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Stable Equivariant Bordism

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§ 0. Introduction

Given a compact Lie group G, there are at least two reasonable approaches one can use in defining a G-equivariant bordism theory. The first of these methods is the geometric approach initiated by Conner and Floyd in [3, 4], which has been extensively studied by Stong (cf., for exemple, [8]). The second approach is to construct an equivariant version of the appropriate Thom spectrum and then to define G-bordism theory to be a suitable homology theory with coefficients in this spectrum. This has been done by tom Dieck in [9, 10], where he considers the unitary case. We propose in this paper to investigate the relationship which exists between these two approaches; for simplicity, we restrict attention to the unoriented case, although similar results would hold in the unitary case. The main result is Theorem (4.1): Stabilized geometric equivariant bordism is isomorphic to homotopy theoretic equivariant bordism.

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§ 1. Homotopy-Theoretic Equivariant Bordism

Let G be a compact Lie Group. Let $RO(G)$ denote the set of real finite dimensional orthogonal representations of G. The set $RO(G)$ is partially ordered by $V<W$ if V is isomorphic to some G-submodule of W. If $V\in RO(G)$, let $|V|=$dimension of V, and $RO_k(G)=\{V\in RO(G)\mid |V|=k\}$.

If $V\in RO(G)$, let $D(V)$, $S(V)$ denote the unit disc and unit sphere in V, and $\Sigma(V)=D(V)/S(V)$ the quotient space, with base point $S(V)/S(V)$.

Let W be any real orthogonal representation of G (possibly infinite dimensional) and let $BO_n(W)$ be the Grassmannian of n-dimensional subspaces of W, with the G-action induced by the linear action on W. There is also the space

$$EO_n(W)=\{(V,x)\in BO_n(W)\times W\mid x\in V\}$$

which is the total space of the "tautological" n-plane bundle $\gamma^n(W)$ over $BO_n(W)$, and there is an action of G on $EO_n(W)$ by bundle maps covering the action on $BO_n(W)$ and such that the projection is equivariant; $\gamma^n(W)$ is a G-vector bundle. Moreover the unit disc bundle $D\gamma^n(W)$ and its boundary sphere bundle $S\gamma^n(W)$ are equivariant subspaces of $EO_n(W)$, such that we have a G-action on the Thom space

$$MO_n(W) := M\gamma^n(W) := D\gamma^n(W)/S\gamma^n(W).$$

If $|W| = n$, then $MO_n(W)$ is just the sphere $\Sigma(W)$.

We now define $\mathbb{R}^\infty(G)$ to be the orthogonal direct sum of countably many copies of each of the irreducible finite dimensional orthogonal representations of G with the obvious action of G. Then, for any non negative integer n, the above construction determines an object MO_n^G in the category $\mathrm{Top}_0(G)$ of pointed G-spaces. The bundle

$$\gamma^n\colon\ EO_n^G := EO_n(\mathbb{R}^\infty(G)) \to BO_n(\mathbb{R}^\infty(G)) =: BO_n^G$$

is known to be a universal equivariant n-plane bundle in the category of G-spaces.

If $W \in RO_k(G)$, one has a suspension map

$$m_{n,k}\colon\ \Sigma(W) \wedge MO_n^G \to MO_{n+k}^G$$

induced by the Whitney sum. Note that

$$MO_{n+k}^G = MO_{n+k}(\mathbb{R}^\infty(G)) = MO_{n+k}(W \oplus \mathbb{R}^\infty(G)).$$

The spaces MO_n^G together with the suspension maps $m_{n,k}$ constitute a G-spectrum, denoted by MO^G.

We may now define the *homotopy-theoretic G-bordism groups* of a space X in $\mathrm{Top}_0(G)$ to be the groups

$$\tilde{N}_n^G(X) := \varinjlim\, [\Sigma V, X \wedge MO_{|V|+n}^G]_0^G, \quad V \in RO(G),$$

where $[\ ,\ ]_0^G$ denotes equivariant homotopy classes of pointed maps, and the limit is taken over the direct system indexed by the partially ordered set $RO(G)$ and the maps induced by suspension.

