Thirty Years of the Research Seminar
Invariantentheorie und algebraische Topologie
Göttingen – September 27 – 29, 2007
Program

Thursday September 27, 2007

4:00 PM Larry Smith
Opening and Closing Remarks
4:45 PM Julia Hartmann
Invariants of Reflection Groups in Characteristic p
6:00 PM Reg Wood
Polynomial Maps of Spheres

A dinner is scheduled for 8:00 PM in the restaurant of the Hotel Kassler Hof.

Friday September 29, 2007

9:30 AM Andreas Röscheisen
Functorial Invariants in Differential Galois theory
11:00 AM Dietrich Notbohm
Depth and Homology Decompositions
12:15 PM Carles Broto
On Fusion Systems of Finite Groups of Lie Type

3 PM Erik Kjær Pederson
Loop Spaces Nonequivalent to Lie Groups
4:30 PM Frank Neumann
On the Algebraic K-Theory of the Category of Unstable Modules over the Steenrod Algebra
5:45 PM William Singer
Rings of Symmetric Functions as Modules over the Steenrod Algebra

Coffee, cakes, and refreshments will be served in the adjacent room from 10:30 – 11:00 AM and 4:00 PM to 4:30 PM. There is a lunch break from 1:15 PM to 3:00 PM.

Saturday September 30, 2007

9:30 AM Martin Raußen
Concurrency in Directed Algebraic Topology
11:00 AM Andrew Raniciki
The Geometric Hopf Invariant
12:15 PM Michael Wibmer
Finite Group Orbits and Coinvariant Algebras

2:30 PM Grant Walker
Peterson's Problem
4:45 PM Nora Seeliger
On the Homology and Cohomology of Certain Free Loop Spaces

The traditional *academic quater hour* will **not** be observed. Alle Zeiten sind **s.t.**.

Abstracts of the Talks

Speaker : Carles Broto : Université Autonome de Barcelona
Title : *On fusion systems of finite groups of Lie type*
Abstract : We use homotopy theoretic methods in order to compare fusion systems of finite groups of Lie type at primes away from the defining characteristic. We will explain the concept of fusion system, its relation to the homotopy type of p-completed classifying spaces of finite groups, and sketch the arguments used in order to derive homotopy equivalences between p-completed classifying spaces, that in turn would imply equivalences of fusion systems of different families of finite groups of Lie type. (Joint work in progress with Jesper Møller and Bob Oliver)

Speaker : Julia Hartmann : Universität Heidelberg
Title : *Invariants of Reflection Groups in Characteristic p*
Abstract : Reflection Groups and their invariants are well understood in characteristic zero or more generally, in the nonmodular case. Many interesting reflection groups (e.g., classical groups) over finite fields do not fall in this category. The talk will explain some of the issues and present results in this direction (joint work with Anne Shepler).

Speaker : Frank Neumann : Leiscester University
Title : *On the Algebraic K-Theory of the Category of Unstable Modules over the Steenrod Algebra*
Abstract : Recently Schwartz calculated the Grothendieck ring of the category of unstable modules over the Steenrod algebra. Naturally one could ask then what can be said about the higher algebraic K-groups of this category. Using the Gabriel-Krull filtration, we construct a local-to-global spectral sequence of homological type converging to the algebraic K-theory of the noetherian objects in the category of unstable modules over the Steedrod algebra. This spectral sequence calculates the global K-theory information out of the local structure of the category of unstable modules given by K-groups of endomorphism rings of indecomposable injective objects. This spectral sequence is in direct analogy with the Brown-Gersten-Quillen spectral sequence converging to the algebraic K-theory of a noetherian scheme via the codimension of support filtration.

Speaker : Dietrich Notbohm : Frei Universität Amsterdam
Title : *Depth and Homology Decompositions*
Abstract : Homology decomposition techniques are a powerful tool first used in the analysis of the homotopy theory of (classifying) spaces. The associated Bousfield-Kan spectral sequences involve higher derived limits of the inverse limit functor. Motivated by this example, we study the impact of depth conditions on the vanishing of higher derived limits and will present several applications. We will show that the depth of the ring of polynomial invariants can be characterized in terms of depth of polynomial invariants of point-wise stabilizer subgroups. This also allows us to reprove the Landweber-Stong conjecture which says that the depth of polynomial invariants is controlled by the Dickson polynomials. Similar applications with respect to group cohomology and Stanley-Reisner algebras will be discussed. In fact, all these applications have a common source and are based on decomposition formulae for algebras over the Steenrod algebra in terms of Lannes' T-functor.

Speaker : Erik Kjær Pederson : Copenhagen University
Title : *Loop Spaces Nonequivalent to Lie Groups*
Abstract : This is joint work with Tilman Bauer and Jesper Grodal. We show that there exists a finite loop space homotopy equivalent to a closed, compact, smooth manifold, but not rationally equivalent to a Lie Group. The example is in the lowest possible rank which is 66.

