MR3617347 57R19 11E04 15-02 37J05
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★**Signatures in algebra, topology and dynamics. (English summary)**
Six papers on signatures, braids and Seifert surfaces, 1–173, *Ensaios Mat.*, 30, *Soc. Brasil. Mat.*, *Rio de Janeiro*, 2016.

In the paper under review the authors provide a wonderful exposition of various aspects of signature, one of the most fundamental and important invariants of quadratic forms. The reviewer thinks that it is better to read the paper than to read the review below—certainly after reading this wonderful paper the reader will learn various fascinating aspects of signature and gain a better understanding and perspective of the subject.

After a concise introduction in Chapter 1, in Chapter 2, entitled "Algebra", the authors provide the basics of quadratic forms, emphasizing historical remarks. This chapter is interesting and quite suggestive. They explain how the theory of quadratic forms was developed, and present various topics which are not familiar to most of us because they are not usually taught or contained in a standard linear algebra course or textbook. Here the reviewer just picks one example: in Section 2.6 the authors discuss Sturm's theorem concerning the number of real roots of a real coefficient polynomial in a given closed interval. It is amazing to learn how Sturm's theorem is connected to the theory of signature.

The rest of the chapters provide a concise overview on topics where the signature enters and plays a fundamental role. They explain the role and appearance of signatures in topology, knot theory, Maslov class (a connection to the cohomology and cocycle of symplectic groups), dynamics, and number theory. These chapters serve as windows for further reading and research.

An appendix provides a brief exposition for an algebraic L-theory, a theory of quadratic forms over non-commutative rings with involution. This notion and extension will be needed in the applications to topology where one needs to study quadratic forms over the group ring of the fundamental groups.

{For the collection containing this paper see MR3587804}

Tetsuya Ito