Space of Penrose tilings ref: Paul Smith 1104.3811

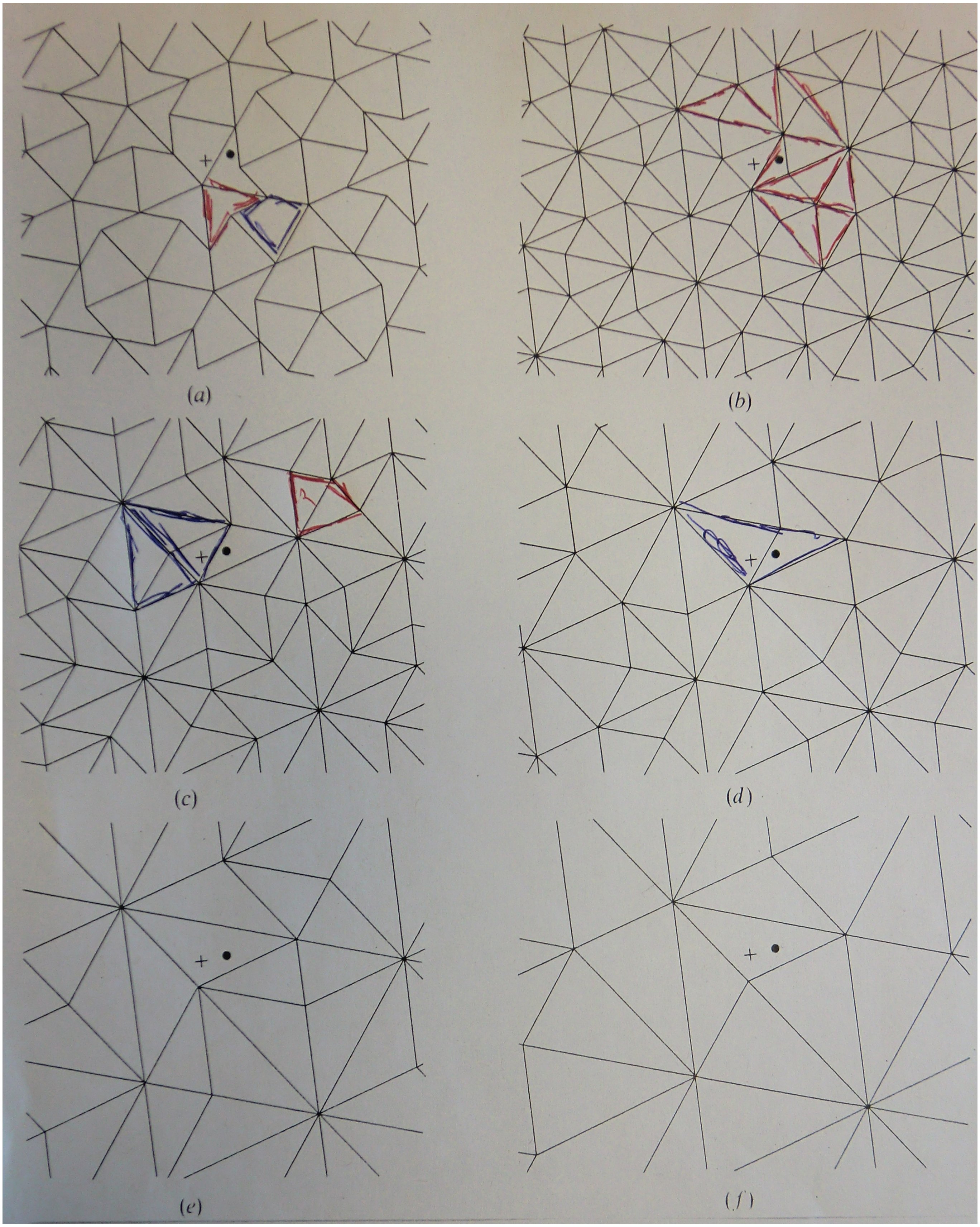
Facts (1) Can tile the plane in countably many ways

