Unexpected connections

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These slides: www.maths.ed.ac.uk/∼tl/perth

Keep yourself open

and don't neglect your larger self

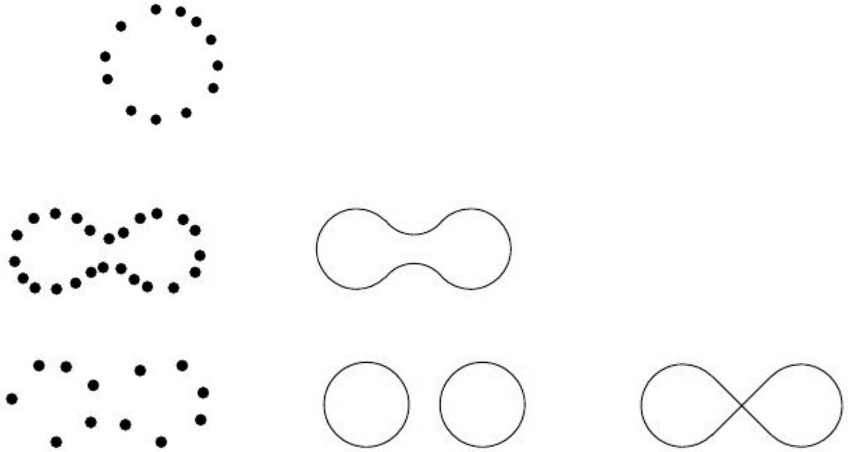
Applied maths

$\neq$ *applied differential equations*

$\neq$ *differential equations applied to physical problems*

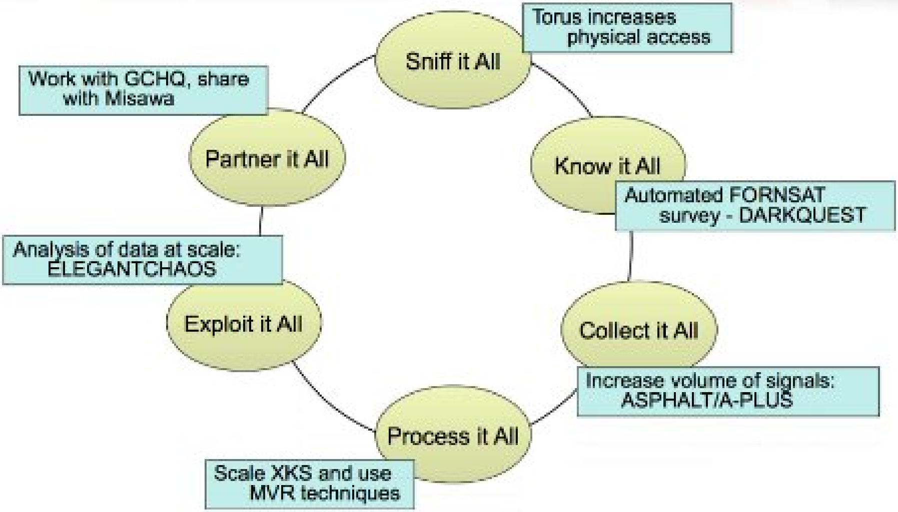
Algebraic topology of data sets

"What's this?" I ask:



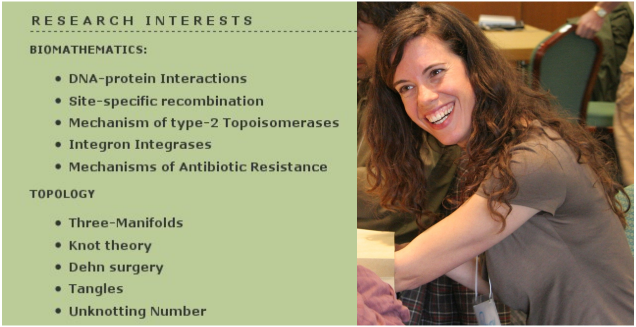
[extracted from article by Vin de Silva]

New Collection Posture



DNA knotting

From the website of Dorothy Buck (Imperial):


RESEARCH INTERESTS

BIOMATHEMATICS:

- DNA-protein Interactions
- Site-specific recombination
- Mechanism of type-2 Topoisomerases
- Integron Integrases
- Mechanisms of Antibiotic Resistance

TOPOLOGY

- Three-Manifolds
- Knot theory
- Dehn surgery
- Tangles
- Unknotting Number

FIGURE 1. In these examples the recombinase complex B meets the substrate in the two crossover sites (highlighted in black).

