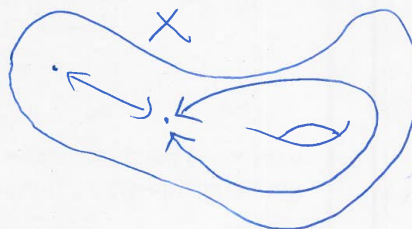


A_∞ -cats.

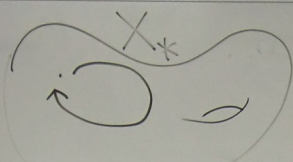
is. (strongly) homotopy assoc. cats

Model path groupoid
of top. space X

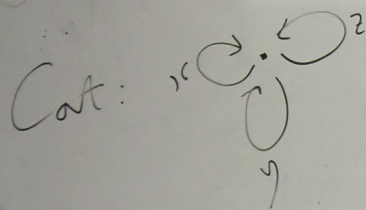
Q: what is the right
struct for $\mathcal{P}X$



Recall



fund. gp. $\pi_1(X_*)$



Cat:

Mor: loops at $*$
homotopy

$$x: I \rightarrow X$$

$$\{0, 1\} \mapsto *$$

compos?

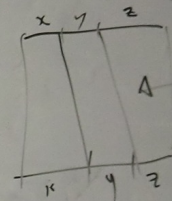
$$xy: I \rightarrow X$$

$$t \mapsto \begin{cases} x(2t) & t \leq \frac{1}{2} \\ y(2t-1) & t > \frac{1}{2} \end{cases}$$

assoc

$$(x \cdot y) \cdot z$$

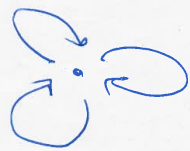
$$x \cdot (y \cdot z)$$



associat-

Want if ... want to keep homotopies?

A_∞ -algebra



composition

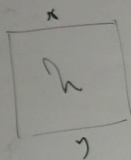
+ more struct ...

\longleftrightarrow alg A

\longleftrightarrow mult =

(eg) $A = C_*(LX_*) \longleftrightarrow \partial$ differential
chains on long sp.

① differential $C_1(LX) \rightarrow C_0(LX)$



$$\partial h = x - y$$

② composite

$$C_0(LX) \xrightarrow{\otimes 2} C_0(LX)$$

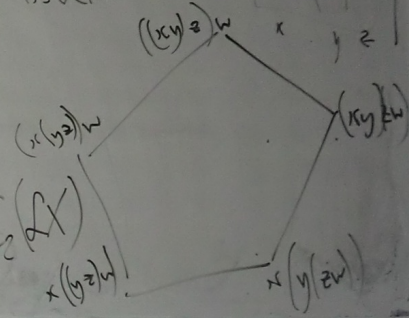
$$x \otimes y \mapsto xy$$

③ associatⁿ $C_2(LX) \rightarrow C_1(LX)$

$$x \otimes y \otimes z \mapsto$$

④ higher associatⁿ

$$C_0(LX) \xrightarrow{\otimes 4} C_0(LX)$$



Homology. perturb?

Take A_∞ -alg A

Homology (wrt m_1) $H_*(A)$

Thm place A_∞ -struct on $H_*(A)$

s.t. $\exists H_*(A) \xrightarrow{q_1} A$

Moral take $H_*(A)$ lose info.

higher prod m_k put it back.
(Massey prod)

Fukaya cat $\text{Fuk}(M)$

non-trivial
non-philosophical symplectic

Obj Lag $L \subset M$

Mor $L_1 \cap L_2 \rightsquigarrow \mathbb{Z}^{L_1, L_2}$

Diff. count ~~holo.~~ disc \rightarrow coeff differential