These are the spectral homology groups of X with coefficients in MO^G, as introduced by tom Dieck [10]. They constitute an equivariant homology theory in the sense of Bredon [1], but in addition have *suspension isomorphisms* for all suspensions with linear G-action; i.e. if $V \in RO_k(G)$, there is a canonical isomorphism

(1.1) $$\sigma(V)\colon\ \tilde{N}_n^G(X) \to \tilde{N}_{n+k}^G(\Sigma(V) \wedge X)$$

(see [2, IV] for the completely analogous proofs in the non equivariant case). Moreover the graded group $N_*^G(X)$ is a natural N_*-module, and the suspensions are maps of N_*-modules (N_* is the unoriented bordism ring).

§ 2. Stable Equivariant Bordism

For any pair of G-spaces (X, A) one defines $\mathfrak{N}_*^G(X, A)$, the G-equivariant bordism of (X, A), as follows (see [8] for a more detailed account): A singular G-manifold of (X, A) is a pair (M, f) where M is a compact differentiable G-manifold with boundary, and $f: (M, \partial M) \to (X, A)$ is an equivariant map. Two singular G-manifolds (M, f), (M', f') are bordant, if there is a triple (V, V_0, F) where V is a compact differentiable G-manifold with boundary, and ∂V is the union of the invariant regularly imbedded G-manifolds M, V_0, M', with $M \cap V_0 = \partial M$; $M' \cap V_0 = \partial M'$; $M \cap M' = \varnothing$; $(M \cup M') \cap V_0 = \partial V_0$; and $F: (V, V_0) \to (X, A)$ is an equivariant map extending f on M and f' on M'. Bordism is an equivalence relation, and on the set $\mathfrak{N}_*^G(X, A)$ of equivalence classes one has a group structure, which is induced by disjoint union of manifolds.

G-equivariant bordism also defines an equivariant homology theory in the sense of Bredon [1]; this theory does not have suspension isomorphisms for suspension with non trivial G-action (e.g. for $G = \mathbb{Z}_2$). One does, however, have a *suspension homomorphism*

$$\sigma(V): \tilde{\mathfrak{N}}_n^G(X) \to \tilde{\mathfrak{N}}_{n+|V|}^G(\Sigma(V) \wedge X) \tag{2.1}$$

assigning to $f: (M, \partial M) \to (X, *)$ the class of

$$(D(V) \times M, \partial(D(V) \times M)) \xrightarrow[f \times 1]{} (D(V) \times X, S(V) \times X \cup D(V) \times *)$$
$$\xrightarrow[\mathrm{pr}]{} (\Sigma V \wedge X, *).$$

Obviously one has $\sigma(V \oplus W) = \sigma(W) \circ \sigma(V)$, hence for a pointed G-space X one has the direct system, indexed by $RO(G)$, of N_*-modules

$$\tilde{\mathfrak{N}}_*^G(\Sigma(V) \wedge X), \qquad V \in RO(G),$$

and suspension homomorphisms, and we define the *stable G-equivariant bordism group*

$$\tilde{\mathfrak{N}}_n^{G:S}(X) := \varinjlim \mathfrak{N}_{n+|V|}^G(\Sigma(V) \wedge X), \qquad V \in RO(G).$$

These groups also form an equivariant homology theory which, by construction, has suspension isomorphisms for all suspensions with linear G action. For pairs of G-spaces one has an analogous stabilization, and the suspension isomorphism take the form

$$\sigma(V): \mathfrak{N}_*^{G:S}(X, A) \cong \mathfrak{N}_{*+|V|}^{G:S}(D(V) \times X, D(V) \times A \cup S(V) \times X).$$

