TWO QUESTIONS ON MAPPING CLASS GROUPS

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Abstract. We show that central extensions of the mapping class group M_g of the closed orientable surface of genus g by $\mathbb{Z}$ are residually finite. Further we give rough estimates of the largest $N = N_g$ such that homomorphisms from M_g to $SU(N)$ have finite image. In particular, homomorphisms of M_g into $SL([\sqrt{g+1}], \mathbb{C})$ have finite image. Both results come from properties of quantum representations of mapping class groups.

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1. Introduction and statements

Set Σ_g^r for the orientable surface of genus g with r punctures. We denote by M_g^r the mapping class group of Σ_g^r, namely the group of isotopy classes of homeomorphisms that fix the punctures.

The following answers Question 6.4 of Farb (see Chapter 2 of [7]).

Proposition 1.1. *The central extensions of the mapping class group M_g (or the punctured mapping class group M_g^1, for $g \geq 4$) by $\mathbb{Z}$ are residually finite.*

Remark 1.1. The universal central extension $\widetilde{M}_g(1)$ surjects onto the universal central extension $\widetilde{Sp(2g,\mathbb{Z})}$ of the (integral) symplectic group, whose class is the Maslov class (generating $H^2(Sp(2g,\mathbb{Z}))$). It is known that $\widetilde{Sp(2g,\mathbb{Z})}$ is the pull-back of $Sp(2g,\mathbb{Z})$ into the universal covering $\widetilde{Sp(2g,\mathbb{R})}$ of the real symplectic group.

By a result of Deligne (see [6]) the extension $\widetilde{Sp(2g,\mathbb{Z})}$, for $g \geq 2$, is *not* residually finite since any finite index subgroup of it contains $2\mathbb{Z}$, where $\mathbb{Z}$ is the central kernel $\widetilde{Sp(2g,\mathbb{Z})} \to Sp(2g,\mathbb{Z})$. The same holds, more generally, for some other arithmetic groups having the congruence subgroup property.

The method of proof uses quantum representations of mapping class groups.

Definition 1.1. *The group Γ has property F_n if all homomorphisms $\Gamma \to PU(n)$ have finite image. Moreover, the group Γ has property F if it has property F for every n.*

Observe that the property F is inherited by finite index subgroups.

Remark 1.2. Let G be a connected, semi-simple, almost $\mathbb{Q}$-simple algebraic $\mathbb{Q}$-group and Γ an arithmetic lattice in G. If $G_{\mathbb{R}}$ has real rank at least 2 and $G^{ad}(\mathbb{R})$ has no compact factor then Γ has property F. This follows from ([17], Chap. VIII, Thm.B) for $K = \mathbb{Q}, l = \mathbb{R}$, S containing only the Archimedian place of $\mathbb{Q}$ and $\mathbf{H} = PO(n)$. In particular, any discrete group Γ commensurable with $Sp(2g,\mathbb{Z})$ for $g \geq 2$ or to $SL(2,\mathcal{O})$, where $\mathcal{O}$ is the ring of integers in a totally real number field of degree at least 2, has property F.

Mapping class groups have not property F. It is therefore interesting to understand whether they have property F_n for some n. This is related to a question of Farb in [7] concerning linear representations in small degree. The previous remark shows that we cannot use unitary representations of M_g that factor through $Sp(2g,\mathbb{Z})$, as the later group has no finite dimensional unitary representations with infinite image. Our second result states as follows:

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Proposition 1.2. *The maximal number N_g for which M_g has property F_{N_g} satisfies:*

$$\sqrt{g+1} \leq N_g < \begin{cases} 5^{g/2}F_{g-1}, \text{if } g \text{ is even}, \\ 5^{(g-1)/2}(F_g + F_{g-2}), \text{if } g \text{ is odd}, \end{cases}$$

where F_j is the Fibonacci sequence, defined by $F_0 = 0, F_1 = 1$ and the recurrence $F_{n+1} = F_n + F_{n-1}$, for $n \geq 1$. Moreover, the upper bounds are valid for any finite index subgroup of M_g.

