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FRIEDRICH HIRZEBRUCH MADE GROUND-BREAKING DISCOVERIES

by Cem Akalin, Bonn General-Anzeiger, 9.6.2012

Friedrich Hirzebruch “related various geometric areas. What he did for modern mathematics was really revolutionary.” Sir Michael Atiyah was himself a ground-breaker in analysis. The 83-year-old had known Hirzebruch for more than 50 years. In the 1960's they were the founders of the so-called K-theory, which deals with "vector bundles on topological spaces". Friedrich Hirzebruch, one of the most important figures in German mathematics after World War II, died on Pentecost Sunday and was buried this week with great participation of scientists from home and abroad.

His work had a pioneering influence on the development of modern mathematics. "It was due to Friedrich Hirzebruch that after the years of war and persecution the reputation of Germany as an internationally recognized center of mathematics was restored," said Professor Don B. Zagier, a director of the Max Planck Institute for Mathematics in Bonn, which was founded by Hirzebruch.

In 1988 Hirzebruch was awarded the Wolf Prize in Israel for "achievements in the interest of mankind and friendly relations among peoples" and spoke before the Knesset, where he recalled the fate of many scientists persecuted by the Nazis. For his achievements Hirzebruch was also awarded the Grand Cross with Star of the Order of Merit of the Federal Republic of Germany, the Japanese Seki-Takakazu Prize, Russia's Lomonosov Gold Medal, the Albert Einstein Medal and the Georg Cantor Medal of the German Mathematical Society. 14 universities and one Academy awarded him an honorary doctorate. He was also a member of numerous academies of science and the Order Pour le Mérite.

Hirzebruch was born on 17 October 1927 in Hamm / Westphalia, studied mathematics from 1945 to 1950 in Munster and Zurich. At 22 he received his PhD from Heinrich Behnke and Heinz Hopf , with a thesis on four-dimensional Riemann surfaces. From 1952 he spent two years at Princeton. He proved the so-called Riemann-Roch theorem at that time. In 1969, he created the Bonn Sonderforschungsbereich of Theoretical Mathematics. In 1980 Hirzebruch initiated the Max Planck Institute for Mathematics in Bonn, which he led until his retirement in 1995.

Quite recently he was asked how he came to prove his signature theorem (in 1953). His answer: "I was looking for a good Riemann-Roch formula- the description of the dimensions of linear systems on algebraic varieties of arbitrary dimension. The signature theorem was a preparation for the Riemann-Roch theorem, I knew how to compute the signature using the Pontrjagin characteristic numbers, which are particular Chern numbers. But I could not prove it. "

Then, one day I read a note by another mathematician. René Thom had written about the so-called cobordism ring, a kind of algebraization of topology. "Thom's work made clear that it was only necessary to verify the signature theorem for the complex projective spaces. As soon as I read that, the proof of the signature theorem block was complete. It was only a matter of a few seconds."

Got that? In order to understand even just a little of Hirzebruch's work, one must go back in time. The basis for our understanding of numbers and geometric figures can be traced to mathematicians like Euclid more than 2,000 years ago. The brilliant Greek was the first to write down the laws governing triangles, rectangles, circles, solid space and surfaces.

Euclid's Elements served as a standard reference work for generations. It has long been known that the sum of the angles of any triangle is always 180 degrees. But what happens when the figures are not flat? What happens for triangles on a curved surface? The formula fails. The sum of the angles is no longer 180 degrees, changing quite suddenly.

Topology is basically the "geometry of position." It considers the shapes of elastic figures such as rubber bands or balloons, which are allowed to change under stretching. The topology looks at how to create different shapes, and how to modify, compare and classify them. In topology, there is no major difference between a circle and a square, because one an easily be stretched to the other.

Stretching is allowed, but not tearing, and holes cannot be cut. As long as a figure can be elastically stretched to another, without tearing or cutting of holes, then the figures are topologically equivalent. Stretching changes distances but not relative position. For example, it is not possible to change the figure 8 into the figure 6, since 8 consists of two loops while 6 is a loop with a handle.

