to K'$ be a homomorphism of fields. Then every polynomial over K' can be expressed as $\psi_*(f)$ for some $f \in K[t]$.

a. False
b. True

(3) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let K and K' be isomorphic fields. If K' is perfect then so is K.

a. True
b. False

(4) **Splitting fields** MULTIPLE CHOICE One answer only

How many automorphisms ϕ of $\mathbb{Q}(\sqrt{2})$ are there such that $\phi(\sqrt{2}) = -\sqrt{2}$?

a. 1
b. nothing in the notes so far gives us the answer
c. 0
d. 2

(5) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The splitting field of $f(t) \in K[t]$ over K is $K[t]/\langle f \rangle$.

a. True
b. False

(6) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The splitting field of $t^3 - 2$ over $\mathbb{Q}$ is $\mathbb{Q}(\sqrt[3]{2})$.

a. True
b. False

(7) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For a polynomial f over an algebraically closed field K, the splitting field of f over K is K.

a. True
b. False

(8) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and $f(t) \in K[t]$. If f has a root in M then f splits in M.

a. True
b. False

(9) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If M is a splitting field of $f(t) \in K[t]$, and f splits in L, then $L = M$.

a. True
b. False

(10) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The splitting field of a cubic over $\mathbb{Q}$ has degree 3 over $\mathbb{Q}$.

a. False
b. True

(11) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every polynomial f over a field K, the splitting field of f over K has degree $\deg(f)!$.

a. True

b. False

(12) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every polynomial f over a field K, the splitting field of f over K has degree $\geq \deg(f)!$.

a. False
b. True

(13) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For every polynomial f over a field K, the splitting field of f over K has degree $\leq \deg(f)!$.

a. False
b. True

(14) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Whenever M is the splitting field of some polynomial over K, there are at most $[M : K]$ automorphisms of M over K.

a. False
b. True

(15) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Whenever M is the splitting field of some polynomial over K, there are at most $[M : K]$ homomorphisms $M \to M$ over K.

a. True
b. False

(16) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The automorphism $a + b\sqrt{5} \mapsto a - b\sqrt{5}$ of $\mathbb{Q}(\sqrt{5})$ $(a, b \in \mathbb{Q})$ can be extended to an automorphism of $\mathbb{Q}(\sqrt{2}, \sqrt{5}, \sqrt{12})$.

a. False
b. True

(17) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? For a polynomial f of degree n over K, there are at most n automorphisms of $\mathrm{SF}_K(f)$ over K.

a. True
b. False

(18) **Splitting fields** Multiple choice One answer only

True or false? For a polynomial f of degree n over K, there are at most $n!$ automorphisms of $\mathrm{SF}_K(f)$ over K.

a. True
b. False

(19) **Splitting fields** Multiple choice One answer only

True or false? Let f be a polynomial over a field K and let M and M' be splitting fields of f over K. Then there is a unique isomorphism $M \to M'$ over K.

a. False
b. True

(20) **Splitting fields** Multiple choice One answer only

True or false? Let f be a polynomial over a field K and let M be a splitting field of f over K. Then there is a unique automorphism of M over K.

a. True
b. False

(21) **Splitting fields** Multiple choice One answer only

True or false? Let f be a polynomial over a field K and let M and M' be splitting fields of f over K. Then there exists an isomorphism $M \to M'$ over K.

a. True
b. False

(22) **Splitting fields** Multiple choice One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$ and let $\alpha \in \mathbb{C}$ be a root of f. Then $\mathrm{SF}_{\mathbb{Q}}(f)$ is a splitting field of f over $\mathbb{Q}(\alpha)$.

a. False

b. True

(23) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$. Let $\alpha \in \mathbb{C}$ be a complex number such that $f(\alpha) = 0$ but $f(-\alpha) \neq 0$. Then $\mathrm{SF}_{\mathbb{Q}}(f)$ is a splitting field of f over $\mathbb{Q}(-\alpha)$.

a. True
b. False

(24) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension. If θ and ϕ are automorphisms of M over K then $\theta \circ \phi$ is an automorphism of M over K.

a. False
b. True

(25) **Splitting fields** MULTIPLE CHOICE One answer only

Let $M : K$ and $M' : K$ be field extensions, and let $\theta : M \to M'$ be an isomorphism over K. Which of the following statements about $\theta^{-1} : M' \to M$ is true?

a. θ^{-1} is an isomorphism over K
b. θ^{-1} is an isomorphism of fields but not necessarily an isomorphism over K
c. θ^{-1} is a bijection but not necessarily a homomorphism of fields
d. none of the other answers is correct
e. θ^{-1} is a homomorphism of fields but not necessarily an isomorphism of fields

(26) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The field extension $\mathbb{C} : \mathbb{R}$ is finite.

a. True
b. False

(27) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? The field extension $\mathbb{C} : \mathbb{R}$ has finite Galois group.

a. False
b. True

(28) **Splitting fields** Multiple choice | One answer only

True or false? The field extension $\mathbb{R} : \mathbb{Q}$ is finite.

a. True
b. False

(29) **Splitting fields** Multiple choice | One answer only

True or false? For a simple extension $K(\alpha) : K$ with $\alpha \notin K$, the Galois group of $K(\alpha) : K$ is nontrivial.

a. False
b. True

(30) **Splitting fields** Multiple choice | One answer only

True or false? For every field extension $M : K$, there exists a polynomial f over K such that M is the splitting field of f over K.

a. False
b. True

(31) **Splitting fields** Multiple choice | One answer only

True or false? Let $M : K$ be a field extension. If $\alpha, \alpha' \in M$ are conjugate and α is algebraic then so is α'.

a. True
b. False

(32) **Splitting fields** Multiple choice | One answer only

True or false? Let $f(t) \in K[t]$ be a polynomial with distinct roots $\alpha_1, \ldots, \alpha_k$ in its splitting field. Then the natural homomorphism $\mathrm{Gal}_K(f) \to S_k$ is injective.