Corollary 1.1. *Every homomorphism $M_g \to SL([\sqrt{g+1}], \mathbb{C})$ has finite image, if $g \geq 1$.*

It is likely that N_g behaves like an exponential for large g. This seems difficult to check because very few unitary representations of M_g are known. On the other hand one might expect that the maximal n with the property than every homomorphism $M_g \to SL(n, \mathbb{C})$ has finite image is a linear function on g.

Notice that groups having homomorphisms with infinite image into $SL(2, \mathbb{C})$ have not the property T of Kazhdan. However, M_g has no such representations, if $g \geq 3$, by the Corollary above.

Results of similar flavor were proved in [10] where it is shown that representations $M_g \to GL(2\sqrt{g-1}, \mathbb{C})$ cannot be faithful and in [3] where it is shown that the image of an element of M_g under a representation into $GL(g, \mathbb{C})$ should have algebraic eigenvalues.

One inequality above is an immediate consequence of a theorem of Bridson ([3]) concerning the property FA_n, which was introduced by Farb in [9]. The second inequality comes from the existence of quantum representations of M_g with infinite image ([13]).

2. Proof of Proposition 1.1

We prove the claim for the universal central extension first. This is known when $g = 1$ since the universal central extension is isomorphic to the braid group B_3.

An important result due independently to Andersen ([1]) and to Freedman, Walker and Zhang ([11]) states that the $SU(2)$ TQFT representation of the mapping class group is *asymptotically faithful*. Specifically, there is a sequence of representations ρ_k (indexed by an integer k, called the level) $\rho_k : M_g \to PU(N(k,g))$ into the projective unitary group of dimension $N(k,g)$ (for some $N(k,g)$ depending exponentially on k) such that the intersection of the kernels $\cap_{k\geq 2} \ker \rho_k$ is trivial for $g \geq 3$, and respectively the center of the mapping class group M_2 (which is a group of order two generated by hyperelliptic involution), when $g = 2$. Moreover, for $g = 2$ we can use the $SU(n)$ TQFT representation, with $n \geq 3$, for which the intersection of the kernels above is trivial (see [1]). When using this result we will say that we make use of the asymptotic faithfulness (of the quantum representations).

Each quantum representation is a projective representation which lifts to a linear representation $\widetilde{\rho}_k : \widetilde{M_g}(12) \to U(N(k,g))$ of the central extension $\widetilde{M_g}(12)$ of the mapping class group M_g by $\mathbb{Z}$. The later representation corresponds to invariants of 3-manifolds with a p_1-structure. Masbaum, Roberts ([18]) and Gervais ([15]) gave a precise description of this extension. Namely, the cohomology class $c_{\widetilde{M_g}(12)} \in H^2(M_g, \mathbb{Z})$ associated to this extension equals 12 times the signature class χ. It is known (see [16]) that the group $H^2(M_g)$ is generated by χ, when $g \geq 2$. Recall that χ is the class of one fourth the Meyer signature cocycle.

Observe that the ρ_k action of the center of $\widetilde{M_g}(12)$ is by roots of unity of order $2k$ (see [18] for the explicit formula). In fact, this action corresponds to the change of the p_1-structure of a 3-manifold and it is well-known that the quantum invariant changes by a root of unity of order $2k$. Thus every element of the center acts non-trivially via $\widetilde{\rho}_k$, for large enough k, so that the representations of $\widetilde{M_g}(12)$ are also asymptotically faithful. This implies that $\widetilde{M_g}(12)$ is residually finite. In fact, let $a \in \widetilde{M_g}(12)$ be any element $a \neq 1$. By the asymptotic faithfulness there exists some level k so that $\widetilde{\rho}_k(a) \in U(N(k,g))$ is non-trivial. The subgroup $\widetilde{\rho}_k(\widetilde{M_g}(12)) \subset U(N(k,g))$ is a discrete linear group and thus, by a classical theorem of Malcev, it is residually finite. In particular, there exists a homomorphism of $\widetilde{\rho}_k(\widetilde{M_g}(12))$ onto some finite group sending $\widetilde{\rho}_k(a)$ into a non-trivial element. This shows that every non-trivial element of $\widetilde{M_g}(12)$ is detected by some homomorphism into some finite group.