Let $\xi\colon E\to X$ be a k-dimensional G-vector bundle and let $A\subset X$; then we have a natural *Thom homomorphism*

$$\tau(\xi)\colon\ \mathfrak{N}_*^G(X,A)\to\mathfrak{N}_{*+k}^G(D(E),D(E|A)\cup S(E))$$

defined as follows: If $f\colon(M,\partial M)\to(X,A)$ represents a bordism element, then one has the induced bundle map

$$\begin{array}{ccc} f^*E & \xrightarrow[\bar f]{} & E \\ {\scriptstyle f^*\xi}\downarrow & & \downarrow{\scriptstyle \xi} \\ M & \xrightarrow[f]{} & X; \end{array}$$

$\tau(\xi)[M,f]$ is represented by

$$(D(f^*E),\partial(D(f^*E)))\xrightarrow[\bar f]{}(D(E),D(E|A)\cup S(E)).$$

If $\xi\colon E\to X$ and $\xi'\colon E'\to X$ are G-vector bundles, then $\xi\oplus\xi'$ is the composite $\xi^*E'\to E\xrightarrow[\xi]{}X$ and

(2.2) $$\tau(\xi\oplus\xi')=\tau(\xi^*\xi')\circ\tau(\xi).$$

If $\pi\colon V\times X\to X$ is the trivial bundle with $V\in RO(G)$, then $\tau(\pi)=\sigma(V)$ is the suspension homomorphism. So by (2.2) in particular the Thom homomorphisms are compatible with suspensions, and passing to the limit we have a Thom homomorphism

(2.3) $$\tau(\xi)\colon\ \mathfrak{N}_*^{G:S}(X,A)\to\mathfrak{N}_{*+k}^{G:S}(D(E),D(E|A)\cup S(E)).$$

(2.4) **Lemma.** *Let ξ be a G-vector bundle over X which is stably invertible. Then the Thom homomorphism* (2.3) *is an isomorphism.*

Proof. One has formula (2.2) also for the stable Thom homomorphism. If ξ' is an inverse bundle of ξ, then $\tau(\xi\oplus\xi')$ is an isomorphism, being a suspension. Thus $\tau(\xi)$ has a left inverse, and $\tau(\xi^*\xi')$ has a left and a right inverse, so $\tau(\xi^*\xi')$ is an isomorphism and $\tau(\xi)$ is an isomorphism. □

The bundle ξ is invertible in particular if X is compact, but in fact it is sufficient to assume that X is a limit of a sequence of closed subspaces X_i, such that $\xi|X_i$ is stably invertible, for stable bordism is compatible with direct limits. So we have a Thom isomorphism for every reasonable G-bundle, in particular for the universal bundle γ^n. In the absolute case one can pass to quotients to get a Thom isomorphism

(2.5) $$\tau(\xi)\colon\ \mathfrak{N}_*^{G:S}(X)\to\mathfrak{N}_{*+k}^{G:S}(D(E),S(E))\cong\tilde{\mathfrak{N}}_{*+k}^{G:S}(M(\xi)).$$

§ 3. The Pontrjagin-Thom Construction

As noted by tom Dieck [10], there is an equivariant *Pontrjagin-Thom construction*

(3.1) $$\Phi\colon \tilde{\mathfrak{N}}_*^G(-)\to \tilde{N}_*^G(-)$$

which will be shown to stabilize to a transformation

(3.2) $$\Phi^S\colon \tilde{\mathfrak{N}}_*^{G:S}(-)\to \tilde{N}_*^G(-).$$

The construction of Φ runs as follows:

Let $f\colon (M,\partial M)\to(X,*)$ represent an element of $\mathfrak{N}_*^G(X,*)$. Choose an equivariant imbedding $e\colon M\to D(V)$, for some $V\in RO(G)$ (see [6, 7]). Then M's tangent bundle $\tau(M)$ is a subbundle of $M\times V$, and the normal bundle ν of the imbedding e is defined to be the orthogonal complement of $\tau(M)$ in this bundle. Then a small disc bundle $D\nu$ of ν can be identified with a compact subset of $D(V)$, called a "tubular neighbourhood" although it is not a neighbourhood, and there is the usual collapsing map

$$k\colon \Sigma(V)=D(V)/S(V)\to D\nu/\big(D(\nu\,|\,\partial M)\cup S\nu\big).$$