a. False
b. True

(33) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in K[t]$ be a polynomial with distinct roots $\alpha_1, \ldots, \alpha_k$ in its splitting field. Then the natural homomorphism $\mathrm{Gal}_K(f) \to S_k$ is surjective.

a. False
b. True

(34) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in K[t]$ be a polynomial with distinct roots $\alpha_1, \ldots, \alpha_k$ in its splitting field. Then the natural homomorphism $\mathrm{Gal}_K(f) \to S_k$ is an isomorphism.

a. True
b. False

(35) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Q}[t]$. Then $\mathrm{Gal}_{\mathbb{Q}(i)}(f)$ embeds naturally as a subgroup of $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. True
b. False

(36) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in \mathbb{Q}[t]$. Then $\mathrm{Gal}_{\mathbb{Q}}(f)$ embeds naturally as a subgroup of $\mathrm{Gal}_{\mathbb{Q}(i)}(f)$.

a. False
b. True

(37) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial of degree n over a field K. Then the order of the Galois group of f over K divides $n!$.

a. True
b. False

(38) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? There is a polynomial of degree 4 over $\mathbb{Q}$ whose Galois group is C_9.

a. True
b. False

(39) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? There is a polynomial of degree 3 over $\mathbb{Q}$ whose Galois group is $C_2 \times C_2$.

a. False
b. True

(40) **Splitting fields** MULTIPLE CHOICE One answer only

True or false? Let $f(t) \in K[t]$, let $\alpha \in \mathrm{SF}_K(f)$ be a root of f, and let $\theta \in \mathrm{Gal}_K(f)$. Then $\theta(\alpha)$ is a root of f.

a. True
b. False

Total of marks: 40

Chapter 7: Preparation for the fundamental theorem

(1) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? For an algebraic extension $M : K$, if M is algebraically closed then $M : K$ is normal.

a. True
b. False

(2) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Every normal extension is finite.

a. False
b. True

(3) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Every finite extension is normal.

a. True
b. False

(4) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Every field extension of prime degree is normal.

a. False
b. True

(5) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? In a normal extension $M : K$, every polynomial over K that has one root in M splits in M.

a. True
b. False

(6) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? In a normal extension $M : K$, every irreducible polynomial over K that has one root in M splits in M.

a. True
b. False

(7) **Preparation for the fundamental theorem** Multiple choice | One answer only

Let ξ be the real cube root of 2. Which of the following statements about the extension $\mathbb{Q}(\xi) : \mathbb{Q}$ is true?

a. It is separable but not normal
b. It is not normal or separable
c. It is normal and separable
d. It is normal but inseparable

(8) **Preparation for the fundamental theorem** Multiple choice | One answer only

Which of the following statements about the extension $\mathbb{Q}(e^{2\pi i/3}) : \mathbb{Q}$ is true?

a. It is separable but not normal
b. It is normal and separable
c. It is normal but inseparable
d. It is not normal or separable

(9) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let $M : K$ be a normal extension and $\alpha \in M$. Then the conjugacy class of α in M has the same cardinality as the conjugacy class of α in the splitting field of its minimal polynomial.

a. False
b. True

(10) **Preparation for the fundamental theorem** Multiple choice | One answer only

Let p be a prime and let $K = \mathbb{F}_p(u)$ be the field of rational expressions over $\mathbb{F}_p$. Extend K by a pth root α of u. Which of the following statements about the extension $K(\alpha) : K$ is true?

a. It is separable but not normal
b. It is normal and separable
c. It is not normal or separable
d. It is normal but inseparable

(11) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K. Then the splitting field extension $\mathrm{SF}_K(f) : K$ is finite, normal and separable.

a. False
b. True

(12) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? In the group of linear automorphisms of $\mathbb{R}^3$, any two rotations by an angle of $\pi/2$ are conjugate to one another.

a. False
b. True

(13) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? In the group of linear automorphisms of $\mathbb{R}^3$, any two rotations are conjugate to one another.

a. True
b. False

(14) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? In the group of rotations of a cube, any two rotations about axes through the midpoints of opposite faces are conjugate to one another.

a. False
b. True

(15) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? In the group of rotations of a cube, any two nontrivial rotations about axes through opposite vertices are conjugate to one another.

a. True
b. False

(16) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be algebraic extensions, with $L : K$ normal. Then L is a union of conjugacy classes in M over K.

a. True
b. False

(17) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Every splitting field extension is algebraic.

a. False
b. True

(18) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? Every splitting field extension is finitely generated.

a. True
b. False

(19) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

True or false? For every normal extension $M : K$, there is some $f \in K[t]$ such that M is a splitting field of f over K.

a. True
b. False

(20) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? For every finite normal extension $M : K$, there is some $f \in K[t]$ such that M is a splitting field of f over K.

a. False
b. True

(21) **Preparation for the fundamental theorem** Multiple choice One answer only

Let $M : L : K$ be field extensions, with $M : K$ finite and normal. Which of the following is true?

a. $M : L$ and $L : K$ are normal
b. neither $M : L$ nor $L : K$ need be normal
c. $L : K$ is normal, but $M : L$ need not be
d. $M : L$ is normal, but $L : K$ need not be

(22) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? For a field extension $M : K$, there is a nontrivial natural action of $\mathrm{Gal}(M : K)$ on K.

a. True
b. False

(23) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? For a field extension $M : K$, there is a natural action of $\mathrm{Gal}(M : K)$ on M.

a. False
b. True

(24) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? Let f be a polynomial over a field K, and let α and α' be two roots of f in its splitting field. Then $\alpha' = \varphi(\alpha)$ for some φ in $\mathrm{Gal}_K(f)$.

