Tips on mathematical writing

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From reading lecture notes and textbooks, you're familiar with the difference between good and bad mathematical writing. The qualities to aim for in your own writing are almost the same as those you appreciate in lecture notes and books.

The main exception to this is motivation. A good lecturer will not only explain *what* she is doing, but also *why*. (Thus, she will not only tell you the definition of continuity, but also explain why it captures a certain intuitive idea.) However, for answers to problem sheets, you don't need to explain why you've chosen to do what you're doing, as long as your answer is correct and crystal clear. No one is born with the skill of writing clear mathematics; it takes work and practice.

Do

Present a coherent argument   Most questions at this level ask you to prove or disprove something. Your answer should be a perfectly precise, watertight argument, leaving the reader in no doubt that you are correct. The reader should never have to guess what you mean.

Make the logic clear   A proof is not just a sequence of statements, any more than a skeleton is just a collection of bones. You have to join everything up. Use logical symbols and words: $\Rightarrow$, $\Leftrightarrow$, $\therefore$, 'if', 'so', 'for all', 'let', 'suppose', etc. Use them correctly. For example, $\Rightarrow$ and $\therefore$ do not mean quite the same thing.

Introduce your variables   If you want to use a variable that isn't mentioned in the question, you need to introduce it first. For instance, suppose you're asked to say what it means for a map $f\colon X \to Y$ of real vector spaces to be linear. Bad answer:

$$f(ax + by) = af(x) + bf(y).$$

(Bad because you haven't said what a, b, x and y are. Are they elements of X? Integers? Monkeys?) OK answer:

$$f(ax + by) = af(x) + bf(y) \quad (a, b \in \mathbb{R},\ x, y \in X).$$

(You've said what types of thing the variables are, but are you saying that this equation holds for *all* a, b, x and y? For *some* a, b, x and y? Or what?) Good answer:

$$f(ax + by) = af(x) + bf(y) \text{ for all } a, b \in \mathbb{R},\ x, y \in X.$$

Don't be afraid to use words   See the previous two tips. You don't need a lot of words—see 'Waffle' opposite—but you do need enough.

Write in full sentences   You're writing English: mathematical English, but English all the same. If you don't write in sentences, using punctuation, your logic is unlikely to be clear. Again, don't make the reader guess what you mean.

Do it in rough first   When you're doing problem sheets, you'll find it really frees up your thinking if you do everything in rough first. First, while you're solving the problem, write down your thoughts in whatever way comes naturally. Then, once it's solved, write out the argument properly, this time with your attention on clear communication. Doing it this way saves you from having to concentrate on two different tasks at once.

Say which hypotheses you're using when   If you're asked to prove that A and B together imply C, it's good style (though not essential) to mention at which points in your argument you're using hypotheses A and B.

Read it out loud   If you're not sure whether what you've written is clear, an excellent test is to read it out loud. For instance, some people tend to write things like this:

$$x + 3 = 10$$
$$x = 7.$$

But you'd never *say* '$x + 3 = 10$ $x = 7$'. You'd say something like '$x + 3 = 10$, so $x = 7$'. Reading it out loud, you'll catch the mistake, prompting you to insert 'so' or whatever logical connector is appropriate in the context.

Check it   Once you've written out your answer, reread it, checking against all the other points here.

Don't

Waffle   Some people interpret 'write a coherent argument' as 'write lots of words'. That's a misunderstanding. Everything you write should be logically clear and unambiguous. Long waffley sentences are unlikely to achieve that.

Start by stating the result to be proved   There's a human tendency to do this. Do it in rough if you want, but not in your final draft. (Or if you must do it, put the word 'Claim:' beforehand.) Otherwise, it looks like you're beginning your proof by assuming the statement that you've been asked to prove.

Write lists of logically disjointed statements   This is one aspect of 'Make the logic clear' (opposite). The pair of equations shown in 'Read it out loud' is an example of what *not* to do.

Confuse functions with their values   Suppose we have a function $f\colon \mathbb{R} \to \mathbb{C}$. Then f is a device for turning real numbers into complex numbers. So, if x is a real number, then $f(x)$ is a complex number. It follows that f and $f(x)$ are different types of thing: f is a function, whereas $f(x)$ is a number. Keep the distinction clear.

Sometimes we allow ourselves a little latitude. For instance, if we're continually dealing with functions $\mathbb{R} \to \mathbb{C}$, and if we're in the habit of denoting real numbers by x, then we might say 'the function $e^{-2\pi i x}$' when we should really say 'the function $f\colon \mathbb{R} \to \mathbb{C}$ defined by $f(x) = e^{-2\pi i x}$'. Strictly speaking, the short version is incorrect, but it's OK as long as there's no possible ambiguity in the context and you understand that you're taking a liberty.

Write nonsense   Sounds obvious... but if you get into the habit of rereading your work, you'll be surprised how often you discover that something you've written makes no sense whatsoever. It happens to all of us.