Jirí Adámek
(Braunschweig, adamek@iti.cs.tu-bs.de)
A logic of injectivity

> Injectivity is an important concept in algebra, homotopy theory and elsewhere. We study the 'injectivity consequence' of morphisms of a category: a morphism h is a consequence of a set H of morphisms if every object injective w.r.t. all members of H is also injective w.r.t. h. We formulate a very simple logic which is always sound, i.e., whenever a proof of h from assumptions in H exists, then h is a consequence of H. In a wide range of categories (including e.g. all locally presentable categories and the category of topological spaces) we prove the completeness: every consequence can be proved. This is a joint work with Michel Hébert and Lurdes Sousa.

Francis Borceux
(Louvain-la-Neuve, borceux@math.ucl.ac.be)

Yemon Choi
(Newcastle, Yemon.Choi@newcastle.ac.uk)
Homological algebra for Banach modules?

Maria Manuel Clementino
(Coimbra, mmc@mat.uc.pt)

Bob Coecke
(Oxford, Bob.Coecke@comlab.ox.ac.uk)
Quantum measurements as Eilenberg-Moore coalgebras

> We showed that the part of quantum mechanics relevant to quantum informatics can be fully expressed in terms of tensor structure alone, and, without loss of expressiveness, admits a very high level of abstraction in terms of a particular kind of symmetric monoidal categories. In contrast with the usual Birkhoff-von Neumann lattice-theoretic 'non-logics' this categorical axiomatization comes with some kind of 'hyper-deductive logic', namely a degenerate version of Lambek–Barr–Seely categorical semantics for multiplicative linear logic. Relying on the work of Kelly & Laplaza and Joyal & Street it also follows that such categories admit a purely graphical calculus which turns out to be a quite substantial two-dimensional extension of Dirac's bra-ket calculus.
>
> A first step towards this is the axiomatixation of the tensor product in terms of dagger-compact categories, due to Abramsky (Oxford) & myself, which provides the theory with the notions of scalar, (self-)adjointness, compound system, Bell-states and map-state duality, unitarity, inner-product, trace, positivity, projector, a Born-rule to calculate probabilities, an abstract counterpart to complex phase, and we are in fact able impose the absence of (the redundant) global phases — the presence of which was the main reason for von Neumann to shift to a lattice-theoretic setting.
>
> The CPM-construction due to Selinger (Dalhousie) extends this list

with the important notions of mixed states, mixed operations and Jamiolkowski map-state duality.

More surprisingly, also quantum measurements do not need explicit sums and can be defined as a 'self-adjoint' Eilenberg-Moore coalgebra with respect to some particular kind of internal commutative comonoid, a result due to Pavlovic (Kestrel Institute) & myself. In conceptual terms, this definition of measurement exactly captures von Neumann's projection postulate together with the distinct abilities to copy and delete in the quantum world as compared to the classical world (a feature which is key to the whole quantum informatic endeavor). A compelling application of this purely multiplicative notion of quantum measurement, due to Paquette (UdMontreal) & myself, is a simple graphical proof of Naimark's theorem, from which the definition of generalized measurement arises.

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[2] S. Abramsky and B. Coecke (2005) Abstract physical traces. Theory and Applications of Categories 14: 111-124.

[3] P. Selinger (2006) Dagger compact closed categories and completely positive maps. Electronic Notes in Theoretical Computer Science (To appear). http://www.mathstat.dal.ca/∼selinger/papers.html#dagger

[4] B. Coecke (2005) Kindergarten quantum mechanics - lecture notes. In: Quantum Theory: Reconsiderations of the Foundations III, pp.81–98. AIP Press. quant-ph/0510032

[5] B. Coecke (2005) Introducing categories to the practising physicist. In: What is category theory? Polimetrica Publishing. Advanced Studies in Mathematics and Logic 30, pp.45-74. Polimetrica Publishing.

[6] B. Coecke and D. Pavlovic (2005) Quantum measurements without sums. In: Mathematics of Quantum Computing and Technology. Taylor and Francis. (to appear)

[7] B. Coecke and E. O. Paquette (2006) Generalized measurements and Naimarks theorem without sums. Submitted.