a. False
b. True

(25) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let f be an irreducible polynomial over a field K, and let α and α' be two roots of f in its splitting field. Then $\alpha' = \varphi(\alpha)$ for some φ in $\mathrm{Gal}_K(f)$.

a. True
b. False

(26) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? For all complex roots α and β of $f(t) = t^6 - 10t^4 + 15$, there is some element of $\mathrm{Gal}_{\mathbb{Q}}(f)$ that maps α to β.

a. True
b. False

(27) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? For all complex roots α and β of $f(t) = t^6 - 10t^4 + 15t$, there is some element of $\mathrm{Gal}_{\mathbb{Q}}(f)$ that maps α to β.

a. False
b. True

(28) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? For every irreducible cubic f over $\mathbb{Q}$, the Galois group $\mathrm{Gal}_{\mathbb{Q}}(f)$ is either A_3 or S_3.

a. False
b. True

(29) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions where $M : K$ and $L : K$ are both finite and normal. Then every element of $\mathrm{Gal}(M : K)$ restricts to an automorphism of L.

a. False
b. True

(30) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions where $M : K$ is finite and normal. Then every element of $\mathrm{Gal}(M : K)$ restricts to an automorphism of L.

a. False
b. True

(31) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions. Then $\mathrm{Gal}(M : L)$ is a subgroup of $\mathrm{Gal}(M : K)$.

a. False
b. True

(32) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions where $M : K$ and $L : K$ are both finite and normal. Then every automorphism of L over K extends to an automorphism of M over K.

a. False
b. True

(33) **Preparation for the fundamental theorem** Multiple choice One answer only

True or false? Let f be an irreducible polynomial over a field K. Then f has $\deg(f)$ distinct roots in its splitting field.

a. False
b. True

(34) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let f be an irreducible polynomial over $\mathbb{Q}$. Then f has $\deg(f)$ distinct roots in its splitting field.

a. True
b. False

(35) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$. Then f has $\deg(f)$ distinct roots in its splitting field.

a. False
b. True

(36) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let f be an irreducible polynomial over a field K. If f is inseparable then K must be an infinite field of positive characteristic.

a. False
b. True

(37) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Formal differentiation of polynomials over an arbitrary field satisfies the product rule (Leibniz rule).

a. True
b. False

(38) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let $f \in \mathbb{Q}[t]$. If f and its formal derivative Df are both divisible by $t^2 + 1$, then f has a repeated root in its splitting field.

a. True
b. False

(39) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? An irreducible polynomial over a field is separable if and only if its formal derivative is the zero polynomial.

a. True
b. False

(40) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? An algebraic extension $M : K$ is separable if and only if every irreducible polynomial over K is separable.

a. True
b. False

(41) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let $M : L : K$ be field extensions. If any two of $M : L$, $L : K$ and $M : K$ are algebraic, then so is the third.

a. False
b. True

(42) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let $M : K$ be a normal separable extension of degree 48. Then $\mathrm{Gal}(M : K)$ is solvable.

a. False
b. True

(43) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let M be a normal extension of $\mathbb{F}_3$. If M has 81 elements then $\mathrm{Gal}(M : \mathbb{F}_3)$ is abelian.

a. True
b. False

(44) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? Let $M : K$ be a normal separable extension. If $[M : K]$ is a prime number then $\mathrm{Gal}(M : K)$ is cyclic.

a. True
b. False

(45) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? There is some subgroup H of $\mathrm{Aut}(\mathbb{C})$ such that $\mathrm{Fix}(H) = \mathbb{Z}$.

a. False
b. True

(46) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? There is some subgroup H of $\mathrm{Aut}(\mathbb{C})$ such that $\mathrm{Fix}(H) = \mathbb{R}$.

a. False
b. True

(47) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? For a field M of characteristic p, the Galois group $\mathrm{Gal}(M : \mathbb{F}_p)$ is $\mathrm{Aut}(M)$.

a. True
b. False

(48) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? For a field M of characteristic 0, the Galois group $\mathrm{Gal}(M : \mathbb{Q})$ is $\mathrm{Aut}(M)$.

a. True
b. False

(49) **Preparation for the fundamental theorem** Multiple choice | One answer only

True or false? For a field M and a finite subgroup H of $\mathrm{Aut}(M)$, the extension $M : \mathrm{Fix}(H)$ is also finite.

a. True
b. False

(50) **Preparation for the fundamental theorem** Multiple choice | One answer only

Let $M : K$ be an algebraic field extension, with M algebraically closed. Which of the following statements is true?

a. $M : K$ need not be normal and need not be separable.
b. $M : K$ need not be normal but is separable.
c. $M : K$ is normal and separable.
d. $M : K$ is normal but need not be separable.

(51) **Preparation for the fundamental theorem** Multiple choice | One answer only

Let $M : K$ be a normal extension. Which of the following statements is true?

a. Every irreducible polynomial over K that has at least one root in M splits in M.
b. Every polynomial over K that has at least one root in M splits in M.
c. Every irreducible polynomial over K splits in M.
d. None of the other statements is true.
e. Every polynomial over K splits in M.

(52) **Preparation for the fundamental theorem** Multiple choice | One answer only

Exactly one of the following statements is *false*. Which one?

a. Every splitting field extension is algebraic.

b. Every splitting field extension is normal.
c. Every splitting field extension is finitely generated.
d. Let $M : L : K$ be algebraic extensions, with $L : K$ normal. Then L is a union of conjugacy classes in M over K.
e. For every normal extension $M : K$, there is some $f \in K[t]$ such that M is a splitting field of f over K.

