TOPOLOGICAL STRUCTURES II, MATHEMATICAL CENTRE TRACTS 115 (1979) 41-49

THE K-THEORY OF ALMOST SYMMETRIC FORMS

F.J.-B.J. Clauwens

INTRODUCTION

To motivate this paper we first recall a few facts.

According to [W1 , chapter 5] a normal map f between manifolds of dimension 2k and fundamental group π gives rise to a (so-called quadratic) form ψ defined on some finitely generated free left B module V, where B denotes the integral group ring $Z[\pi]$. The appropriate equivalence class of ψ in $L_{2k}(B)$ is the obstruction s(f) for changing f into a homotopy equivalence by surgery (for $k > 2$).

According to [C] a closed manifold P of dimension 2q and fundamental group ρ gives rise to a (so-called almost symmetric) form σ defined on some finitely generated free left A module K, where A is $Z[\rho]$. The main theorem there states that $\sigma \otimes \psi$ represents the obstruction for doing surgery on $id_P \times f$ if ψ does so for f.

In this paper we will study the algebra of almost symmetric forms; therefore we first recall the main things about quadratic forms from [W2].

Orientability considerations give rise to a homomorphism $w: \pi \to \{\pm 1\}$. The map $-: B \to B$ defined by the formula $\overline{\Sigma n_g g} = \Sigma n_g w(g) g^{-1}$ satisfies $\overline{x+y} = \bar{x}+\bar{y}$, $\overline{xy} = \bar{y}\,\bar{x}$ and $\bar{\bar{x}} = x$. For such an involuted ring B the dual $V^d = \mathrm{Hom}_B(V,B)$ of a left B-module V inherits the structure of a left B-module by $(af)(v) = f(v)\bar{a}$; the canonical map $\hat{}: V \to V^{dd}$ defined by $\hat{x}(f) = \overline{f(x)}$ is an isomorphism provided V is finitely generated projective. A form ζ on V can be viewed as a homomorphism $V \to V^d$; then $\zeta^* = \zeta^d \circ \hat{}: V \to V^{dd} \to V^d$ is one such too.

<u>DEFINITION</u>. Let ε be a sign. An ε-quadratic form over B consists of a finitely generated free left B-module V and a class of forms ψ on V defined up to the equivalence $\psi \sim \psi + \zeta - \varepsilon\zeta^*$. It is called nonsingular if the symmetrisation $\lambda = \psi + \varepsilon\psi^*$ is an isomorphism $V \to V^d$. We call $(W, \phi^d\psi\phi)$ isomorphic to

(V,ψ) if ϕ is a module isomorphism $W \to V$.

If F is f.g. free the quadratic form ψ on $F \oplus F^d$ defined by $\psi_F(x,f) = (f,0)$ is nonsingular; any quadratic form of this isomorphism type is called standard. Now $L_{2k}(B)$ is defined as the quotient of the Grothendieck group of nonsingular $(-1)^k$ quadratic forms over B by the subgroup generated by standard such forms.

DEFINITION. Let η be a sign, A an involuted ring. A nonsingular almost η-symmetric form over A consists of a finitely generated free left A module K and an isomorphism $\sigma\colon K \to K^d$ such that $\sigma^* = \eta\sigma(1+N)$, where N is nilpotent (compare [C; §9]). Again $\phi^d\sigma\phi$ is considered to be isomorphic to σ for any module isomorphism ϕ.

ALMOST SYMMETRIC FORMS ARE QUADRATIC

Let A be an involuted ring, $\eta = (-1)^q$. We consider quadratic forms over the polynomial ring $A[s]$ over A equipped with the involution $\overline{}$ such that $\overline{\Sigma a_j s^j} = \Sigma \overline{a_j}\,(1-s)^j$.