The universal central extension is $\widetilde{M_g}(1)$, where $\widetilde{M_g}(n)$ denotes the central extension by $\mathbb{Z}$ whose class is $c_{\widetilde{M_g}(n)} = n\chi$. It is immediate from their explicit presentations (see [15]) that $\widetilde{M_g}(d)$ embeds into $\widetilde{M_g}(n)$ if d divides $n \neq 0$. Such an embedding sends the generator z of the center into $z^{n/d}$. In particular, $\widetilde{M_g}(1)$ embeds in $\widetilde{M_g}(12)$ and thus the universal central extension is residually finite.

Now, an arbitrary central extension of M_g by $\mathbb{Z}$ is either trivial and hence residually finite, or else isomorphic to $\widetilde{M_g}(n)$, for some $n \in \mathbb{Z}\setminus\{0\}$. We observed above that there is an injective homomorphism $\widetilde{M_g}(1) \to \widetilde{M_g}(n)$, which sends the central element z into z^n. Moreover, the image is a normal subgroup of $\widetilde{M_g}(n)$. In particular, we have $\widetilde{M_g}(n)/\widetilde{M_g}(1) = \mathbb{Z}/n\mathbb{Z}$. This implies that $\widetilde{M_g}(n)$ is residually finite. In fact, any element of $\widetilde{M_g}(n)$ which is not detected by the homomorphism onto $\mathbb{Z}/n\mathbb{Z}$ belongs to $\widetilde{M_g}(1)$. Inducting finite groups representations from $\widetilde{M_g}(1)$ to $\widetilde{M_g}(n)$ we obtain finite group representations of the later detecting every non-trivial element of $\widetilde{M_g}(1)$. This proves the claim.

Remark 2.1. Freedman, Walker and Zhang already observed in [11] that a simple consequence of the asymptotic faithfulness is that M_g is residually finite.

Remark 2.2. This proof works more generally for the punctured mapping class group M_g^1 and for those extensions whose cohomology classes are of the form $n\chi + e$, for some $n \in \mathbb{Z}$. Recall that $H^2(M_g^1) = \mathbb{Z}\chi \oplus \mathbb{Z}e$, where χ is the signature class and e is the class associated to the puncture, for $g \geq 4$ (see [16]).

Remark 2.3. Notice that there exist quantum type representations of $Sp(2g, \mathbb{Z})$, for instance those associated to the monodromy of level k theta functions in the $U(1)$ gauge theory (see e.g. [12, 14]). Again these are only projective unitary representations which lift to unitary representations of some central extension $\rho_{Sp,k} : \widetilde{Sp(2g, \mathbb{Z})}(4) \to U(k^g)$. Here $\widetilde{Sp(2g, \mathbb{Z})}(4)$ is the central extension of $Sp(2g, \mathbb{Z})$ by $\mathbb{Z}$ whose class is 4 times the Maslov class. However, these representations factor through the integer metaplectic group. Further the generator of the kernel of $\widetilde{Sp(2g, \mathbb{Z})}(4) \to Sp(2g, \mathbb{Z})$ acts as the multiplication by a root of unity of order 8, for any level k. Thus the intersection of $\cap_{k\geq 2}\rho_{Sp,k}$ is $2\mathbb{Z}$, and the result of Deligne cited above shows that this is sharp.

3. Proof of Proposition 1.2

We consider first the following notion introduced by Farb in [9]:

Definition 3.1. *Let $n \geq 1$. A group Γ has property FA_n if any isometric action on any n-dimensional CAT(0) cell complex X has a fixed point.*

Observe that property FA_1 corresponds to Serre's property FA, which asks that any action without inversions of Γ on a real tree should fix a vertex. Notice that Kazhdan groups have property FA. Moreover if a group has property FA_n then it has property FA_k for all $k < n$. It is known (see [9]) that a group Γ with property FA_{n-1} has n-integral representation type, namely the eigenvalues of matrices in $\rho(\Gamma)$, for a homomorphism $\rho : \Gamma \to GL(n, K)$ with K a field, are algebraic integers if $\mathrm{char}(K) = 0$. Moreover, there are only finitely many conjugacy classes of irreducible representations of Γ into $GL(n, K)$, for an algebraically closed field K.

Culler and Vogtmann proved that M_g has property FA_1 in [5]. In [7] one asks to estimate the maximal $n = n(g)$ for which M_g has property FA_n.