Tomas Everaert
(Brussels, teveraer@vub.ac.be)
Relative commutator theory in varieties of omega-groups

We introduce a new notion of commutator which depends on a choice of subvariety in any variety of Ω-groups. This notion encompasses Higgins's commutator, Fröhlich's central extensions and the Peiffer commutator of precrossed modules. Furthermore, we discuss the example of the variety of groups and the subvariety of n-nilpotent groups for a fixed n.

Emmanuel Galatoulas
(Athens, galas@tee.gr)
Bicategorical notions of quantum mechanical processes: beyond quantaloids

Julia Goedecke
(Cambridge, julia.goedecke@cantab.net)

Miles Gould
(Glasgow, mg@maths.gla.ac.uk)
Coherence for categorified algebraic theories

> It has long been known that every weak monoidal category A is equivalent via monoidal functors and monoidal natural transformations to a strict monoidal category $\mathbf{st}\,A$. We extend this result to weak P-categories, for any strongly regular (operadic) theory P.

Panagis Karazeris
(Patras, pkarazer@upatras.gr)
Flatness of functors into sites

> Consider a functor from a small category to a possibly large site with suitable colimits. We prove a (slightly more general) Diaconescu's theorem in this context, as suggested in some unpublished work of A. Kock. More precisely, we show that left Kan extension of the functor, along the Yoneda embedding, preserves finite limits if the functor satisfies, in the internal logic of the site, that the category of its elements is filtered. The proof relies on a representation of equalizers of presheaves as colimits of presheaves represented by cones for certain connected finite diagrams in the small category. The classical Diaconescu's theorem follows as a corollary. Unlike other proofs which rely on the local cartesian closedness of the recipient categories, the proposed proof can dispense on some occasions with that requirement. Thus the left exactness of geometric realization, as a functor into Kelley spaces, is another instance of our result.

Rudger Kieboom
(Brussels, rkieboom@vub.ac.be)

Anders Kock
(Aarhus, kock@imf.au.dk)

Jürgen Koslowski
(Braunschweig, koslowj@iti.cs.tu-bs.de)
What is the correct notion of morphism for interpolative semigroups?

> Such semigroups are defined by requiring every element to be a product. Clearly, monoids have this property, and it is well-known that semigroup morphisms between monoids need not preserve the neutral element. So the question is how to strengthen the notion of semigroup morphism in the interpolative case in order to guarantee the preservation of neutral elements, if these are present. This problem is a special case of the question how to generalize the notion of functor to taxonomies, i.e., categories without identities where the associativity diagram for the composition operation is in fact a coequalizer.

Alexander Kurz
(Leicester, kurz@mcs.le.ac.uk)
Relating algebras on Ind and Pro completions

(Joint work with Jiří Rosický)

We prove a general representation theorem for algebras for a functor. In detail: Consider a small, finitely complete and cocomplete category $\mathbf{C}$ and functors H on the Ind-completion of $\mathbf{C}$ and K on the Pro-completion of $\mathbf{C}$ (where Ind-completion refers to the completion with filtered colimits, Pro-completion being defined dually). In this situation there are adjoint functors $S : \mathrm{Ind}\,\mathbf{C} \longrightarrow \mathrm{Pro}\,\mathbf{C}$ and $P : \mathrm{Pro}\,\mathbf{C} \longrightarrow \mathrm{Ind}\,\mathbf{C}$. Theorem: If H preserves filtered colimits and K does so weakly and both H and K agree on $\mathbf{C}$, then, S/P lift to operations on H/K-algebras and for any H-algebra A, the unit $A \longrightarrow PSA$ is an H-algebra morphism. This generalises well-known presentation theorems of, for example, Stone and Jónsson–Tarski.

Tom Leinster
(Glasgow, tl@maths.gla.ac.uk)
A universal Banach space

Philipp Reinhard
(Glasgow, pmr@maths.gla.ac.uk)

Mehrnoosh Sadr-zadeh
(Southampton, ms6@ecs.soton.ac.uk)
Between algebra and coalgebra: an application

Algebra part joint work with A. Baltag and B. Coecke, Coalgebra part joint work with C. Cirstea.