(53) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

Exactly one of the following statements is true. Which one?

a. None of the other statements is true.
b. Let f be a polynomial over a field K, and let α and α' be two roots of f in its splitting field. Then $\alpha' = \phi(\alpha)$ for some $\phi \in \mathrm{Gal}_K(f)$.
c. For every field extension $M : K$, there is a nontrivial natural action of $\mathrm{Gal}(M : K)$ on K.
d. Let $M : L : K$ be field extensions, with $M : K$ finite and normal. Then every element of $\mathrm{Gal}(M : K)$ restricts to an automorphism of L.
e. For all complex roots α and β of $f(t) = t^6 - 10t^4 + 15$, there is some element of $\mathrm{Gal}_{\mathbb{Q}}(f)$ that maps α to β.

(54) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

Exactly one of the following statements is *false*. Which one?

a. Let $f \in \mathbb{Q}[t]$. Then f has $\deg(f)$ distinct roots in its splitting field.
b. Let f be an irreducible polynomial over a field K. If f is inseparable then K must have positive characteristic.
c. Let $f \in \mathbb{Q}[t]$. If f and its formal derivative Df are both divisible by $t^2 + 1$, then f has a repeated root in its splitting field.
d. Formal differentiation of polynomials over an arbitrary field satisfies the product rule (Leibniz rule).
e. Let f be an irreducible polynomial over a field K. If f is inseparable then K must be infinite.

(55) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

Which one of the following statements is *false*?

a. Let $M : L : K$ be field extensions. If any two of $M : L$, $L : K$ and $M : K$ are algebraic, then so is the third.
b. An irreducible polynomial over a field is separable if and only if its formal derivative is the zero polynomial.
c. Let $M : K$ be a normal separable extension of degree 48. Then $\mathrm{Gal}(M : K)$ is solvable.
d. Let M be a normal extension of $\mathbb{F}_3$. If M has 81 elements then $\mathrm{Gal}(M : \mathbb{F}_3)$ is abelian.

(56) **Preparation for the fundamental theorem** MULTIPLE CHOICE One answer only

Which one of the following statements is *false*?

a. There is a subgroup H of $\mathrm{Aut}(\mathbb{C})$ such that $\mathrm{Fix}(H) = \mathbb{Z}$.
b. For a field M and a finite subgroup H of $\mathrm{Aut}(M)$, the extension $M : \mathrm{Fix}(H)$ is finite.
c. For a field M of characteristic 0, the Galois group $\mathrm{Gal}(M : \mathbb{Q})$ is $\mathrm{Aut}(M)$.
d. For a field M of characteristic p, the Galois group $\mathrm{Gal}(M : \mathbb{F}_p)$ is $\mathrm{Aut}(M)$.

Total of marks: 56

Chapter 8: The fundamental theorem of Galois theory

(1) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? The order of the Galois group of a polynomial of degree n is divisible by n.

a. True
b. False

(2) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? The order of the Galois group of an irreducible polynomial of degree n is divisible by n.

a. False
b. True

(3) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For a field extension $M : K$, the Galois correspondence associates with each subgroup H of $\mathrm{Gal}(M : K)$ the intermediate field $\mathrm{Fix}(H)$.

a. True
b. False

(4) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For a field extension $M : K$, the Galois correspondence associates with each intermediate field L the group $\mathrm{Gal}(M : L)$.

a. False
b. True

(5) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For a field extension $M : K$, the Galois correspondence associates with each intermediate field L the group $\mathrm{Gal}(L : K)$.

a. True
b. False

(6) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? For a field extension $M : K$ and subgroups $H_1 \subseteq H_2 \subseteq \mathrm{Gal}(M : K)$, we have $\mathrm{Fix}(H_1) \subseteq \mathrm{Fix}(H_2)$.

a. True
b. False

(7) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? For a field extension $M : K$ and subgroups $H_1 \subseteq H_2 \subseteq \mathrm{Gal}(M : K)$, we have $\mathrm{Fix}(H_2) \subseteq \mathrm{Fix}(H_1)$.

a. True
b. False

(8) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(M : L_1) \subseteq \mathrm{Gal}(M : L_2)$.

a. True
b. False

(9) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(M : L_2) \subseteq \mathrm{Gal}(M : L_1)$.

a. True
b. False

(10) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(L_1 : K) \subseteq \mathrm{Gal}(L_2 : K)$.

a. True
b. False

(11) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For field extensions $M : L_1 : L_2 : K$, we have $\mathrm{Gal}(L_2 : K) \subseteq \mathrm{Gal}(L_1 : K)$.

a. False
b. True

(12) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For field extensions $M : L : K$, where $M : K$ is finite and normal, the group $\mathrm{Gal}(L : K)$ is a quotient of the group $\mathrm{Gal}(M : K)$.

a. True
b. False

(13) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For field extensions $M : L : K$, where $M : K$ and $L : K$ are finite and normal, the group $\mathrm{Gal}(L : K)$ is a quotient of the group $\mathrm{Gal}(M : K)$.

a. False
b. True

(14) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? Every splitting field extension is separable.

a. False
b. True

(15) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? Every splitting field extension is algebraic.

a. False
b. True

(16) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Every splitting field extension in characteristic 0 is separable.

a. True
b. False

(17) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Every splitting field extension in characteristic > 0 is separable.

a. False
b. True

(18) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions and let H be a subgroup of $\mathrm{Gal}(M : K)$. If $L \subseteq \mathrm{Fix}(H)$ then $H \subseteq \mathrm{Gal}(M : L)$.

a. True
b. False

(19) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions and let H be a subgroup of $\mathrm{Gal}(M : K)$. If $\mathrm{Fix}(H) \subseteq L$ then $\mathrm{Gal}(M : L) \subseteq H$.

a. True
b. False

(20) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. Then $L \subseteq \mathrm{Fix}(\mathrm{Gal}(M : L))$.

a. False
b. True

(21) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. Then $\mathrm{Fix}(\mathrm{Gal}(M : L)) \subseteq L$.

a. True
b. False

(22) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let H be a subgroup of $\mathrm{Gal}(M : K)$. Then $H \subseteq \mathrm{Gal}(M : \mathrm{Fix}(H))$.