On the other hand ν has a classifying map $u\colon M\to BO^G_{|V|-n}$ covered by $\bar{u}\colon D\nu\to D\gamma^{|V|-n}$, and the map $(f\circ\nu,\bar{u})\colon D\nu\to X\times D\gamma^{|V|-n}$ induces an equivariant map $D\nu/\big(D(\nu\,|\,\partial M)\cup S\nu\big)\to X\wedge MO^G_{|V|-n}$; if we compose this with the collapsing map k, we get a map which represents the element $\Phi[M,f]\in\tilde{N}_*^G(X,*)$.

One shows as in the classical case, that we have produced in this manner a well-defined natural transformation $\Phi\colon \tilde{\mathfrak{N}}_*^G(-)\to\tilde{N}_*^G(-)$, which preserves degrees and respects the N_*-module structures present.

In order to be able to stabilize this transformation we have to show:

(3.3) **Lemma.** *The natural transformation Φ is compatible with the suspension homomorphisms* (2.1) *and* (1.1).

Proof. We have to follow an element $[M,f]\in\tilde{\mathfrak{N}}_m^G(X)$ around the diagram

$$\begin{array}{ccc} \tilde{\mathfrak{N}}_m^G(X) & \xrightarrow[\sigma(W)]{} & \tilde{\mathfrak{N}}_{m+k}^G(\Sigma(W)\wedge X) \\ \Phi\big\downarrow & & \big\downarrow\Phi \\ \tilde{N}_m^G(X) & \xrightarrow[\sigma(W)]{} & \tilde{N}_{m+k}^G(\Sigma(W)\wedge X), \end{array}\qquad W\in RO_k(G).$$

If one constructs $\Phi[M,f]$ by means of an imbedding $e\colon M\to D(V)$, one may use the imbedding $1\times e\colon D(W)\times M\to D(W)\times D(V)=D(W\oplus V)$ for the construction of $\Phi\sigma(W)[M,f]$, and then the verification is straight forward. □

As a consequence we get by passing to the limit a natural transformation of N_*-modules

$$\Phi^S\colon \tilde{\mathfrak{N}}_*^{G:S}(-)\to \tilde{N}_*^G(-),$$

which preserves degrees and is compatible with suspension isomorphisms.

Remark. Our presentation of the Pontrjagin-Thom construction in this section differs from that in [5], chiefly in being more flexible, though both result in the same natural transformation.

§ 4. The Isomorphism Theorem

Call a pointed G-space *admissible*, if the inclusion $*\subset X$ of the basepoint is an equivariant cofibration. Then our main result is

(4.1) **Theorem.** *For any compact Lie group G and any admissible G-space X the natural transformation*

$$\Phi^S\colon \tilde{\mathfrak{N}}_*^{G:S}(X)\to \tilde{N}_*^G(X)$$

is an isomorphism.

Proof. One may suppose that X is compact because both theories are determined by their values on compact spaces, and one may suppose that the basepoint is disjoint, by looking at the exact sequences of the cofibration

$$\{*\}\cup\{+\}\subset X^+\to X.$$

We construct an inverse transformation Ψ for Φ^S using the following idea: By definition the homotopy-bordism of X^+ may be considered as stable equivariant homotopy groups of the spectrum $MO^G\wedge X^+$. If we denote such homotopy groups by $\pi_*^{G:S}(-)$ – without defining them – the transformation Ψ will be defined by the following diagram:

$$\begin{array}{ccc}
\pi_*^{G:S}(X^+\wedge MO^G) & \xrightarrow{\ \kappa\ } & \tilde{\mathfrak{N}}_*^{G:S}(X^+\wedge MO^G)\\
\| & & \downarrow{\scriptstyle \tau^{-1}}\\
N_*^G(X) & & \mathfrak{N}_*^{G:S}(X\times BO^G)\\
& {\scriptstyle \Psi}\searrow \quad \swarrow{\scriptstyle \mathrm{pr}_{1*}} & \\
& \mathfrak{N}_*^{G:S}(X) &
\end{array}$$

here κ is the canonical map which looks at a sphere as a particular manifold (which bounds when projected to the point), and τ is essentially the Thom isomorphism for the bundles $X\times\gamma^n$, where γ^n is the universal G-bundle.