There is a version of FA_n, namely the strong FA_n (which implies FA_n), in which one considers complete CAT(0) spaces and semi-simple actions. It is proved by Bridson in ([3], see also [2]) that M_g has strong FA_g. Moreover it is known that M_g acts (faithfully if $g > 2$) by semi-simple isometries on the completion of the Teichmüller space with the Weil-Petersson metric, which has dimension $6g - 6$. Thus $g \leq n(g) \leq 6g - 7$.

The key point is to relate the property FA_n to the finiteness of unitary representations. Specifically, we have the following:

Proposition 3.1. *If Γ is a finitely generated group with property FA_{n^2-1} then the representations $\Gamma \to SL(n,\mathbb{C})$ have finite image.*

Proof. Let $\overline{\Gamma}$ be the image of Γ under some homomorphism into $SL(n,\mathbb{C})$. A finitely generated subgroup $\overline{\Gamma}$ of $SL(n,\mathbb{C})$ lies into some $SL(n,A)$, where A is a finitely generated $\mathbb{Q}$-algebra contained in $\mathbb{C}$. Let $\varphi : A \to \overline{\mathbb{Q}}$ be a specialization of A, which induces a morphism $\varphi : SL(n,A) \to SL(n,\overline{\mathbb{Q}})$. The image $\varphi(\overline{\Gamma})$ belongs then to some $SL(n,K)$, where K is a finite extension of $\mathbb{Q}$.

Lemma 3.1. *If all specializations $\varphi(\overline{\Gamma})$ are finite then $\overline{\Gamma}$ is finite.*

Proof. Jordan's theorem says that there is some $f(n)$ such that any finite subgroup of $GL(n,K)$ has a normal abelian subgroup of index at most $f(n)$. The intersection of all subgroups of $\overline{\Gamma}$ of index at most $f(n)$ is then a finite index subgroup $U \subset \overline{\Gamma}$ such that $\varphi([U,U]) = 1$ for every specialization φ. Since specializations φ separate the points of A we have $[U,U] = 1$. Therefore there exists a finite index normal abelian subgroup U of $\overline{\Gamma}$. If U is finite then $\overline{\Gamma}$ will be finite and we are done.

Let us assume from now on that U is infinite. Since U is finitely generated abelian there is an infinite order element $Z \in U$. The following lemma will show that there exists a specialization φ such that $\varphi(Z)$ is of infinite order, contradicting our assumptions. Thus U should be finite abelian and the result will follow.

Lemma 3.2. *Let Z be a matrix with entries in a finitely generated $\mathbb{Q}$-algebra A contained in $\mathbb{C}$. Suppose that for every number field K and any ring homomorphism $\varphi : A \to K$ the image $\varphi(Z)$ is a matrix of finite order. Then Z has finite order.*

Proof. By Noether's normalization lemma (see [19], p.63) there exist algebraically independent elements $\xi_1, \xi_2, \ldots, \xi_p \in A$ such that A is an integral extension of the purely transcendental extension $B = \mathbb{Q}[\xi_1, \ldots, \xi_p]$. Moreover $\xi_1, \ldots, \xi_p$ form a transcendence basis for the field of fractions of A over $\mathbb{Q}$.

Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of the matrix Z. We will prove that λ_j are roots of unity. First λ_j are integral over A because they are the roots of the characteristic polynomial of Z, which is a monic polynomial with coefficients in A. The integrality is transitive and hence λ_i are integral over B. Thus λ_j satisfies an algebraic equation $P_j(\lambda_j) = 0$, where $P_j \in B[X]$ is the minimal polynomial of λ_j over B. The polynomial P_j is monic and irreducible because B is a unique factorization domain. This implies that, if we consider P_j as a polynomial from $\mathbb{Q}[\xi_1, \ldots, \xi_p, X]$ then it is still an irreducible polynomial in the $p+1$ variables $\xi_1, \ldots, \xi_p, X$.

If $p = 0$ then the fractions field of A is a number field and thus λ_i should be roots of unity.