a. True
b. False

(23) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension and let H be a subgroup of $\mathrm{Gal}(M : K)$. Then $\mathrm{Gal}(M : \mathrm{Fix}(H)) \subseteq H$.

a. False
b. True

(24) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $f \in \mathbb{Q}[t]$ be a reducible polynomial of degree 5. Then $|\mathrm{Gal}_{\mathbb{Q}}(f)| \leq 24$.

a. True
b. False

(25) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? For a finite separable field extension $M : K$, the number of automorphisms of M over K is greater than or equal to the number of intermediate fields of $M : K$.

a. False
b. True

(26) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. If $L : K$ is also normal then $\mathrm{Gal}(M : K)/\mathrm{Gal}(L : K) \cong \mathrm{Gal}(M : L)$.

a. True
b. False

(27) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. If $L : K$ is also normal then $\mathrm{Gal}(M : K)/\mathrm{Gal}(M : L) \cong \mathrm{Gal}(L : K)$.

a. False
b. True

(28) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. Then L is a normal extension of K if and only if $\mathrm{Gal}(L : K)$ is a normal subgroup of $\mathrm{Gal}(M : K)$.

a. False
b. True

(29) **The fundamental theorem of Galois theory** Multiple choice One answer only

True or false? Let $M : L : K$ be field extensions with $M : K$ finite, normal and separable. Then L is a normal extension of K if and only if $\mathrm{Gal}(M : L)$ is a normal subgroup of $\mathrm{Gal}(M : K)$.

a. True
b. False

(30) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L_1 : L_2 : K$ be field extensions. If $[M : L_1] = [M : L_2] < \infty$ then $L_1 = L_2$.

a. True
b. False

(31) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K, with splitting field M. Then any two distinct automorphisms of M over K must take different values on some root of f in M.

a. False
b. True

(32) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K, with splitting field M. Let L be a subfield of M containing K, such that $L : K$ is normal. Then $|\mathrm{Gal}(L : K)| \leq |\mathrm{Gal}(M : K)|$.

a. False
b. True

(33) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let f be a polynomial over a field K, with splitting field M. Let L be a subfield of M containing K, such that $L : K$ is normal. Then every automorphism of L over K can be extended to an automorphism of M.

a. False
b. True

(34) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? The intermediate fields of a finite normal separable extension $M : K$ correspond one-to-one with the elements of $\mathrm{Gal}(M : K)$.

a. False
b. True

(35) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? The Galois correspondence preserves inclusions.

a. False
b. True

(36) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? Let $f \in K[t]$ where K is a field of characteristic 0. Then $|\mathrm{Gal}_K(f)| = \deg(f)$.

a. False
b. True

(37) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? Let K be a field and $f \in K[t]$. If $Df = 0$ then f is constant.

a. False
b. True

(38) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? The polynomial $t^3 - 2$ is separable over $\mathbb{F}_7$.

a. True
b. False

(39) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? Every normal separable extension of prime degree has abelian Galois group.

a. False
b. True

(40) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For nonzero polynomials $f, g \in \mathbb{Q}[t]$, if $\mathrm{Gal}_{\mathbb{Q}}(f) \cong \mathrm{Gal}_{\mathbb{Q}}(g)$ then f is a constant multiple of g.

a. False
b. True

(41) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For $f, g \in \mathbb{Q}[t]$, if $\mathrm{Gal}_{\mathbb{Q}}(f) \cong \mathrm{Gal}_{\mathbb{Q}}(g)$ then both sides are isomorphic to $\mathrm{Gal}_{\mathbb{Q}}(fg)$.

a. True
b. False

(42) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For $f \in \mathbb{Q}[t]$, the groups $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(f^2)$ are isomorphic.

a. True
b. False

(43) **The fundamental theorem of Galois theory** Multiple choice | One answer only

True or false? For $f \in \mathbb{Q}[t]$, the groups $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(tf)$ are isomorphic.

a. True
b. False

(44) **The fundamental theorem of Galois theory** MULTIPLE CHOICE | One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$ be irreducible polynomials. Then the order of $\mathrm{Gal}_{\mathbb{Q}}(fg)$ is the product of the orders of $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(g)$.

a. True
b. False

(45) **The fundamental theorem of Galois theory** MULTIPLE CHOICE | One answer only

True or false? Let $f \in \mathbb{Q}[t]$. If not all the complex roots of f are real then $|\mathrm{Gal}_{\mathbb{Q}}(f)|$ is even.

a. False
b. True

(46) **The fundamental theorem of Galois theory** MULTIPLE CHOICE | One answer only

True or false? Rotation by $\pi/2$ about the origin defines an automorphism of the field $\mathbb{Q}(i)$.

a. False
b. True

(47) **The fundamental theorem of Galois theory** MULTIPLE CHOICE | One answer only

True or false? Let $f \in \mathbb{Q}[t]$ be a polynomial of degree 5 and M its splitting field. Then $M : \mathbb{Q}$ has at most 120 intermediate fields.

a. True
b. False

(48) **The fundamental theorem of Galois theory** MULTIPLE CHOICE | One answer only

True or false? Let $f \in \mathbb{Q}[t]$ be a polynomial and M its splitting field. Then $M : \mathbb{Q}$ has at most $\deg(f)$ intermediate fields.

a. True
b. False

(49) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : L$ and $L : K$ are finite, normal and separable, then so is $M : K$.

a. False
b. True

(50) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : K$ is finite, normal and separable, then so is $L : K$.

a. False
b. True

(51) **The fundamental theorem of Galois theory** MULTIPLE CHOICE One answer only

True or false? Let $M : L : K$ be field extensions. If $M : K$ is finite, normal and separable, then so is $M : L$.

a. True
b. False

Total of marks: 51

Chapter 9: Solvability by radicals

(1) **Solvability by radicals** Multiple choice | One answer only

True or false? $\sqrt{i}$ is a respectable mathematical expression.