For a more explicite construction, suppose $\alpha\in\tilde{N}_n^G(X^+)$ is represented by $a\colon (D(V), S(V))\to(X^+\wedge M\gamma^{|V|-n}(Q), +)$ for some V, $Q\in RO(G)$; here we use the compactness of $D(V)$ to replace $\mathbb{R}^{\infty}(G)$ by the finite-dimensional representation Q (this in fact is unnecessary by the remark after (2.4)). The map a then also represents an element

$$\kappa(\alpha)=[D(V),a]\in\tilde{\mathfrak{N}}^{G:S}_{|V|}(X^+\wedge M\gamma^{|V|-n}(Q)).$$

The space in brackets is the Thom space of the bundle $\xi:=X\times\gamma^{|V|-n}$; thus we can apply to the element $\kappa(\alpha)$ the composite

$$\tilde{\mathfrak{N}}^{G:S}_{|V|}(M(\xi))\xrightarrow[\tau(\xi)^{-1}]{}\mathfrak{N}^{G:S}_n(X\times BO_{|V|-n}(Q))\xrightarrow[\mathrm{pr}_{1*}]{}\tilde{\mathfrak{N}}^{G:S}_n(X^+)$$

where $\tau(\xi)$ is the Thom isomorphism (2.5). Then it is easy to verify that the element $\Psi(\alpha):=\mathrm{pr}_{1*}\circ\tau(\xi)^{-1}\circ\kappa(\alpha)$ only depends on α and not on the choices involved in the definition, so that we have a well-defined degree-preserving function

$$\Psi:\ \tilde{N}^G_*(X^+)\to\tilde{\mathfrak{N}}^{G:S}_*(X^+)$$

which, it is claimed, inverts Φ^S.

It is easily seen that $\Psi\circ\Phi^S$ is the identity of $\tilde{\mathfrak{N}}^{G:S}_*(X)$. For the proof let $[M^n,f]\in\tilde{\mathfrak{N}}^{G:S}_n(X)$ where we may assume that the range of f is actually X, by passing to the appropriate suspension. To obtain $\Phi^S[M^n,f]$ we chose an imbedding e: $M\to D(V)$ with normal bundle ν which has a bundle map $\bar{u}$: $D\nu\to D\gamma^{|V|-n}(Q)$ for some $Q\in RO(G)$. These define the map of pairs

$$(f\circ\nu,\bar{u}):\ (D\nu, D(\nu|\partial M)\cup S\nu)$$
$$\to(X\times D\gamma^{|V|-n}(Q),\ \{*\}\times D\gamma^{|V|-n}(Q)\cup X\times S\gamma^{|V|-n}(Q)),$$

and there is the canonical projection

$$p:\ (X\times D\gamma^{|V|-n}(Q),\ \{*\}\times D\gamma^{|V|-n}(Q)\cup X\times S\gamma^{|V|-n}(Q))$$
$$\to(X\wedge M\gamma^{|V|-n}(Q),*).$$

Now in $\mathfrak{N}^{G:S}_n(M,\partial M)$ there is the fundamental class ι, represented by the identity of M, and $\tau(\nu)(\iota)\in\mathfrak{N}^{G:S}_{|V|}(D\nu, D(\nu|\partial M)\cup S\nu)$ is also the fundamental class. Therefore by an obvious bordism [3, p. 12f.] the element

$$p_*\circ(f\circ\nu,\bar{u})_*\,\tau(\nu)(\iota)\quad\text{represents }\ \kappa\Phi[M,f],$$

and by naturality of the Thom isomorphism

$$\tau(\xi)^{-1}(p_*\circ(f\circ\nu,\bar{u})_*\,\tau(\nu)(\iota))=\tau(\xi)^{-1}\,\kappa\Phi[M,f]$$

is represented by

$$(f,\bar{u}|M):\ (M,\partial M)\to X\times BO_{|V|-n}(G).$$

This establishes that $\Psi\circ\Phi^S$ is the identity.