Let us assume henceforth that $p \geq 1$. Observe that any specialization $\varphi : B \to \overline{\mathbb{Q}}$ can be lifted (not uniquely) to a specialization $\varphi : D \to \overline{\mathbb{Q}}$ of a finite extension D of B. First, specializations of B can be extended to possibly infinite specializations (see [20], Thm.6, p.31) of any extension D of B. Moreover, the extended specialization is finite on any element of D which is integral over B (see [20], Prop.22, p.41). In particular, any specialization of B extends to $A[\lambda_1, \ldots, \lambda_n]$. On the other hand, observe that any specialization φ of B corresponds to prescribing the values of $\varphi(\xi_j) \in \overline{\mathbb{Q}}$ arbitrarily.

Hilbert's irreducibility theorem states that there exist infinitely many (actually a Zariski dense set of) specializations $\varphi : B \to \mathbb{Q}$ such that the polynomials $\varphi(P_j) \in \mathbb{Q}[X]$ are still irreducible. Since $\varphi(Z)$ is of finite order, each $\varphi(\lambda_j)$ is a root of unity so that $\varphi(P_j)$ should be a cyclotomic polynomial. The degree of $\varphi(P_j)$ is the degree d_j of P_j, since these are monic polynomials. But there are only finitely many cyclotomic polynomials of given degree. Let S be the finite family of coefficients of all cyclotomic polynomials of degree smaller or equal to $\max(d_1, \ldots, d_n)$. It suffices then to choose some specialization φ of B for which one coefficient of some $\varphi(P_j)$ does not belong to S. For instance it suffices to choose a specialization for which some coefficient of $\varphi(P_j)$ is not in $\mathbb{Z}$, because cyclotomic polynomials have coefficients in $\mathbb{Z}$. This is possible unless all coefficients of the polynomials P_j are independent on the $\xi_1, \ldots, \xi_p$. This might happen only if $P_j \in \mathbb{Q}[X]$, namely if all its coefficients, which are elements of $\mathbb{Q}[\xi_1, \ldots, \xi_p]$, are actually constant. But in this case all λ_j are algebraic integers. This contradicts the fact that the transcendence degree of the fractions field of A was supposed to be $p \geq 1$. Therefore all eigenvalues λ_j of Z are roots of unity.

An alternative argument is as follows. The set of $\mathbb{C}$-valued specializations $\varphi : A[\lambda_1, \ldots, \lambda_n] \to \mathbb{C}$ is an irreducible affine algebraic variety of dimension p and λ_j is a rational function on it. If λ_j is a root of unity for any $\overline{\mathbb{Q}}$-valued specialization then $|\lambda_j|$ is identically 1. But a bounded regular function on a irreducible complex algebraic variety should be constant. This implies that all λ_j are algebraic integers and we conclude as above.

Eventually, it suffices to show that Z is diagonalizable. Consider the Jordan-Chevalley decomposition $Z = D + N$, where D is semi-simple, N is nilpotent and $DN = ND$. The entries of the matrices D and N belong to the field of fractions of A (see [4], Thm.7, p.71-72). Let $a \in A$ be the least common multiple of denominators arising in the entries of D and N. Every specialization φ of A with the property that $\varphi(a) \neq 0$ extends uniquely to a specialization, still denoted φ, of the localization of A at a. In particular, it makes sense to consider $\varphi(D)$ and $\varphi(N)$. Therefore $\varphi(Z) = \varphi(D) + \varphi(N)$ is the Jordan-Chevalley decomposition of $\varphi(Z)$. But the minimal polynomial of $\varphi(Z)$ divides $X^s - 1$, where s is the order of $\varphi(Z)$. This implies that the minimal polynomial has distinct roots and so $\varphi(Z)$ is semi-simple. The uniqueness of the Jordan-Chevalley decomposition yields then $\varphi(N) = 0$. Since this holds for any specialization φ such that $\varphi(a) \neq 0$ and such specializations separate the points of A we derive that $N = 0$. Thus Z is diagonalizable and hence of finite order, as claimed. □

Remark 3.1. We could also use ([3], Prop. 6.1) which says that the image in $GL(g, \mathbb{C})$ of an element of a finitely generated group with strong property FA_g has algebraic eigenvalues. However, lemma 3.2 can be applied to more general situations, since there is no assumption on Z.