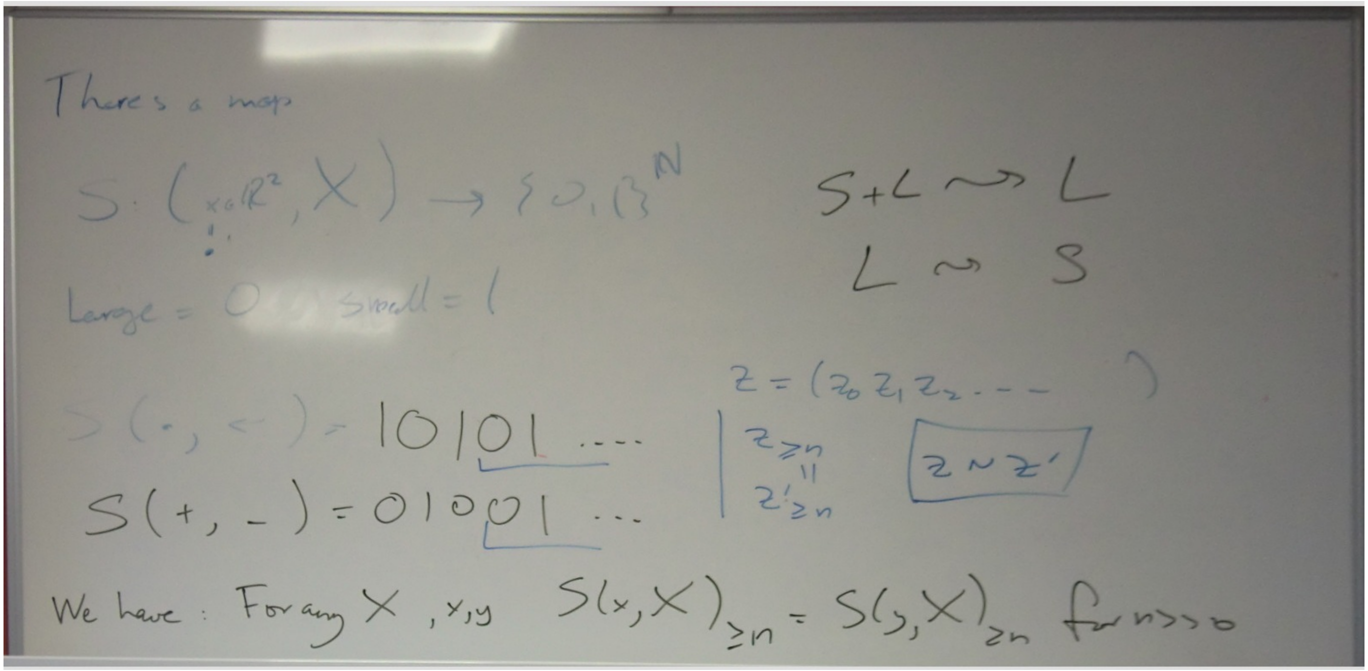
(2) Every tiling is aperiodic

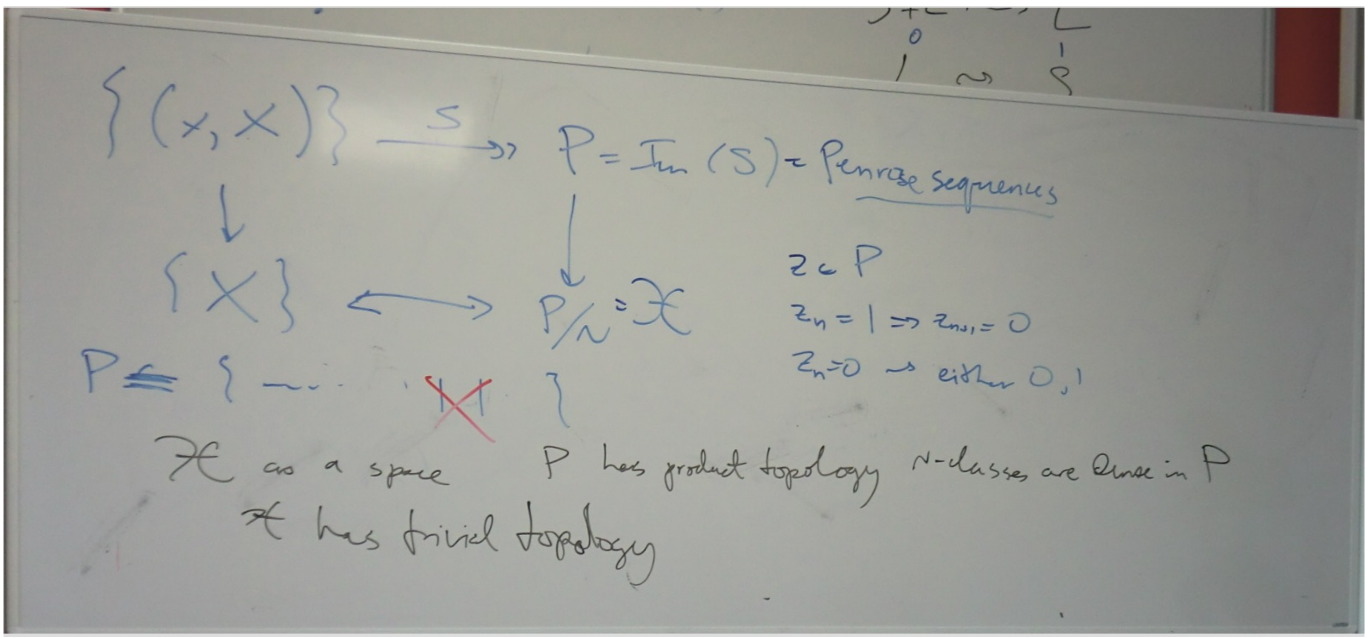