The starting point is a simple algebraic setting consisting of a quantale, its right module, and a family of their lax-endomorphisms. The setting has applications to reasoning about information flow in multi-agent systems and can analyze complex puzzles and security protocols.

The quantale contains communication actions and its right module, propositions. The adjoints to the lax-endomorphisms stand for the knowledge of agents and the action of quantale on the module represents information update as a result of communication. We can view these endomorphisms and their adjoints as factorizations of the usual closure-like knowledge modalities of epistemic logic.

Equivalently, one may choose to move to a coalgebraic setting, use projective predicate lifting on a polynomial functor, and obtain a similar logic.

Richard Steiner
(Glasgow, rjs@maths.gla.ac.uk)

Paul Taylor
(Manchester, pt@cs.man.ac.uk)
Computable real analysis without set theory or Turing machines

The many schools of computable or constructive analysis accept without question the received notion of set with structure. They rein in the wild behaviour of set-theoretic functions using the double bridle of topology and recursion theory, adding encodings of explicit numerical representations to the epsilons and deltas of metrical analysis. Fundamental conceptual results such as the Heine–Borel theorem can only be saved by set-theoretic tricks such as Turing tapes with infinitely many non-trivial symbols.

It doesn't have to be like that.

When studying computable continuous functions, we should never consider uncomputable or discontinuous ones, only to exclude them later. By the analogy between topology and computation, we concentrate on open subspaces. So we admit $+$, $-$, $\times$, $\div$, $<$, $>$, $\neq$, $\wedge$ and $\vee$, but not $\leq$, $\geq$, $=$, $\neg$ or $\Rightarrow$. Universal quantification captures the Heine–Borel theorem, being allowed over *compact* spaces. Dedekind completeness can also be presented in a natural logical style that is much simpler than the constructive notion of Cauchy sequence, and also more natural for both analysis and computation.

Since open subspaces are defined as continuous functions to the Sierpiński space, rather than as subsets, they enjoy a "de Morgan" duality with closed subspaces that is lost in intuitionistic set-, type- or topos theories. Dual to $\forall$ compact spaces is $\exists$ over "overt" spaces. Classically, all spaces are overt, whilst other constructive theories use explicit enumerations or distance functions instead. Arguments using $\exists$ and overtness are both dramatically simpler and formally dual to familiar ideas about compactness.

Christopher Townsend
(Royal Bank of Canada, Christopher.Townsend@rbccm.com)
A representation theorem for geometric morphisms

We show that geometric morphisms between elementary toposes can be represented as adjunctions between the corresponding categories of locales. These adjunctions are characterized as those that preserve the order enrichment and have right adjoints commuting with the power locale constructions. They can also be characterized in terms of a modified Frobenius condition.

As application a new categorical account is given of the theory of geometric morphisms for which the pullback stability of localic geometric morphisms and the hyperconnected-localic factorization results are recovered.

Tim Van der Linden
(Brussels, tvdlinde@vub.ac.be)
On the second cohomology group in semi-abelian categories

(Joint work with Marino Gran)

In the paper [1] we develop some new aspects of cohomology in the context of semi-abelian categories: we establish a Hochschild–Serre 5-term exact sequence extending the classical one for groups and Lie algebras; we prove that an object is perfect if and only if it admits a universal central extension; we define the Baer sum of two central extensions and show how the second Barr–Beck cohomology group classifies them; we then prove a universal coefficient theorem to explain the relationship with homology.

The aim of this talk is giving an overview of the paper's main results, and explaining some of the techniques used to prove them.

[1] M. Gran and T. Van der Linden, *On the second cohomology group in semi-abelian categories*, preprint math.KT/0511357, 2005.

Jirí Velebil
(Prague, velebil@math.feld.cvut.cz)
Definable operations in monads

Jon Woolf
(Liverpool, Jonathan.Woolf@liverpool.ac.uk)
Derived categories 1, 2, 3

Sam Zoghaib
(ENS (Paris), Cambridge, zoghaib@clipper.ens.fr)