Speaker : Andrew Raniciki : University of Edinburgh
Title : *The Geometric Hopf Invariant*
Abstract : The talk will report on a joint project with Michael Crabb. The geometric Hopf invariant of a stable map $F : \Sigma^\infty X \to \Sigma^\infty Y$ is the stable $\mathbb{Z}_2$-equivariant map

$$h(F) = (F \wedge F)\Delta_X - \Delta_Y F : X \to Y \wedge Y.$$

The stable $\mathbb{Z}_2$-equivariant homotopy class of $h(F)$ is the primary obstruction to the desuspension of F. If F is the geometric Umkehr map of an immersion of manifolds then $h(F)$ counts the double points. The π_1-equivariant version of $h(F)$ for the geometric Umkehr map of a normal map determines the Wall nonsimply connected surgery obstruction.

Speaker : Martin Raußen : Aalborg Universitæt
Title : *Concurrency and Directed Algebraic Topology*
Abstract : Concurrency theory in computer science studies the effects and problems arising when sevral executing processes run simultaneously sharing common resources. In recent years models with a combinatorial/geometric flavour have been introduced and investigated as tools in the analysis of concurrent processes. A common feature of these models is, that an execution corresponds to a directed path (abreviated to d-path), and that homotopies preserving the directions have equivalent computations as a result.

As an algebraic topologist one is tempted to apply the methods of the subject. What are the counterparts of the connected components, the fundamental group, homology, induced maps, etc.? Answers are not at all obvious. Directedness models irreversibility of the time flow; this loss of symmetry has serious effects that are at the heart of *directed algebraic topology*: Topological spaces have to be equiped with (non-symmetric) path spaces that d-maps have to respect. One can no longer formulate answers using (homotopy, homology) groups; instead one needs to investigate (homotopy, homology) functors from a certain category associated to the model space. Birth and death of homology classes indicate structural changes at various levels. Moreover, a directed homotopy equivalence between model spaces is not just a homotopy equivalence respecting the time flow.

Speaker : Andreas Röscheisen : Universität Heidelberg
Title : *Functorial Invariants in Differential Galois theory*
Abstract : In classical Galois theory, for a given finite Galois extension E/F one has a Galois correspondence between the subgroups of the Galois group and the intermediate fields of E/F by taking invariants with respect to the subgroup. In differential Galois theory, there is an analogous correspondence for so called Picard-Vessiot extensions. However, if the characteristic of the fields is positive, there exist intermediate (differential) fields over which the extensions field is inseparable and these intermediate fields cannot be obtained as fields of invariants.

In this talk, we give an introduction to group functors (and group schemes) as well as functorial invariants and show some properties related to them. Moreover, we show how this leads to a Galois correspondence which also takes into account the intermediate fields over which the extension field is inseparable.

Speaker : Nora Seeliger : Aberdeen University
Title : *On the Homology and Cohomology of Certain Free Loop Spaces*
Abstract : In this talk we will compute the homology and the cohomollogy of the free loop space of a complex projective space by computing the Serre spectral sequence of the fibration $\Omega(\mathbb{CP}(n) \longrightarrow \Lambda(\mathbb{CP}(n)) \downarrow \mathbb{CP}(n)$. This leads to a discussion of the spaces M for which the Serre spectral sequence of the free-loop fibration $\Omega(M) \longrightarrow \Lambda(M) \downarrow M$ collapses.

Speaker : William Singer : Fordham University
Title : *Rings of Symmetric Functions as Modules over the Steenrod Algebra*
Abstract : Let P_s be the polynomial algebra on s variables over the field $\mathbb{Z}/2$. Let the symmetric group Σ_s act on this algebra by permuting the variables. This talk is about the problem of determining a minimal set of generators for the invariant ring $P_s^{\Sigma_s}$ as a module over the Steenrod algebra $\mathscr{A}$. That is, we would like to determine the graded vector space:

$$(\divideontimes) \qquad \mathbb{Z}/2 \otimes_{\mathscr{A}} (P_s^{\Sigma_s}).$$

Work on this difficult problem was begun by Janfada and Wood in their papers [1],[2].