THEOREM 1. *Any element in $L_{2q}(A[s])$ can be represented by a quadratic form Ψ which is linear in s. Any such linear Ψ can be viewed as an almost $(-1)^q$ symmetric form.*

PROOF. Let the element be represented by a quadratic form $\Psi = \Sigma\Psi_i s^i$ of degree M in s. By the addition of a standard form and the use of an isomorphism we get (in matrix notation)

$$\begin{pmatrix}1 & -s & \Psi_M^*(1-s)^{M-1}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Psi & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ -1+s & 1 & 0\\ \Psi_M s^{M-1} & 0 & 1\end{pmatrix} = \begin{pmatrix}\Psi-\Psi_M s^M & 0 & -s\\ \Psi_M s^{M-1} & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$$

a form of degree $M-1$ if $M \geq 2$; so we can make that $M = 1$.
We can get rid of the constant term by using the equivalence

$$\Psi_0 + \Psi_1 s \sim \Psi_0 + \Psi_1 s - \Psi_0(1-s) + \eta\Psi_0^* s = (\Psi_1 + \Psi_0 + \eta\Psi_0^*)s.$$

To prove the last clause we consider the linear $\Psi = \Psi_1 s$ and write σ for $\eta\Psi_1^*$. Then the symmetrisation $\Lambda = \Psi + \eta\Psi^*$ of Ψ becomes $\sigma + (\eta\sigma^* - \sigma)s$ which is invertible exactly if σ is invertible and $\eta\sigma^{-1}\sigma^{*}-1$ is nilpotent. Q.E.D.

THEOREM 2. *There is a well defined biadditive pairing*

$$L_{2q}(A[s]) \times L_{2k}(B) \to L_{2q+2k}(A \otimes B)$$

which assigns to the quadratic form $\Psi = \Sigma\Psi_i s^i$ *over* $A[s]$ *with symmetrisation* Λ *and the quadratic form* ψ *over* B *with symmetrisation* λ *the quadratic form*

$$\lambda\Psi(\lambda^{-1}\psi) = \Sigma\Psi_i \otimes \lambda\cdot(\lambda^{-1}\psi)^i$$

over $A \otimes B$. *In particular it extends the familiar product of a symmetric form with a quadratic form.*

PROOF. Again write $\eta = (-1)^q$, $\varepsilon = (-1)^k$.

We start with the observation that for a general form $\Gamma = \Sigma\Gamma_i s^i$ over $A[s]$ we have

$$\begin{aligned}\lambda\Gamma^{*}(\lambda^{-1}\psi) &= \Sigma\Gamma_i^{*} \otimes \lambda(1-\lambda^{-1}\psi)^i = \Sigma\Gamma_i^{*} \otimes \lambda(\varepsilon\lambda^{-1}\psi^{*})^i \\ &= \varepsilon\Sigma\Gamma_i^{*} \otimes \lambda^{*}((\lambda^{*})^{-1}\psi^{*})^i = \varepsilon\Sigma\Gamma_i^{*} \otimes (\psi^{*}(\lambda^{*})^{-1})^i\lambda^{*} \\ &= \varepsilon\{\Sigma\Gamma_i \otimes \lambda(\lambda^{-1}\psi)^i\}^{*} = \varepsilon\{\lambda\Gamma(\lambda^{-1}\psi)\}^{*}\end{aligned}$$

Hence the symmetrisation of the image is

$$\lambda\Psi(\lambda^{-1}\psi) + \varepsilon\eta\{\lambda\Psi(\lambda^{-1}\psi)\}^{*} = \lambda\Psi(\lambda^{-1}\psi) + \eta\lambda\Psi^{*}(\lambda^{-1}\psi) = \lambda\Lambda(\lambda^{-1}\psi)$$

which is invertible since both λ and Λ are. Furthermore if we change Ψ into the equivalent $\Psi + Z - \eta Z^{*}$ the image changes into

$$\begin{aligned}&\lambda\Psi(\lambda^{-1}\psi) + \lambda Z(\lambda^{-1}\psi) - \eta\lambda Z^{*}(\lambda^{-1}\psi) = \\ &= \lambda\Psi(\lambda^{-1}\psi) + \lambda Z(\lambda^{-1}\psi) - \eta\varepsilon\{\lambda Z(\lambda^{-1}\psi)\}^{*}\end{aligned}$$

which is equivalent to $\lambda\Psi(\lambda^{-1}\psi)$.

If we change Ψ into the isomorphic $\Phi^{d}\Psi\Phi$ the image changes into

$$\lambda\Phi^d(\lambda^{-1}\psi)\Psi(\lambda^{-1}\psi)\Phi(\lambda^{-1}\psi) = \{\Phi(\lambda^{-1}\psi)\}^d\lambda\Psi(\lambda^{-1}\psi)\Phi(\lambda^{-1}\Psi)$$

which is isomorphic to $\lambda\Psi(\lambda^{-1}\psi)$. Finally if Ψ is standard then $\lambda\Psi(\lambda^{-1}\psi)$ is also standard: in fact such a Ψ is induced from a quadratic form over A for which this statement is well-known. Since our pairing obviously respects direct sums we have proven that the class of the image in $L_{2q+2k}(A\otimes B)$ is independent of the choice of the representing element for the class in $L_{2q}(A[s])$.

