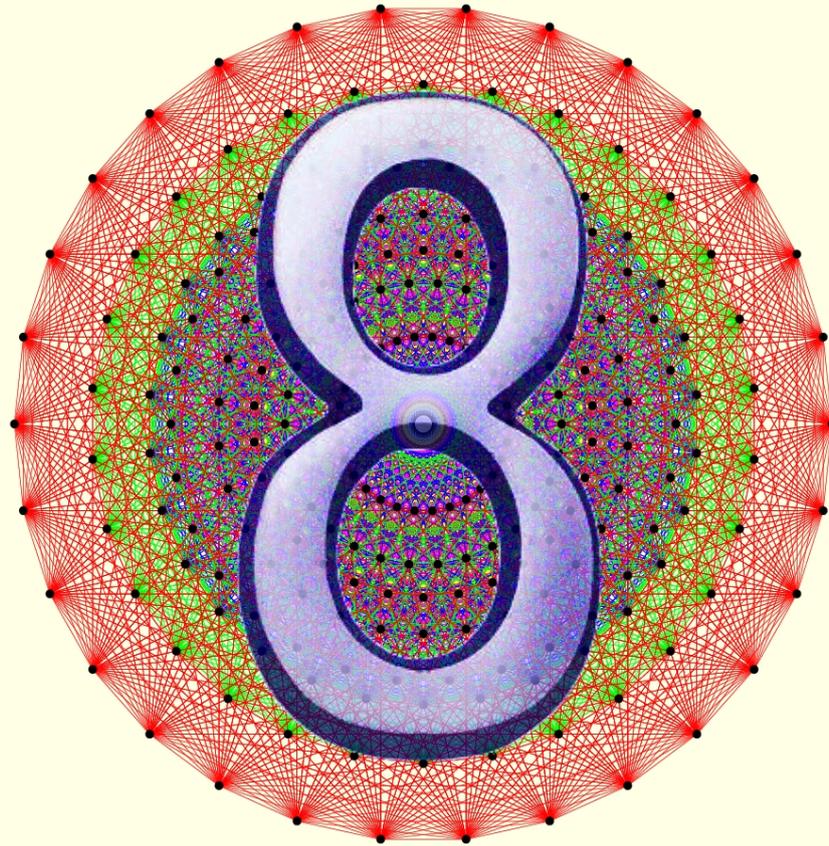


The Rankin Lectures 2008



The number 8 plays a special role in mathematics due to the ‘octonions’, an 8-dimensional number system where one can add, multiply, subtract and divide, but where the commutative and associative laws for multiplication — $ab = ba$ and $(ab)c = a(bc)$ — fail to hold.

The octonions were discovered by Hamilton’s friend John Graves in 1843 after Hamilton told him about the ‘quaternions’. While much neglected, they stand at the crossroads of many interesting branches of mathematics and physics.

For example, superstring theory works in 10 dimensions because $10 = 8 + 2$: the 2-dimensional worldsheet of a string has 8 extra dimensions in which to wiggle around, and the theory crucially uses the fact that these 8 dimensions can be identified with the octonions. Or: the densest known packing of spheres in 8 dimensions arises when the spheres are centered at certain ‘integer octonions’, which form the root lattice of the exceptional Lie group E_8 . The octonions also explain the curious way in which topology in dimension n resembles topology in dimension $n + 8$.

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