(2') no nontrivial translation of any tiling

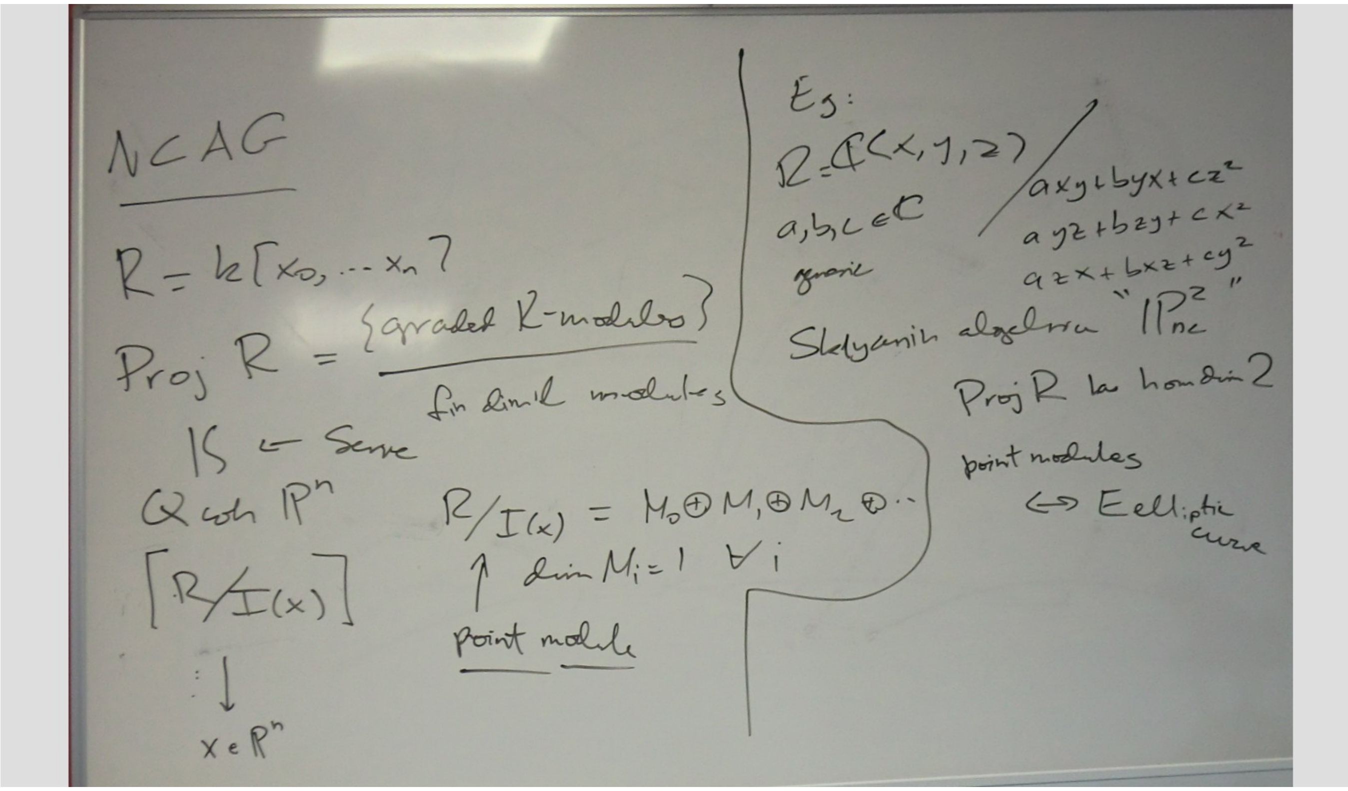
(3) Let X, Y be tilings. Every finite region of X occurs in Y