Now by Theorem 1 we may from now on assume that Ψ is of the type σs, where σ is nonsingular, almost η symmetric; so $\lambda\Psi(\lambda^{-1}\psi)$ is just $\sigma\otimes\psi$.

Firstly if we change ψ by an isomorphism ϕ into $\phi^*\psi\phi$ then $\sigma\otimes\psi$ changes by the isomorphism $1\otimes\phi$.

Secondly the isomorphism $K\otimes(F\oplus F^d)\cong(K\otimes F)\oplus(K\otimes F)^d$ which maps $a\otimes(x,f)$ to $(a\otimes x,\ \sigma(a)\otimes f)$ lets $\sigma\otimes\psi_F$ correspond with $\psi_{K\otimes F}$. So standard forms are mapped to standard forms.

It remains to be shown that the equivalence $\psi\sim\psi+\zeta-\varepsilon\zeta^*$ changes $\sigma\otimes\psi$ into something in the same class; this will be a consequence of the following lemma.

<u>LEMMA</u>. *For every integer* $p\geq 0$ *there is an isomorphism* Φ_p *and there are forms* Z_p *and* H_p *over* $A\otimes B$ *such that*

$$\Phi_p^d(\sigma\otimes\psi)\Phi_p = \sigma\otimes(\psi+\zeta-\varepsilon\zeta^*) + Z_p - \varepsilon\eta Z_p^* + H_p(N^{p+1}\otimes 1)$$

where N *is* $\eta\sigma^{-1}\sigma^* - 1$ *and thus nilpotent.*

<u>PROOF</u>. We apply induction. For $p = 0$ we take

$$\Phi_0 = 1,\qquad Z_0 = -\sigma\otimes\zeta\qquad H_0 = -\varepsilon\sigma\otimes\zeta^*.$$

In general Φ_p will be of the form $1+N\otimes\phi_1+\ldots+N^p\otimes\phi_p$ and H_p of the form $\sigma\otimes\theta_{p0}+\sigma N\otimes\theta_{p1}+\ldots$. If we assume all this for p then $\Phi_{p+1}^d(\sigma\otimes\psi)\Phi_{p+1}$ becomes

$$\sigma\otimes(\psi+\zeta-\varepsilon\zeta^*) + Z_p - \varepsilon\eta Z_p^* + H_p(N^{p+1}\otimes 1) +$$

$$+ (N^d)^{p+1}\sigma\otimes\phi_{p+1}^d\psi + \sum_{j=1}^{p}(N^d)^{p+1}\sigma N^j\otimes\phi_{p+1}^d\psi\phi_j +$$

$$+ \sigma N^{p+1}\otimes\psi\phi_{p+1} + \sum_{j=1}^{p+1}(N^d)^j\sigma N^{p+1}\otimes\phi_j^d\psi\phi_{p+1}.$$

Now we rewrite $(N^d)^{p+1}\sigma$

$\{(N^d)^{p+1}\sigma\otimes\phi$

$+\ \varepsilon(\eta\sigma^*-\sigma)N^p$

and we want the last ter
of $H_p(N^{p+1}\otimes 1)$ hence we
Z by defining $Z_{p+1} = Z_p$
The remaining term
as are the remaining ter
sible because $N^d\sigma$ can be
and H_{p+1} of the right fc

By viewing almost s
classifying the latter u
lence relation on them.
According to Theore
formulation of the produ
sufficiently coarse to d
Poincaré complexes in th
plexes: As explained in
σ to a 2q-dimensional al
class in $L_{2q}(A[s])$. The
as taking the tensor prc
σ = 1.
Both the inherent p
able for L_{2q} make it prc
tions then $L^{2q}(A)$ is.
One could hope that
to an honest $(-1)^q$ symme
shows that this is not t
the ring A contains a ce
Dedekind domain.
The two-dimensiona
and hence to an element
$\pi_1(T^2) = \mathbb{Z}\times\mathbb{Z}$. Suppose
symmetric form σ; then

