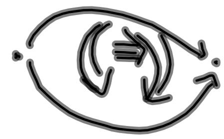
1. n-categories

Let $n \in \mathbb{N} \cup \{\infty\}$.

An n-cat consists of:

- some objects or 0-morphisms $\bullet$
- some 1-morphisms $\bullet \longrightarrow \bullet$
- some 2-morphisms
$$\begin{tikzcd} \bullet \arrow[r, bend left, ""{name=U}] \arrow[r, bend right, ""{name=D}] & \bullet \arrow[from=U, to=D, Rightarrow] \end{tikzcd}$$
- some 3-morphisms 

$\vdots$

- some n-morphisms

plus various ways of composing, e.g.

$A \xrightarrow{f} B \xrightarrow{g} C$ gives $A \xrightarrow{g \circ f} C$,

$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, bend right, "g"'] \arrow[r, phantom, "\Downarrow\alpha"] & B \arrow[r, bend left, "f'"] \arrow[r, bend right, "g'"'] \arrow[r, phantom, "\Downarrow\alpha'"] & C \end{tikzcd} \quad \text{gives} \quad \begin{tikzcd} A \arrow[r, bend left, "f' \circ f"] \arrow[r, bend right, "g' \circ g"'] \arrow[r, phantom, "\Downarrow"] & C \end{tikzcd}$$

$$\begin{tikzcd}[column sep=large] \cdot \arrow[r, bend left=60] \arrow[r] \arrow[r, bend right=60] \arrow[r, phantom, bend left=30, "\Downarrow"] \arrow[r, phantom, bend right=30, "\Downarrow"] & \cdot \end{tikzcd} \quad \text{gives} \quad \begin{tikzcd}[column sep=large] \cdot \arrow[r, bend left=60] \arrow[r, bend right=60] \arrow[r, phantom, "\Downarrow"] & \cdot \end{tikzcd}$$

such ~~that~~ every diagram like

$$\begin{tikzcd} \cdot \arrow[r, bend left=80] \arrow[r, bend left=25] \arrow[r, bend right=25] \arrow[r, bend right=80] \arrow[r, phantom, bend left=55, "\Downarrow\alpha"] \arrow[r, phantom, "\Downarrow\beta"] \arrow[r, phantom, bend right=55, "\Downarrow\gamma"] & \cdot \arrow[r, "g"] & \cdot \arrow[r, bend left=50] \arrow[r] \arrow[r, bend right=50] \arrow[r, phantom, bend left=25, "\Downarrow\delta"] \arrow[r, phantom, bend right=25, "\Downarrow\varepsilon"] & \cdot \end{tikzcd}$$

f

g

~~fog~~, fog

has an essentially unique composite

$$\begin{tikzcd}[column sep=large] \cdot \arrow[r, bend left=60] \arrow[r, bend right=60] \arrow[r, phantom, "\Downarrow"] & \cdot \end{tikzcd}$$

E.g. ∞-cat with:

- objs are spaces
- 1-morphisms are maps
- 2 ——— htys
- 3 ——— htys between htys

inv

$\vdots$

(n,k)-cat:

n-cat where i-morphisms $(i>k)$ are inv.

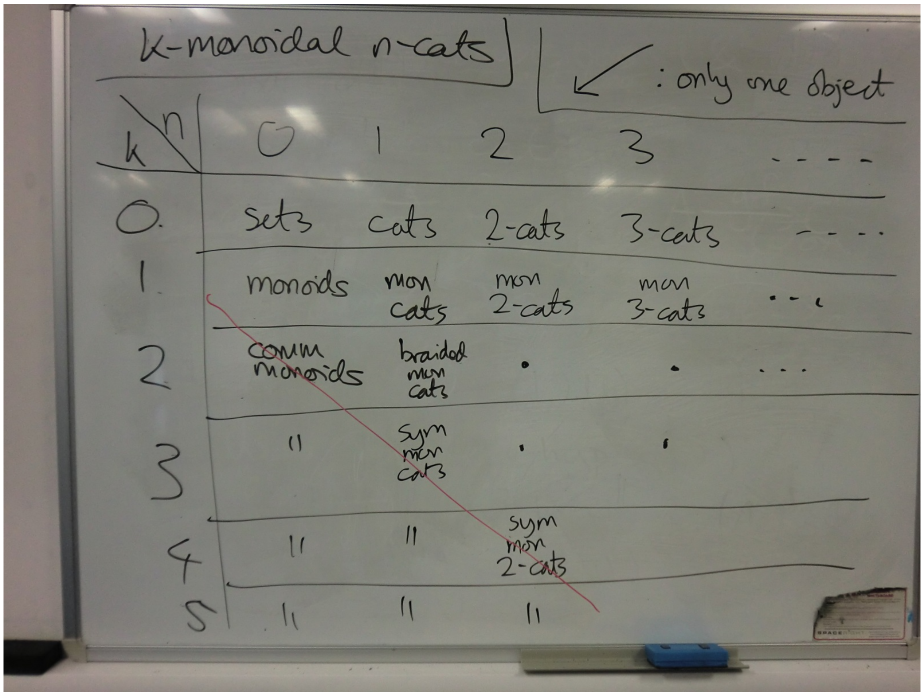
Lurie-∞-cat = $(\infty,1)$-cat

Eg 2-cat with

- objs are rings
- 1-morphisms $A \to B$ are (A,B)-bimods
- 2- ——— are homms.

$_A M_B \otimes_B {}_B N_C = {}_A (M \otimes_B N)_C$

Eg n-cats form an $(n+1)$-cat.



k-monoidal n-cats

↙ : only one object

k \ n	0	1	2	3	....
0	sets	cats	2-cats	3-cats	....
1	monoids	mon cats	mon 2-cats	mon 3-cats	...
2	comm monoids	braided mon cats	.	.	...
3	"	sym mon cats	.	.	
4	"	"	sym mon 2-cats		
5	"	"	"		