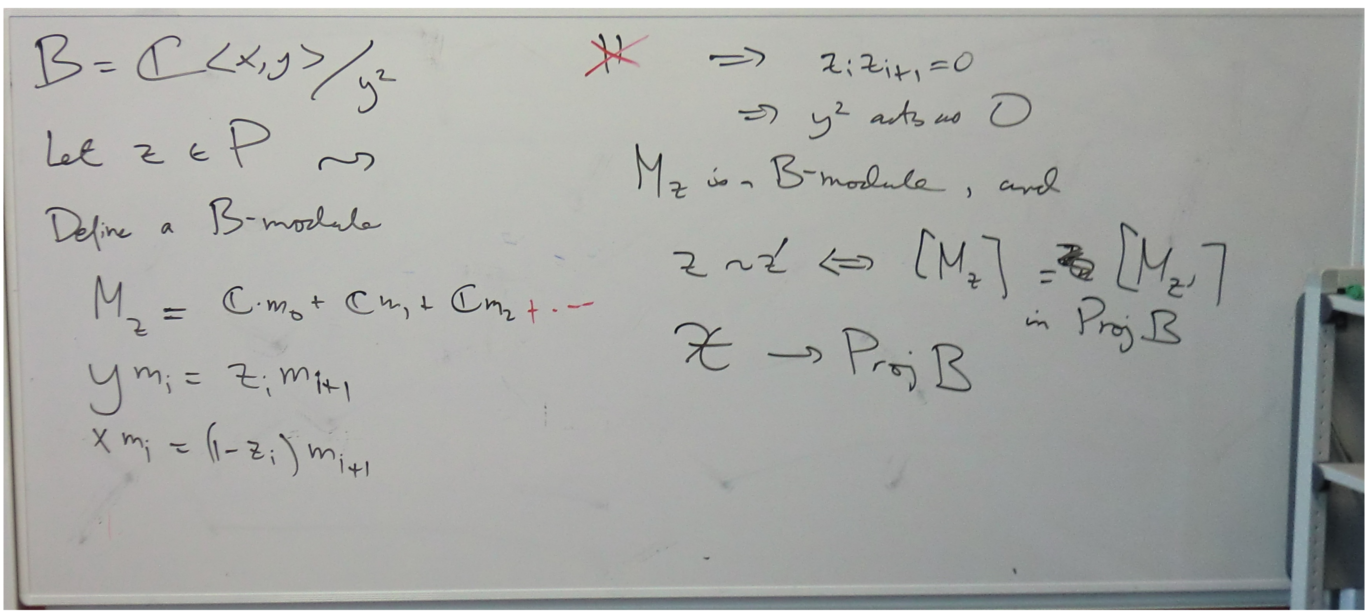
(∞ often)