With the help of the Riemannian geometry, the branch of differential geometry named after Bernhard Riemann (1826-1866), one can measure angles, lengths, distances and volumes of spaces of arbitrary dimensions and curved surfaces. Without the Riemannian geometry Einstein would never have been able to describe his general relativity theory. Riemann showed how to calculate the curvature of a space by a careful analysis of the distances between its points..

This mathematical discovery enabled Einstein to show that the curvature of space-time represents the gravitational force. A simple example can show how mathematicians think of this: If you are on a trampoline, then the elastic sheet is elastically deformed by your the weight. The strain is greatest, where there is most weight, and diminishes at the edges. Roughly speaking, this was Hirzebruch's area of research Hirzebruchs, and it was also important to determine numerical invariants of these mult-dimensional worlds.

Hirzebruch obtained such invariants. This rather complicated, complex geometric question has had a major influence on mathematical physics. The aim was to find a signature that is an invariant. The search for finite representations of infinity is one of the great mysteries of Mathematics, and underlies the basic question of the nature of the universe. There are not many things which can be reduced to a single number.

As strange as it sounds: The mathematician Leonhard Euler (1707-1783) discovered an apparently obvious constant, which is however a work of genius. He discovered a very simple relationship between the respective edges, corners and faces of polyhedra: the number of vertices (E) plus the number of faces (F) minus the number of edges (K) is always 2:

$E + F - K = 2$

This applies to a cube as much as to a pyramid. The pyramid has five vertices, 5 faces and eight edges: 5 +5-8 = 2 The cube: 8 +6-12 = 2. The Hirzebruch signature formula is similar, but for higher dimensions. Mathematicians call a number "invariant" if it does not change. Somewhat poetically, Zagier calls them "immortal numbers".

"Try to explain music to an alien, using only words. You can explain to him that the vibrations are caused for example by strings, which are amplified by a resonance box, and that these vibrations can be perceived through the senses. But what does this say about the music? That's how difficult it is to represent what Hirzebruch has done for mathematics, "says Zagier.

"Hirzebruch's discoveries have given a better understanding of the relationship between pure mathematics and theoretical physics," says Professor Andrew Ranicki of the School of Mathematics, University of Edinburgh. "Many spectacular insights of modern physics would not have been possible without Hirzebruch's discoveries."

Hirzebruch had an office in the Max-Planck-Institute for Mathematics, over the main post office in downtown Bonn. Researchers gather daily at the 4 o'clock tea under the glass dome on the top floor, a ritual to which the professor attached a great significance. He had a strong belief that scientists should not spend all day working in their offices. Dialogue was very important for him.

Finally, it was thanks to him that the leading mathematicians of the world met in Bonn for the annual “Arbeitstagung”. This still carries on today, but only every two years. The meetings were in the nature of family reunions, including a boat outing on the Rhine, a short hike and the traditional strawberry cake. Serre, Atiyah, Bott, Grothendieck, Deligne - they all came.

Hirzebruch created in Bonn a unique working environment, promoting young people. People know each other. So it was last Tuesday at the funeral reception in the University Club. "Hirzebruch was a gregarious man with a tremendous amount of humor and a profound human feeling," said Zagier, who first met Hirzebruch when he was an 18-year-old graduate student.

"I was studying at Oxford. When my research faltered, I wrote to Hirzebruch asking if I could study with him. He invited me to Bonn, and we talked for hours about mathematics. Although I was so young, he treated me as a real mathematician . " Zagier came to Hirzebruch, and stayed on. Hirzebruch carried on researching, travelling and lecturing to the end. He delivered his last lecture on 23rd April. It was called: "The shape of planar algebraic curves defined over the reals."

Caption for Photo #1. A gift in the Professor's office: artistic expressions of the legendary formulae of Friedrich Hirzebruch.

Caption for Photo #2: Berlin 1981.Friedrich Hirzebruch traveled widely, lecturing about his research.

Caption for Photo #3: A mathematical communion: in 2010 Ida Thompson baked Hirzebruch's signature formula for Hirzebruch (left) and Michael Atiyah (right). [Photo taken in the house of Ida Thompson and Andrew Ranicki in Edinburgh].