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Mikhail Mikhailovich Postnikov (on his 60th birthday)

27 October 1987 was the sixtieth birthday of Mikhail Mikhailovich Postnikov, Doctor of Mathematical and Physical Sciences, Professor, Laureate of the Lenin prize.

Postnikov was born in Shatur (near Moscow) and came from an engineering family. He was interested in mathematics from an early age and on completing his secondary education in 1942 he entered the first course of the mathematics faculty of the Perm State University. At that time the lectures on mathematical analysis were given by Professor Sof'ya Aleksandrovna Yanovskaya, who was evacuated from Moscow and was an expert in the area of mathematical logic and set theory. She also led the seminar attended by Postnikov.

In the spring of 1943 Professor Yanovskaya returned to Moscow, and in the autumn of 1943 Postnikov transferred to course II of the Faculty of Mechanics and Mathematics of Moscow State University. He completed the programme of courses II and III in one academic year. In 1945 he graduated from the faculty and became a postgraduate student of L.S. Pontryagin, after which he took up a research post in the Steklov Institute of the Academy of Sciences of the USSR, where he is still employed.

In 1949 Postnikov defended his Ph.D. dissertation and in 1953 his D.Sc. In 1961 he was awarded the Lenin prize for his investigations into algebraic topology.

The main scientific achievements of Postnikov are in connection with the study of the homotopy properties of topological spaces and continuous maps. Here the most important problems are the classification of spaces to within homotopy equivalence and the classification of maps to within homotopy. The early results in this field are due to Hopf and Whitney, who classified maps from an n-dimensional polyhedron X^n to the n-sphere S^n. Very soon after this, Whitehead noted that Whitney's method provided the possibility of classifying maps from a polyhedron X^n to any Y with $\pi_i(Y) = 0$ for $i < n$ (here π_i is the homotopy group). In 1945 Pontryagin classified the maps $X^3 \to S^2$, while Steenrod classified the maps $X^{n+1} \to S^n$, $n > 2$, which gave rise to the notion of cohomology operations. In his Ph.D. dissertation Postnikov classified maps from a polyhedron X^3 to a simply-connected space Y on the basis of his cohomology operation, namely the Postnikov square.

The space $K(\pi, n)$ with just one non-trivial homotopy group π in dimension n (the so-called Eilenberg–MacLane space) is homotopically uniquely characterized by π and n. For spaces with just two non-zero homotopy groups the corresponding assertion is no longer true: with isomorphic homotopy groups they can have different homotopy types, and these types differ by the so-called Eilenberg–MacLane invariant. Postnikov considered the maximum possible generalization of this problem: there exists a space with known homotopy groups; how many further invariants are necessary in order to describe its homotopy type completely? It turns out that there is a countable set of invariants (one in each dimension), later called the higher cohomology operations. Together with the homotopy groups, these invariants (which have long since been called the Postnikov invariants of the space) completely determine the homotopy type of a finite cellular space. En route he also solved the problem of the classification of maps. Namely, let $\{k_n\}$ be a system of Postnikov invariants of a space Y. For any two maps $f, g : X \to Y$ the value $k_1(f, g)$ of the first invariant k_1 is defined on the pair (f, g). If it is non-trivial, then f and g are not homotopic. If it is trivial, then the invariant $k_2(f, g)$ is defined. If the latter is non-trivial, then f and g are non-homotopic, while if it is trivial, then $k_3(f, g)$ can be defined, and so on. If all the values of $k_n(f, g)$ are trivial, then f and g are homotopic. Thus the Postnikov invariants appear here as obstructions to the homotopy of maps.

H. Cartan and J.-P. Serre restated the results of Postnikov from the language of simplicial sets to the language of bundles, which is more convenient and now more generally accepted. In these terms, any space can, to within homotopy, be decomposed into a tower of bundles whose fibres are Eilenberg–MacLane spaces. For more than thirty years this tower has been known as the Postnikov tower of a given space. Here the Postnikov invariants are characteristic classes of the corresponding bundles, that is, they show how the "storeys" of these towers fit on to one another.

Postnikov's results very quickly became widely known internationally. He was highly rated by the English topologist J.H.C. Whitehead who had been visiting the USSR and who, incidentally, was instrumental in the wide dissemination of Postnikov's papers in the USSR.

Postnikov played an enormous role in establishing and developing algebraic topology in the USSR. He worked in the Faculty of Mechanics and Mathematics at Moscow State University for more than twenty years and directed the scientific research seminar (in the last ten years, together with A.V. Chernavskii). Almost all the Soviet books on algebraic topology that have been published since the war have appeared with his active participation (direct authorship, translation, editing, recommendation for translation, and so on) and almost all the Moscow algebraic topologists are either students of Postnikov or students of his students, or students of students of his students.

The first generation of Postnikov's students were the students of the mid-fifties. These include S.P. Novikov, now an active member of the Academy of Sciences of the USSR and head of the department of topology of the Mathematics Institute of the Academy of Sciences of the USSR, the head of the department of higher geometry and topology of Moscow State University, B.G. Averbukh, and L.N. Ivanovskii. The second generation includes the students of the Faculty of Mechanics and Mathematics of the year 1971–72; these include Yu.B. Rudyak, A.F. Kharshiladze, N.Ya. Gozman, and A.A. Bolibrukh. Quite recently A.V. Pazhitnov and A.V. Khokhlov obtained their Ph.D.'s. It can truly be said that Postnikov is the expert at raising qualified topologists.

In speaking of the teaching activities of Postnikov (he has been the professor of the department of higher geometry and topology in the Faculty of Mechanics and Mathematics of Moscow State University for more than 20 years), one must not fail to mention his brilliant lectures, which one recalls at the end of a 'Mech-Math.' course as one of the adornments of student life. Postnikov is currently working on an extensive course of lecutres on geometry (some of these lectures have already been published by "Nauka").

M.M. Postnikov in his sixtieth year is engaged in active research, full of creative projects, surrounded by the admiration of students, the gratitude of pupils, the esteem of colleagues. We wish him an active and creative longevity. May life give him joy as he has given joy to us.

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