[source: Buck and Flapan]

A few other applications of mathematics

- Algebraic topology and real algebraic geometry for robotics [link]
- Group-theoretic classification of insect gaits [link]
- Quantum field theory can predict patterns of biodiversity [link]

Applied maths
can help
pure maths

Random matrices and the Riemann zeta function

The year: 1972.

The scene: Afternoon tea at the Institute for Advanced Study, Princeton.

Freeman Dyson, dapper British physicist: 'So tell me, Montgomery, what have you been up to?'

Hugh Montgomery, boyish American mathematician: 'Well, lately I've been looking into the distribution of the zeros of the Riemann zeta function.'

Dyson: 'Yes? And?'

Montgomery: 'It seems the two-point correlations go as...' *(turning to write on a nearby blackboard)*

Dyson: Extraordinary! Do you realize that's the pair-correlation function for the eigenvalues of a random Hermitian matrix? It's also a model of the energy levels in a heavy nucleus — say uranium 238.

[adapted from web page by Bob McLeod]

Random matrices and the Riemann zeta function

[source: Bob McLeod]

Two more examples of applied helping pure

- The *Gruppenpest* (plague of groups) [link]

- My collaborator Mark Meckes: [link]

We end this section by considering a quantity related to magnitude which is in some ways better behaved. For a compact (not necessarily positive definite) metric space A, the **maximum diversity** of A is

$$|A|_+ = \sup_{\mu \in P(A)} \left(\int \int e^{-d(a,b)} \, d\mu(a) \, d\mu(b) \right)^{-1}, \tag{4.3}$$

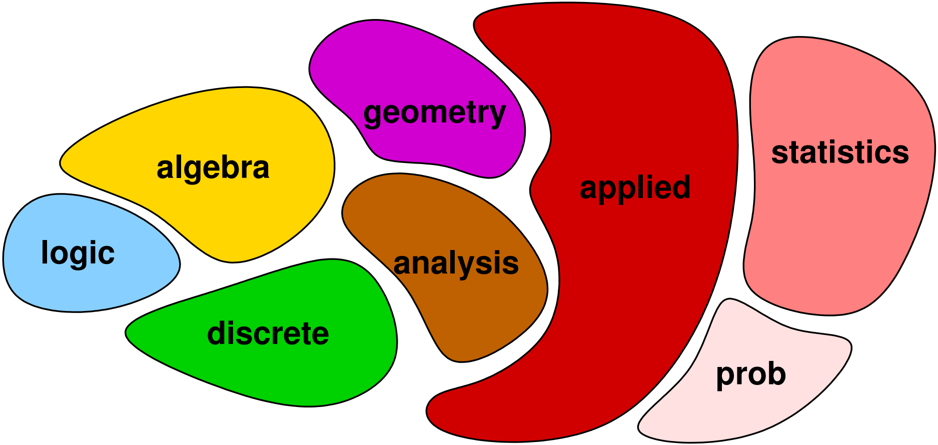
where $P(A)$ denotes the space of Borel probability measures on A. By renormalization, this is simply what one obtains by restricting the supremum in (3.5) to positive measures; thus we trivially have

$$|A|_+ \le |A| \tag{4.4}$$

for any compact PDMS A. The name stems from the following interpretation of the quantity

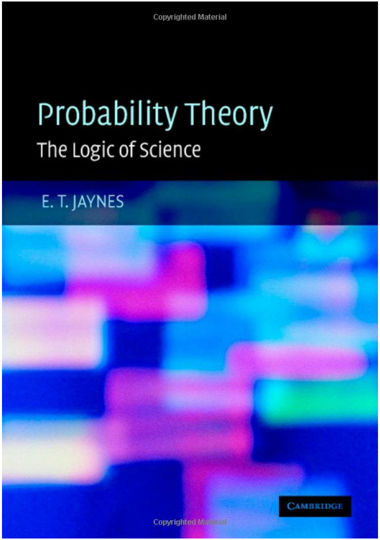
Prepare to rewire your brain

We all have a mental map of mathematics. . .