We find it convenient to work in terms of the vector space dual to $(\divideontimes)$

$$(\Phi) \qquad (\Gamma_s/\Sigma_s)^{\mathscr{A}}.$$

Here we are writing Γ_s for the algebra of divided powers on s variables, and $(\Gamma_s/\Sigma_s)^{\mathscr{A}}$ for the $\mathscr{A}$-annihilated elements of (Γ_s/Σ_s). We define a bigraded vector space $(\Gamma/\Sigma)^{\mathscr{A}}$ by writing:

$$(\bigstar) \qquad ((\Gamma/\Sigma)^{\mathscr{A}})_{s,*} = ((\Gamma_s/\Sigma_s)^{\mathscr{A}})_*$$

Our aim is to show that this bigraded vector space has additional structure that is useful for computation. In particular, we will review the definition of the **bigraded Steenrod algebra** $\mathscr{A}$, introduced by Liulevicius in [3], and studied further by May [4] and the current writer [5]. This Steenrod algebra $\mathscr{A}$ is generated by symbols $\{\ sq^k \mid k \geq 0\ \}$, and is subject to a set of relations that superficially resemble the classical Adem relations that hold in $\mathscr{A}$. But there is a crucial difference. The symbol sq^0 represents in $\mathscr{A}$ an independent generator; not the unit of the algebra. Thus, the *Adem relations* in $\mathscr{A}$ are homogeneous of length two in the generators (for example, $sq^1sq^2 = sq^3sq^0$), and $\mathscr{A}$ acquires a bigrading in which degree$(sq^k) = (k,\ 1)$.

Our main result will be the construction of an action of $\mathscr{A}$ upon $(\Gamma/\Sigma)^{\mathscr{A}}$, under which $(\Gamma/\Sigma)^{\mathscr{A}}$ becomes an $\mathscr{A}$-algebra. We will interpret some of the results of Janfada and Wood in terms of this action, and show some new computations as well.

[1] A.J. Janfada and R.M.W. Wood, *The hit problem for symmetric polynomials over the Steenrod algebra,* Math. Proc. Cam. Phil. Soc. **133** (2002), 295-303.

[2] A.J. Janfada and R.M.W.Wood, *Generating $H^*(BO(3),\ F_2)$ as a module over the Steenrod algebra,* Math. Proc. Cam. Phil. Soc. **134** (2003), 239-258.

[3] A. Liulevicius, *The factorization of cyclic reduced powers by secondary cohomology operations,* Mem. Amer. Math. Soc. **42** (1962), American Mathematical Society, Providence, RI.

[4] J.P. May, *A general algebraic approach to Steenrod Operations,* in: The Steenrod Algebra and its Applications, Lecture Notes in Math. **168**, Springer Verlag, Berlin-Heidelberg-New York, (1970), 153-231.

[5] W.M. Singer, *On the algebra of operations for Hopf cohomology,* Bull. London Math. Soc. **37** (2005), 627-635.

Speaker : Grant Walker : Manchester University
Title : *Hit Problems*
Abstract : The *Hit Problem* of F. P. Peterson in algebraic topology asks for a minimal generating set for the polynomial algebra $P = \mathbb{F}_2[t_1, \ldots, t_n]$ as a module over the Steenrod algebra $\mathscr{A}_2$. An equivalent problem is to find a basis for the set K of elements u in the dual Hopf algebra D of P (a divided power algebra) such that $\mathrm{Sq}^k(u) = 0$ for all $k > 0$. From the point of view of modular representation theory, it is natural to work over a general finite field $\mathbb{F}_q$ and to study $K = \sum_{d\geq 0} K^d$ as a $\mathbb{F}_q(G)$-module, where $G = GL(n, \mathbb{F}_q)$. We study an embedding in K^d of the Steinberg representation of G for $d = \big((q^n - 1)/(q - 1)\big) - n$ and an embedding in K^d for $d = \big((q^{2n} - 1)/(q^2 - 1)\big) - n$ of the permutation representation of G on complete flags in a vector space of dimension n over $\mathbb{F}_q$.

Speaker : Michael Wibmer : Universität Heidelberg
Title : *Finite Group Orbits and Coinvariant Algebras*
Abstract : Let V be a finite dimensional vector space over a field k of characteristic zero and G a finite subgroup of $\mathrm{Gl}(V)$. The natural action of G on V induces an action of G on $k[V]$, the algebra of polynomials functions on V. The set of all G-stable ideals I of $k[V]$ such that $k[V]/I$ is the regular representation of G, can be identified with the underlying point set of a certain algebraic variety G–$\mathrm{Hilb}(V)$, called the G-orbit Hilbert scheme. It comes equipped with a natural morphism $\pi : G\text{–}\mathrm{Hilb}(V) \to V/G$, which under certain conditions on G (e.g. $G \leq \mathrm{Sl}(V)$ and $\dim(V) = 2$ or 3) is a resolution of singularities of the quotient variety V/G. The fibre of π over the origin can be described quite explicitly in terms of the coinvariant algebra of G.

In this talk we will review the above mentioned results of Reid, Nakamura, et al. and discuss relations to the vanishing ideals of full length G-orbits in V.

Speaker : Reg Wood : Manchester University
Title : *Polynomial Maps of Spheres*
Abstract : We discuss the homotopy classes represented by polynomial maps between spheres.