Now we rewrite $(N^d)^{p+1}\sigma \otimes \phi^d_{p+1}\psi + \sigma N^{p+1} \otimes \psi\phi_{p+1}$ as

$$\{(N^d)^{p+1}\sigma \otimes \phi^d_{p+1}\psi - \varepsilon\eta\sigma^* N^{p+1} \otimes \psi^*\phi_{p+1}\} +$$

$$+ \varepsilon(\eta\sigma^* - \sigma)N^{p+1} \otimes \psi^*\phi_{p+1} + \sigma N^{p+1} \otimes (\psi + \varepsilon\psi^*)\phi_{p+1}$$

and we want the last term $\sigma N^{p+1} \otimes \lambda\phi_{p+1}$ to cancel the first term $\sigma N^{p+1} \otimes \theta_{p0}$ of $H_p(N^{p+1} \otimes 1)$ hence we define $\phi_{p+1} = -\lambda^{-1}\theta_{p0}$. The first term we absorb in Z by defining $Z_{p+1} = Z_p + (N^d)^{p+1}\sigma \otimes \phi^d_{p+1}\psi$.

The remaining term $\varepsilon\sigma N^{p+2} \otimes \psi^*\phi_{p+1}$ will be absorbed in $H_{p+1}(N^{p+2} \otimes 1)$, as are the remaining terms of $H_p(N^{p+1} \otimes 1)$ and the Σ-terms. The last is possible because $N^d\sigma$ can be rewritten as $-\sigma N(1+N)^{-1}$. So there exists Φ_{p+1}, Z_{p+1} and H_{p+1} of the right form. Q.E.D.

By viewing almost symmetric forms A as quadratic forms over A[s] and classifying the latter up to stable isomorphism we have defined an equivalence relation on them.

According to Theorem 2 this relation is sufficiently fine to admit the formulation of the product formula (for surgery obstructions). It is also sufficiently coarse to define a bordism invariant of algebraic symmetric Poincaré complexes in the sense of [R], hence one of geometric Poincaré complexes: As explained in [C] we can associate an almost $(-1)^q$ symmetric form σ to a 2q-dimensional algebraic symmetric Poincaré complex and then take its class in $L_{2q}(A[s])$. The result is well-defined on $L^{2q}(A)$ since it can be seen as taking the tensor product with the element of $L_0(Z[s])$ represented by $\sigma = 1$.

Both the inherent periodicity in q and the wealth of techniques available for L_{2q} make it probable that $L_{2q}(A[s])$ is better suited for calculations then $L^{2q}(A)$ is.

One could hope that an almost $(-1)^q$ symmetric form is always equivalent to an honest $(-1)^q$ symmetric one; the following example, due to A. Ranicki shows that this is not the case. However we will see that it is the case if the ring A contains a central element t such that $t + \bar{t} = 1$ or if it is a Dedekind domain.

The two-dimensional torus $T^2 = S^1 \times S^1$ gives rise to an element in $L^2(A)$, and hence to an element in $L_2(A[s])$, where A is the integral group ring of $\pi_1(T^2) = \mathbb{Z} \times \mathbb{Z}$. Suppose that this element could be represented by an antisymmetric form σ; then σ could be written as $\phi - \phi^*$; the result $\sigma \otimes \psi$ of its action on a (-1)-quadratic form ψ would be equivalent to $\phi \otimes (\psi - \psi^*)$, hence would depend only on the symmetrisation $\psi - \psi^*$ of ψ. In particular it would kill the Arf nontrivial element in $L_2(\mathbb{Z})$. On the other hand it follows from [SH] that multiplication with a circle induces a split injection on L-groups and hence the product with T^2 gives a split injection $L_2(\mathbb{Z}) \to L_4(A)$.

SOME CALCULATIONS

THEOREM 3. *If there exists a central element* t *of* A *such that* $t + \bar{t} = 1$ *then the canonical map* $L_{2q}(A) \to L_{2q}(A[s])$ *is an isomorphism.*

PROOF. The map $A[s] \to A$ substituting t for s gives a left inverse so we must show that for any integer $p \geq 0$ there is an isomorphism ϕ_p and there are forms ζ_p and θ_p such that

$$\phi_p^d(\sigma s)\phi_p = \sigma t + \zeta_p - \eta\zeta_p^* + \sigma N^{p+1}\theta_p.$$

For $p = 0$ we take $\phi_0 = 1$, $\zeta_0 = \sigma s(1-t)$, $\theta_0 = (1-s)t$. In general ϕ_p will be of the form $1 + \alpha_1 N + \ldots + \alpha_p N^p$ and $\theta_p = \theta_{p0} + \theta_{p1}N + \theta_{p2}N^2 + \ldots$ where the α_i and θ_{ij} are polynomial in s and t with Z coefficients, hence central.