□

It suffices now to show that, for any specialization φ, the image $G = \varphi(\overline{\Gamma})$ is finite. Observe that, if Γ has property FA_{n-1}, then $G = \varphi(\overline{\Gamma})$ has also property FA_{n-1}. We will show that:

Lemma 3.3. *Let K be a number field. Then a finitely generated subgroup $G \subset SL(n, K)$ with property FA_{n^2-1} should be finite.*

Proof. We prove that, for any embedding of K into a local field K_v the image of G in $SL(n, K_v)$ is precompact.

If $G \subset SL(n, K)$ is a finitely generated subgroup with property FA_{n-1} then its image in $SL(n, K_v)$ is precompact, for each non-Archimedian valuation v of K. In fact, G acts on the Bruhat-Tits building associated to $SL(n, K_v)$, which is a $(n-1)$-dimensional CAT(0) cell complex. The G-action has a fixed point because G has property FA_{n-1} and hence G is contained in the stabilizer of a vertex, which is a compact subgroup.

In what concerns the Archimedian valuations it suffices to consider the complex ones. But $SL(n, \mathbb{C})$ acts on the symmetric space $SL(n, \mathbb{C})/SU(n)$ of non-compact type and real dimension n^2-1. Since this space is CAT(0) and G has property FA_{n^2-1} it follows that the image of G into $SL(n, \mathbb{C})$ is contained in the stabilizer $U(n)$ for any complex valuation inducing an embedding $K \to \mathbb{C}$.

Eventually recall that $SL(n, K)$ embeds as a discrete subgroup of the special linear group $SL(n, A_K)$ over the adèles ring A_K of K. By above G is discrete and precompact into $SL(n, A_K)$ and hence finite. □

This proves Proposition 3.1. □

Remark 3.2. If G is a subgroup of $U(n) \cap SL(n, \mathbb{Q})$ with property FA_{n-1} then G is finite. This follows from above by using the fact that there is an unique complex Archimedian valuation on $\mathbb{Q}$, and one knows already that G is contained in the compact group $U(n)$. In particular, if Γ has property FA_{n-1} then the image of any homomorphism $\Gamma \to U(n) \cap SL(n, \mathbb{Q})$ is finite.

End of the proof of Proposition 1.2. The result of proposition 3.1 holds also for representations into $PSL(n, \mathbb{C})$ and a fortiori for representations into $PU(n)$. Since M_g has property FA_g we derive the lower bound inequality.

Consider now the smallest (projective) quantum representation $M_g \to PU(d_g)$ with infinite image, for $g \geq 2$. This is the $SO(3)$ quantum representation in level 5 (see e.g. [13]), whose dimension d_g is given by the Verlinde formula:

$$d_g = \left(\frac{5}{4}\right)^{g-1} \sum_{j=1}^{4} \left(\sin\frac{2\pi j}{5}\right)^{2-2g} = \begin{cases} 5^{g/2}F_{g-1}, \text{ if } g \text{ is even,} \\ 5^{(g-1)/2}(F_g + F_{g-2}) \end{cases}$$

where F_j is the Fibonacci sequence $F_0 = 0, F_1 = 1, F_{n+1} = F_n + F_{n-1}$. For instance $d_2 = 5$. These mapping class group representations come from the so-called Fibonacci TQFT.

Moreover, it is clear that the upper bound holds for any finite index subgroup of M_g. In fact the image of a finite index subgroup of the mapping class group by the quantum representation is still infinite. This proves the claim.

Remark 3.3. The property FA_{n-1} is not inherited by the finite index subgroups. Actually M_2 has a finite index subgroup which surjects onto a free non-abelian group and hence it has not property FA_1. The situation is subtler for $g \geq 3$ and it seems unknown whether finite index subgroups of M_g have property FA_1. Bridson proved in [2] that for any normal subgroup H of index n in M_g, for $g \geq 3$, and any homomorphism $\phi : H \to G$ to a group G acting by hyperbolic isometries on some complete CAT(0) space – in particular, to $G = \mathbb{Z}$ – the n-th powers of Dehn twists (which belong to H) lie in the kernel of ϕ. Such homomorpisms ϕ have therefore striking similarities with the quantum representations.

Corollary 1.1 follows from Proposition 3.1 above and Bridson's result from [3] saying that M_g has strong FA_g.

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