a. True
b. False

(2) **Solvability by radicals** Multiple choice | One answer only

True or false? Every root of a quadratic with rational coefficients is radical.

a. False
b. True

(3) **Solvability by radicals** Multiple choice | One answer only

True or false? Let $\alpha \in \mathbb{C}$ and $n \geq 1$. If α is radical then so is α^n.

a. True
b. False

(4) **Solvability by radicals** Multiple choice | One answer only

True or false? Let $\alpha \in \mathbb{C}$ and $n \geq 1$. If α^n is radical then so is α.

a. False
b. True

(5) **Solvability by radicals** Multiple choice | One answer only

True or false? Every polynomial over $\mathbb{Q}$ that splits in $\mathbb{Q}$ is solvable by radicals.

a. True
b. False

(6) **Solvability by radicals** Multiple choice | One answer only

True or false? For all $n \geq 1$, the Galois group of $t^n - 1$ over $\mathbb{Q}$ is cyclic.

a. True
b. False

(7) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $K(\alpha)$ be a simple extension of a field K, let $L : K$ be any extension of K, and let $\phi, \psi \colon K(\alpha) \to L$ be homomorphisms over K. Then $\phi = \psi$ if and only if $\phi(\alpha) = \psi(\alpha)$.

a. True
b. False

(8) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For all $n \geq 1$ and $a \in \mathbb{Q}$, the Galois group of $t^n - a$ over $\mathbb{Q}$ is abelian.

a. True
b. False

(9) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension. Then every element of $\mathrm{Gal}(M : K)$ is a linear operator on the vector space M over K.

a. True
b. False

(10) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every finite normal extension of $\mathbb{Q}$ is solvable.

a. True
b. False

(11) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There is a subfield M of $\mathbb{C}$ that is solvable over $\mathbb{Q}$ and contains every other subfield of $\mathbb{C}$ solvable over $\mathbb{Q}$.

a. False
b. True

(12) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let M be a subfield of $\mathbb{C}$ such that $M : \mathbb{Q}$ is finite, normal and solvable. Let $a \in M$ and $n \geq 1$. Then $\mathrm{SF}_M(t^n - a) : \mathbb{Q}$ is also solvable.

a. True
b. False

(13) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The symmetric group S_n is generated by σ and τ, for any n-cycle σ and transposition τ.

a. False
b. True

(14) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The symmetric group S_n is generated by σ and τ, for any n-cycle σ and transposition $\tau = (i\ j)$ such that $\sigma(i) = j$.

a. False
b. True

(15) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every $f \in \mathbb{Q}[t]$, the degree of f divides the order of $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. False
b. True

(16) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every irreducible $f \in \mathbb{Q}[t]$, the degree of f divides the order of $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. True
b. False

(17) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In the symmetric group S_n, every power of an n-cycle is either an n-cycle or the identity.

a. False
b. True

(18) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In the symmetric group S_n, every power of a p-cycle (where p is prime) is either a p-cycle or the identity.

a. True
b. False

(19) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In the symmetric group S_n, every power of a cycle is a cycle.

a. True
b. False

(20) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If f and g are solvable by radicals then so is fg.

a. False
b. True

(21) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If fg is solvable by radicals then so are f and g.

a. True
b. False

(22) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If f is solvable by radicals then so is fg.

a. True
b. False

(23) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If the group $\mathrm{Gal}_{\mathbb{Q}}(fg)$ is solvable then so are $\mathrm{Gal}_{\mathbb{Q}}(f)$ and $\mathrm{Gal}_{\mathbb{Q}}(g)$.

a. False

b. True

(24) **Solvability by radicals** Multiple choice One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. Then $\mathrm{Gal}_{\mathbb{Q}}(fg)$ is isomorphic to the direct product $\mathrm{Gal}_{\mathbb{Q}}(f) \times \mathrm{Gal}_{\mathbb{Q}}(g)$.

a. True
b. False

(25) **Solvability by radicals** Multiple choice One answer only

True or false? Every reducible polynomial over $\mathbb{Q}$ of degree 5 is solvable by radicals.

a. False
b. True

(26) **Solvability by radicals** Multiple choice One answer only

True or false? Every reducible polynomial over $\mathbb{Q}$ of degree 6 is solvable by radicals.

a. True
b. False

(27) **Solvability by radicals** Multiple choice One answer only

True or false? Every radical number is algebraic.

a. False
b. True

(28) **Solvability by radicals** Multiple choice One answer only

True or false? There is a field extension with Galois group A_5.

a. False
b. True

(29) **Solvability by radicals** Multiple choice One answer only

True or false? Let f be a polynomial over $\mathbb{Q}$ that is solvable by radicals. Then the splitting field of f is a solvable extension of $\mathbb{Q}$.

a. True

b. False

(30) **Solvability by radicals** Multiple choice | One answer only

True or false? Let $f \in \mathbb{Q}[t]$. If $\mathrm{Gal}_{\mathbb{Q}}(f)$ is not solvable then none of the complex roots of f are radical.

a. True
b. False

(31) **Solvability by radicals** Multiple choice | One answer only

True or false? Let $\alpha, \beta \in \mathbb{C}$. If α and β are radical then so is $\alpha\beta$.

a. False
b. True

(32) **Solvability by radicals** Multiple choice | One answer only

True or false? For $\alpha \in \mathbb{C}$, if α is algebraic then α is radical.

a. False
b. True

(33) **Solvability by radicals** Multiple choice | One answer only

True or false? Every splitting field extension of a polynomial over $\mathbb{Q}$ is solvable.

a. False
b. True

(34) **Solvability by radicals** Multiple choice | One answer only

What is the smallest number k with the following property: for every $n \geq 3$, there is some k-element subset of S_n that generates S_n?