If we assume all this for p then $\phi_{p+1}^d(\sigma s)\phi_{p+1}$ becomes

$$\sigma t + \zeta_p - \eta\zeta_p^* + \sigma N^{p+1}\theta_p +$$
$$+ \bar{\alpha}_{p+1}(N^d)^{p+1}\sigma s + \sum_{j=1}^{p} \bar{\alpha}_{p+1}(N^d)^{p+1}\sigma s\alpha_j N^j +$$
$$+ \sigma s\alpha_{p+1}N^{p+1} + \sum_{j=1}^{p+1} \bar{\alpha}_j (N^d)^j \sigma s\alpha_{p+1}N^{p+1}.$$

We rewrite $\bar{\alpha}_{p+1}(N^d)^{p+1}\sigma s + \sigma s\alpha_{p+1}N^{p+1}$ as

$$\{\bar{\alpha}_{p+1}(N^d)^{p+1}\sigma s - \eta\sigma^* N^{p+1}\alpha_{p+1}(1-s)\}$$
$$+ (\eta\sigma^* - \sigma)N^{p+1}\alpha_{p+1}(1-s) + \sigma(s + (1-s))\alpha_{p+1}N^{p+1}$$

Then we let the last term cancel the first term of $\sigma N^{p+1}\theta_p$ by defining $\alpha_{p+1} = -\theta_{p0}$ and absorb the first term in ζ by defining $\zeta_{p+1} = \zeta_p + \bar{\alpha}_{p+1}(N^d)^{p+1}\sigma s$.

The middle term $\sigma N^{p+2}\alpha_{p+1}(1-s)$ is absorbed in $\sigma N^{p+2}\theta_{p+1}$, as are the Σ terms and the remaining terms of $\sigma N^{p+1}\theta_p$. Q.E.D.

THEOREM 4.

$$L_0(Z[s]) \cong Z, \quad L_2(Z[s]) \cong (0).$$

PROOF. According to Theorem 1 we may restrict attention to η-quadratic forms ψ of the type $\eta\sigma^* s$, where σ is an almost η-symmetric form on some f.g. free Z-module K. Thus $N = \eta\sigma^{-1}\sigma^* - 1$ satisfies $N^e = 0$ for some e. Then $N^{e-1}K$ is of finite index h in some direct summand L of K.

For $x \in L^\perp = \{x \mid \sigma(x)(L) = 0\}$ we have also $\sigma(L)(x) = 0$ and vice versa, since $\eta\sigma^* N^{e-1} = (\sigma + \sigma N)N^{e-1} = \sigma N^{e-1}$ implies

$$\sigma(N^{e-1}y)(x) = \eta\, \overline{\sigma(x)(N^{e-1}y)}, \qquad \text{for } y \in K.$$

Furthermore $L \subset L^\perp$ since $\sigma N = -\eta N^d \sigma^*$ implies

$$\sigma(N^{e-1}y)(N^{e-1}x) = -\eta\overline{\sigma(N^e x)(N^{e-2}y)} = 0.$$

So σ induces a well-defined form $\tilde\sigma$ on $L^\perp/L$, and $\tilde N = \eta\tilde\sigma^{-1}\tilde\sigma^* - 1$ satisfies $\tilde N(x+L) = Nx + L$ hence $N^{e-1}K \subset L$ implies $\tilde N^{e-1} = 0$.

Now $L \otimes Z[s]$ is a direct summand of $K \otimes Z[s]$ which is isotropic for ψ. If $x = \Sigma x_j s^j \in K \otimes Z[s]$ is in $(L \otimes Z[s])^\perp$ for the symmetrisation $\lambda = \sigma + \sigma Ns$ of ψ then we have for all $\ell \in L$ that

$$0 = \lambda(\ell \otimes 1, \Sigma x_j s^j) = \Sigma\sigma(\ell, x_j)s^j + \Sigma\sigma(N\ell, x_j)(1-s)s^j = \Sigma\sigma(\ell, x_j)s^j$$

hence $x_j \in L^\perp$. We see that $(L \otimes Z[s])^\perp/(L \otimes Z[s])$ is just $(L^\perp/L) \otimes Z[s]$ and obviously the induced quadratic form $\tilde\psi$ on it is just $\eta\tilde\sigma^* s$.