(a)
(b)
(c)
(d)
(e)
(f)


There's a map
$S: (x \in \mathbb{R}^2, X) \to \{0,1\}^{\mathbb{N}}$
Large = 0, small = 1
$S+L \rightsquigarrow L$
$L \rightsquigarrow S$
$z = (z_0 z_1 z_2 \dots)$
$S(\cdot,\cdot) = 10101 \dots$
$z_{\geq n} = z'_{\geq n}$
$\boxed{z \sim z'}$
$S(+,-) = 01001 \dots$
We have: For any X, x,y $S(x,X)_{\geq n} = S(y,X)_{\geq n}$ for $n \gg 0$


$\{(x,X)\} \xrightarrow{S} P = \mathrm{Im}(S) =$ Penrose sequences
$\{X\} \longleftrightarrow P/\sim = \mathcal{X}$
$z \in P$
$z_n = 1 \Rightarrow z_{n+1} = 0$
$z_n = 0 \rightsquigarrow$ either $0, 1$
$P \subseteq \{ \dots \}$
$\mathcal{X}$ as a space P has product topology $\sim$-classes are dense in P
$\mathcal{X}$ has trivial topology


NCAG
$R = k[x_0, \dots x_n]$
$\mathrm{Proj}\, R = \dfrac{\{\text{graded } R\text{-modules}\}}{\text{fin dim'l modules}}$
$\cong$ ← Serre
Qcoh $\mathbb{P}^n$
$[R/I(x)]$
$x \in \mathbb{P}^n$
$R/I(x) = M_0 \oplus M_1 \oplus M_2 \oplus \cdots$
$\dim M_i = 1 \ \forall i$
point module
Eg:
$R = \mathbb{C}\langle x,y,z\rangle \Big/ \begin{matrix} axy+byx+cz^2 \\ ayz+bzy+cx^2 \\ azx+bxz+cy^2 \end{matrix}$
$a,b,c \in \mathbb{C}$
generic
Sklyanin algebra "$\mathbb{P}^2_{nc}$"
Proj R has hom dim 2
point modules
⟺ Elliptic curve


$B = \mathbb{C}\langle x,y\rangle / y^2$
Let $z \in P$ ⇝
Define a B-module
$M_z = \mathbb{C}\cdot m_0 + \mathbb{C}m_1 + \mathbb{C}m_2 + \cdots$
$y\, m_i = z_i\, m_{i+1}$
$x\, m_i = (1-z_i)\, m_{i+1}$
$\Rightarrow z_i z_{i+1} = 0$
$\Rightarrow y^2$ acts as 0
M_z is a B-module, and
$z \sim z' \iff [M_z] = [M_{z'}]$ in Proj B
$\mathcal{X} \to \mathrm{Proj}\, B$

Facts:

(1) $\mathrm{Ext}^1_{\mathrm{Proj}\,B}(M_z, M_{z'}) \neq 0 \leftrightsquigarrow z \in \overline{\{z'\}}$

(2) Alain Connes $\mathfrak{X} \rightsquigarrow S$ C^*-alg of "functions on $\mathfrak{X}$"

$\mathrm{Proj}\, B \simeq \mathrm{Mod}\text{-}S$ "Proj B is affine"

(3) In Proj B have $\mathcal{O} \cong \mathcal{O}(-1) \oplus \mathcal{O}(-2)$