a. 2
b. 1
c. none of the other answers is correct.
d. 3

(35) **Solvability by radicals** Multiple choice | One answer only

True or false? No irreducible polynomial over $\mathbb{Q}$ of degree ≥ 5 is solvable by radicals.

a. False
b. True

(36) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? An irreducible cubic over $\mathbb{Q}$ with exactly one real root has Galois group of order 6.

a. False
b. True

(37) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? $(-2)^{1/4}$ is a respectable mathematical expression.

a. True
b. False

(38) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every root of a cubic over $\mathbb{Q}$ is a radical number.

a. False
b. True

(39) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let K be a subfield of $\mathbb{C}$, let $a \in K$, and let $n \geq 1$. Let $M \subseteq \mathbb{C}$ be a splitting field of $t^n - a$ over K. If every element of K is radical then so is every element of M.

a. False
b. True

(40) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For all $n \geq 1$, the Galois group of $t^n - 1$ over $\mathbb{Q}$ is abelian.

a. True
b. False

(41) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $L : K$ be a field extension and $Y \subseteq L$ with $L = K(Y)$. Let $M : K$ be another field extension, and let $\phi, \psi \colon L \to M$ be homomorphisms over K. If $\phi(y) = \psi(y)$ for all $y \in Y$ then $\phi = \psi$.

a. True
b. False

(42) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let K be a field, $0 \neq a \in K$, and $n \geq 1$. Then $t^n - a$ has n distinct roots in its splitting field over K.

a. True
b. False

(43) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For all $n \geq 1$, the Galois group of $t^n - 1$ over $\mathbb{Q}$ has order n.

a. True
b. False

(44) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every finite normal extension of $\mathbb{Q}$ is separable.

a. False
b. True

(45) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? A polynomial over $\mathbb{Q}$ with exactly 4 distinct roots in $\mathbb{C}$ is solvable by radicals.

a. False
b. True

(46) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? A polynomial over $\mathbb{Q}$ with Galois group A_5 is solvable by radicals.

a. True
b. False

(47) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? A polynomial over $\mathbb{Q}$ with Galois group of order 60 is unsolvable by radicals.

a. True
b. False

(48) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $f, g \in \mathbb{Q}[t]$. If every complex root of f is a root of g and the group $\mathrm{Gal}_{\mathbb{Q}}(g)$ is solvable then so is the group $\mathrm{Gal}_{\mathbb{Q}}(f)$.

a. False
b. True

(49) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every extension of $\mathbb{Q}$ of degree 2 is solvable.

a. False
b. True

(50) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every normal extension of $\mathbb{Q}$ of prime degree is solvable.

a. True
b. False

Total of marks: 50

Chapter 10: Finite fields

(1) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every $q \geq 2$ that is an integer power of a prime number, there is a field of order q.

a. True
b. False

(2) **Solvability by radicals** MULTIPLE CHOICE One answer only

Up to isomorphism, how many fields are there of order ≤ 20?

a. none of the other answers is correct
b. 13
c. 8
d. 10
e. 20

(3) **Solvability by radicals** MULTIPLE CHOICE One answer only

Up to isomorphism, how many fields are there whose order is ≥ 80 and ≤ 90?

a. 1
b. 4
c. 2
d. 3
e. none of the other answers is correct

(4) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $M : K$ is separable.

a. True
b. False

(5) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $M : K$ is normal.

a. True

b. False

(6) **Solvability by radicals** Multiple choice One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then M is the splitting field of some polynomial over K.

a. True
b. False

(7) **Solvability by radicals** Multiple choice One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $\mathrm{Gal}(M : K)$ is abelian.

a. True
b. False

(8) **Solvability by radicals** Multiple choice One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $\mathrm{Gal}(M : K)$ is simple.

a. False
b. True

(9) **Solvability by radicals** Multiple choice One answer only

True or false? Let p be a prime and let $q \geq 1$ be an integer. Then $(\alpha + \beta)^q = \alpha^q + \beta^q$ for all elements α, β of any field of characteristic p.

a. True
b. False

(10) **Solvability by radicals** Multiple choice One answer only

True or false? Let p be a prime and let $q \geq 1$ be an integer power of p. Then $(\alpha + \beta)^q = \alpha^q + \beta^q$ for all elements α, β of any field of characteristic p.

a. False
b. True

(11) **Solvability by radicals** Multiple choice | One answer only

True or false? For every field K of characteristic $p > 0$, the function $K \to K$ defined by $\alpha \mapsto \alpha^p$ is an isomorphism.

a. False
b. True

(12) **Solvability by radicals** Multiple choice | One answer only

True or false? For every finite field K of characteristic $p > 0$, the function $K \to K$ defined by $\alpha \mapsto \alpha^p$ is an isomorphism.

a. False
b. True

(13) **Solvability by radicals** Multiple choice | One answer only

True or false? For every field K of characteristic $p > 0$, the Frobenius map $K \to K$ has trivial kernel.

a. True
b. False

(14) **Solvability by radicals** Multiple choice | One answer only

True or false? For every prime p, the Frobenius map $\mathbb{F}_p(t) \to \mathbb{F}_p(t)$ is bijective.

a. False
b. True

(15) **Solvability by radicals** Multiple choice | One answer only

True or false? The field of four elements is $\mathbb{Z}/\langle 4\rangle$.

a. True
b. False

(16) **Solvability by radicals** Multiple choice | One answer only

True or false? In a finite field of characteristic 5, every element has a unique 5th root.

a. True

b. False

(17) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In a field of characteristic 5, every element has a unique 5th root.

a. False
b. True

(18) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? In a finite field of characteristic 5, every element has a unique 25th root.

a. True
b. False

(19) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? For every prime p and integer $n \geq 0$, there is precisely one field of order p^n (up to isomorphism).

a. True
b. False

(20) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The splitting field of $t^{p^n} - t$ over $\mathbb{F}_p$ has order n.