. . . but it can be misleading.

Statistical inference as a branch of logic



If A is true, then B becomes more plausible

B is true

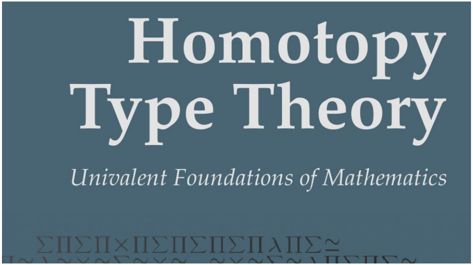
---

therefore, A becomes more plausible.

[Jaynes link]

Knowing unusual combinations of subjects gives you an advantage

Logic + topology + computer programming



Peter Aczel
Benedikt Ahrens
Thorsten Altenkirch
Steve Awodey
Bruno Barras
Andrej Bauer
Yves Bertot
Marc Bezem
Thierry Coquand
Eric Finster
Daniel Grayson
Hugo Herbelin
André Joyal
Dan Licata
Peter Lumsdaine
Assia Mahboubi
Per Martin-Löf
Sergey Melikhov
Alvaro Pelayo
Andrew Polonsky
Michael Shulman
Matthieu Sozeau
Bas Spitters
Benno van den Berg
Vladimir Voevodsky
Michael Warren
Noam Zeilberger

e were also the following students, whose participation was no less valuable.

Carlo Angiuli
Anthony Bordg
Guillaume Brunerie
Chris Kapulkin
Egbert Rijke
Kristina Sojakova

ddition, there were the following short- and long-term visitors, including student visit
se contributions to the Special Year were also essential.

Jeremy Avigad
Cyril Cohen
Robert Constable
Pierre-Louis Curien
Peter Dybjer
Martín Escardó
Kuen-Bang Hou
Nicola Gambino
Richard Garner
Georges Gonthier
Thomas Hales
Robert Harper
Martin Hofmann
Pieter Hofstra
Joachim Kock
Nicolai Kraus
Nuo Li
Zhaohui Luo
Michael Nahas
Erik Palmgren
Emily Riehl
Dana Scott
Philip Scott
Sergei Soloviev

Theorem	Status
$\pi_1(S^1)$	✔✔
$\pi_{k<n}(S^n)$	✔✔
long-exact-sequence of homotopy groups	✔✔
total space of Hopf fibration is S^3	✔
$\pi_2(S^2)$	✔✔
$\pi_3(S^2)$	✔
$\pi_n(S^n)$	✔✔
$\pi_4(S^3)$	✔
Freudenthal suspension theorem	✔✔
Blakers–Massey theorem	✔✔
Eilenberg–Mac Lane spaces $K(G,n)$	✔✔
van Kampen theorem	✔✔
covering spaces	✔✔
Whitehead's principle for n-types	✔✔

Table 8.2: Theorems from homotopy theory proved by hand (✔) and by computer (✔✔).

You can't learn everything yourself. . .

Books I've bought for one project

Elements of information theory
Geometric measure theory
Model theory
Matrix analysis
Measuring biological diversity
Fourier analysis in convex geometry
Lectures on functional equations and their applications
Convex geometry: the Brunn–Minkowski theory
Topics in matrix analysis
Advanced course in integral geometry and valuations
Introduction to geometric probability
Introduction to the theory of distributions
Metric structures for Riemannian and non-Riemannian spaces
A guide to distribution theory and Fourier transforms
Generalized functions (vols 1–3)
Functional analysis
On measures of information and their characterizations

Metric spaces, convexity and nonpositive curvature
Enumerative combinatorics vol 1
Mathematical foundations of information theory
Information and coding theory
Probability theory: the logic of science
Information theory, inference and learning algorithms
Inequalities
Geometry of sets and measures in Euclidean spaces
Mathematical theory of entropy
Probability with martingales
Real analysis
Fourier analysis
Fourier analysis
Maximum entropy and ecology
Real analysis and probability
Introducing genetics
A primer of ecology
Introduction to conservation ecology
Algebraic graph theory

You can't learn everything yourself. . .

. . . but you need to know enough to be able to communicate with your collaborators

Expect the unexpected!