It is well known that ψ is stably equivalent to $\tilde\psi$ and we have just seen that the latter is associated to an almost η-symmetric form $\tilde\sigma$ which has a better e. We can go on inductively until e = 1 which means that we get an η-symmetric form.

It is also well known [SE] that a (-1)-symmetric form is stably trivial and a (+1)-symmetric form stably isomorphic to some multiple m of the form (1) of rank one. Finally m can be detected by taking the signature of the quadratic form over $\mathbb{R}$ which we get by mapping s to $\frac{1}{2}$. Q.E.D.

Now some general remarks about torsion are necessary. If we start with a finite Poincaré complex P our module K gets a natural basis (see §6 of [C]).

The symmetrization λ of the associated quadratic form is $\sigma(1+Ns)$ and according to Lemma 9 of [C] we have $N^2 = 0$ and $1 + Ns$ has a resolution by automorphisms $1 + ((-1)^1\Sigma^{-1}\Sigma^* - 1)s$ of the E_i which are simple; in particular the isomorphisms involving N in the proofs of Theorems 2 and 3 are simple. So the torsion of λ lives in $\tilde{K}_1(A) \subset \tilde{K}_1(A[s])$ and the appropriate L groups $L^X_{2q}(A[s])$ have $X = Wh(\rho)$ in the general case and (0) in the case of simple Poincaré complexes.

At the time this is written we do not have theorems as the above for the odd-dimensional case. Note however, that if we did, we could use the long exact sequence 9.4 of [R] for the L_n groups to calculate $L_n(Z[\rho][s])$ for ρ the cyclic group of prime order $p > 2$. If ω denotes $\exp(2\pi i/p)$ and F_p is the field of p elements, there are maps from $Z[\rho][s]$ to $Z[\omega][s]$ and $Z[s]$ and from these to $F_p[s]$ satisfying all necessary conditions. Since $K_2(F_p[s]) = 0$ according to Theorem 11 of [Q] and 9.13 of [M] the map

$$\tilde{K}_1(Z[\rho][s]) \to \tilde{K}_1(Z[\omega][s]) \oplus \tilde{K}_1(Z[s])$$

is injective, so we may use the "simple" L-groups throughout and we get an exact seqeunce

$$\dots\ L_{n+1}(F_p[s]) \to L_n(Z[\rho][s]) \to L_n(Z[\omega][s]) \oplus L_n(Z[s]) \to L_n(F_p[s])\ \dots$$

But $L_n(Z[\omega][s]) \cong L_n(Z[\omega])$ by Theorem 3, hence is known, and similarly $L_n(F_p[s]) \cong L_n(F_p)$.

The author has now calculated $L_n(Z[\rho][s])$ for ρ cyclic.

REFERENCES

[C] CLAUWENS, F.J.B.J., *Surgery on products*, Indag. Math., 82 (1979) 121-144.

[M] MILNOR, J., *Introduction to algebraic K-theory*, Annals of Math. Studies 72, Princeton University Press, Princeton 1971.

[Q] QUILLEN, D., *Hig*
London Mat

[R] RANICKI, A., *An*
1975.

[SE] SERRE, J.P., *Co*
1970.

[SH] SHANESON, J.L.,
of Math.

[W_1] WALL, C.T.C., *S*
London.

[W_2] WALL, C.T.C., *O*
forms, Pr

[Q] QUILLEN, D., *Higher K-theory for categories with exact sequences*, London Math. Soc. Lecture Note Series 11, 1974, 95-103.

[R] RANICKI, A., *An algebraic theory of surgery*, preprint, Cambridge, 1975.

[SE] SERRE, J.P., *Cours d'Arithmetique*, Presses Universitaires de France, 1970.

[SH] SHANESON, J.L., *Wall's surgery obstruction groups for* G × Z, Annals of Math. 90 (1969) 296-334.

[W_1] WALL, C.T.C., *Surgery on compact manifolds*, Academic Press 1970, London.

[W_2] WALL, C.T.C., *On the axiomatic foundation of the theory of hermitian forms*, Proc. Cambridge Philos. Soc. 67 (1970), 243-250.

F.J.-B.J. Clauwens
Department of Mathematics
Catholic University
Toernooiveld
6525 ED Nijmegen
The Netherlands