a. False
b. True

(21) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The splitting field of $t^{p^n} - t$ over $\mathbb{F}_p$ has degree p^n over $\mathbb{F}_p$.

a. True
b. False

(22) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $n \geq 1$. The splitting field of $t^{p^n} - t$ over $\mathbb{F}_p$ has degree n over $\mathbb{F}_p$.

a. False
b. True

(23) **Solvability by radicals** Multiple choice | One answer only

True or false? Let p be a prime and $n \geq 1$. The field extension $\mathrm{SF}_{\mathbb{F}_p}(t^{p^n} - t) : \mathbb{F}_p$ is simple.

a. True
b. False

(24) **Solvability by radicals** Multiple choice | One answer only

True or false? Let M be a finite field of order p^n, where p is prime and $n \geq 1$. Then $\alpha^p = \alpha$ for all $\alpha \in M$.

a. False
b. True

(25) **Solvability by radicals** Multiple choice | One answer only

True or false? Up to isomorphism, the number of fields of order 27 is equal to the number of monic irreducible cubic polynomials over $\mathbb{F}_3$.

a. False
b. True

(26) **Solvability by radicals** Multiple choice | One answer only

True or false? Let K be a field. Then every subgroup of the multiplicative group $K^\times$ is cyclic.

a. True
b. False

(27) **Solvability by radicals** Multiple choice | One answer only

True or false? Let K be a finite field. Then every subgroup of the multiplicative group $K^\times$ is cyclic.

a. True
b. False

(28) **Solvability by radicals** Multiple choice One answer only

True or false? Let K be a field and let H be a finite subgroup of $K^\times$. Then every element of H has finite order in the group $K^\times$.

a. False
b. True

(29) **Solvability by radicals** Multiple choice One answer only

True or false? Let p be a prime. Every element of $\mathbb{F}_p$ apart from 0 and ± 1 is a generator (in the group theory sense) of the multiplicative group $\mathbb{F}_p^\times$.

a. False
b. True

(30) **Solvability by radicals** Multiple choice One answer only

True or false? The field extension $\mathbb{F}_{343} : \mathbb{F}_7$ is simple.

a. False
b. True

(31) **Solvability by radicals** Multiple choice One answer only

True or false? Let p be a prime and $1 \leq m \leq n$. There are exactly m subfields of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(32) **Solvability by radicals** Multiple choice One answer only

True or false? Let p be a prime and $1 \leq m \leq n$. There is exactly one subfield of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(33) **Solvability by radicals** Multiple choice One answer only

True or false? Let p be a prime and $m, n \geq 1$ with $m|n$. There is exactly one subfield of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(34) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $m, n \geq 1$ with $m|n$. There are exactly m subfields of $\mathbb{F}_{p^n}$ of order p^m.

a. False
b. True

(35) **Solvability by radicals** MULTIPLE CHOICE One answer only

How many subfields does $\mathbb{F}_{16}$ have?

a. 3
b. none of the other answers is correct
c. 2
d. 4
e. 5

(36) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime. Then every polynomial in t^p over $\mathbb{F}_p$ has a pth root in $\mathbb{F}_p[t]$.

a. True
b. False

(37) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Every irreducible polynomial over a finite field is separable.

a. False
b. True

(38) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let $M : K$ be a field extension, with M finite. Then $\mathrm{Gal}(M : K)$ is cyclic of order $[M : K]$.

a. True
b. False

(39) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? $\mathbb{F}_{125}$ has a subfield with 25 elements.

a. True
b. False

(40) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? $\mathbb{F}_{625}$ has a subfield with 25 elements.

a. True
b. False

(41) **Solvability by radicals** MULTIPLE CHOICE One answer only

How many subgroups does C_{45} have?

a. 5
b. 3
c. 2
d. 45
e. none of the other answers is correct

(42) **Solvability by radicals** MULTIPLE CHOICE One answer only

How many subgroups of order 9 does C_{45} have?

a. 1
b. none of the other answers is correct
c. 2
d. 3
e. 0

(43) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $m, n \geq 1$, with $m|n$. Then the unique subfield of $\mathbb{F}_{p^n}$ of order p^m consists of the roots of $t^{p^m} - t$.

a. False
b. True

(44) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? Let p be a prime and $m, n \geq 1$, with $m|n$. Then the unique subfield of $\mathbb{F}_{p^n}$ of order p^m consists of the roots of $t^{p^{n/m}} - t$.

a. False
b. True

(45) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The unique subfield of $\mathbb{F}_{32}$ of order 4 consists of the elements x such that $x^4 = x$.

a. False
b. True

(46) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? The unique subfield of $\mathbb{F}_{32}$ of order 4 consists of the elements x such that $x^8 = x$.

a. True
b. False

(47) **Solvability by radicals** MULTIPLE CHOICE One answer only

Let p be a prime and $m, n \geq 1$, with $m < n$. How many homomorphisms $\mathbb{F}_{p^m} \to \mathbb{F}_{p^n}$ are there?

a. more than 1
b. depends on m and n
c. 1
d. none
e. none of the other answers is correct

(48) **Solvability by radicals** MULTIPLE CHOICE One answer only

Let p be a prime and $m, n \geq 1$, with $m|n$. How many homomorphisms $\mathbb{F}_{p^m} \to \mathbb{F}_{p^n}$ are there?

a. none of the other answers is correct
b. depends on m and n
c. 1
d. more than 1
e. none

(49) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There are exactly 6 automorphisms of the field with 64 elements.

a. True
b. False

(50) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There are exactly 6 homomorphisms $\mathbb{F}_{64} \to \mathbb{F}_{64}$

a. False
b. True

(51) **Solvability by radicals** MULTIPLE CHOICE One answer only

True or false? There are exactly 6 surjective homomorphisms $\mathbb{F}_{64} \to \mathbb{F}_{64}$

a. False
b. True

Total of marks: 51