We now consider the composition $\Phi^S\circ\Psi$. Given $\alpha\in\tilde{N}^G_n(X^+)$ represented by f: $(DV,SV)\to(M\xi,*)$, we can find a G-manifold with boundary

$L \subset \operatorname{Int} D(V)$ of dimension $|V|$, such that $f^{-1}(B\xi) \subset L - \partial L$ and $f^{-1}(*) \cap L = \varnothing$ (see [3,(3.1)]). If we vary f within its G-homotopy class, we get a commutative diagram

$$
(4.2) \qquad
\begin{array}{ccccc}
 & & (D(V), S(V)) & & \\
 & & \downarrow & \overset{f}{\searrow} & \\
(\Sigma V, *) & \longrightarrow & (L/\partial L, *) & \longrightarrow & (M\xi, *) \\
 & & \uparrow & & \uparrow{\scriptstyle q} \\
 & & (L, \partial L) & \xrightarrow[f|L]{} & (D\xi, S\xi),
\end{array}
$$

therefore $\kappa[D(V), f] = \kappa[L, q \circ f|L] \in \mathfrak{N}^{G:S}_{|V|}(M\xi)$.

Now let $\xi \oplus \xi' = \pi: Q \times X \to X$ for $Q \in RO(G)$; then $\sigma(Q) = \tau(\xi^* \xi') \circ \tau(\xi)$ or equivalently

$$
(4.3) \qquad \sigma(Q) \circ \tau(\xi)^{-1} = \tau(\xi^* \xi').
$$

The right side of this equation describes $\tau(\xi)^{-1}$ up to suspension. We use this fact to define the manifold M to be $D((f|L)^* \xi^* \xi')$ which comes provided with an equivariant map into $D(Q) \times B(\xi)$; if we compose this map with the projection onto $D(Q) \times X$ and the canonical map into $\Sigma(Q) \wedge X^+$, we receive an equivariant map h: $M \to \Sigma(Q) \wedge X^+$, and by definition of Ψ and (4.3) $\sigma(Q) \circ \Psi(\alpha) = [M, h] \in \tilde{\mathfrak{N}}^{G:S}_{n+|Q|}(\Sigma(Q) \wedge X^+)$, so that to compute $\sigma(Q) \circ \Phi^S \circ \Psi(\alpha)$, we have to apply the Pontrjagin-Thom construction to the map h.

For this purpose, note first that M is equipped with an equivariant imbedding into

$$
D((f|L)^* \xi^*(\xi \oplus \xi')) = D(Q) \times L \subset D(Q) \times D(V) = D(Q \oplus V),
$$

so we may use this imbedding in performing the construction. Moreover, a "tubular neighbourhood" of this imbedding is just $D(Q) \times L$ and this is the "neighbourhood" we employ in the construction. Now consider the diagram

$$
(4.4) \qquad
\begin{array}{ccccccc}
D(Q) \times L & \xrightarrow[1 \times f|L]{} & \multicolumn{5}{l}{D(Q) \times D(\xi) = D(Q) \times X \times D\gamma^{|V|-n}(Q)} \\
\downarrow & & \downarrow & & \swarrow & \overset{pr_3}{\searrow} & \\
M & \longrightarrow & D(\xi^* \xi') = D(Q) \times B\xi & & & & D\gamma^{|V|-n}(Q) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
L & \xrightarrow[f|L]{} & D\xi & \xrightarrow[\xi]{} & B\xi & \xrightarrow[pr_2]{} & BO_{|V|-n}(Q)
\end{array}
$$

in which the unnamed maps are either bundle projections or induced by the pullback construction. This diagram is commutative and the map $M \to BO_{|V|-n}(Q)$ classifies the normal bundle of M in $D(Q)\times D(V)$. By an examination of diagrams (4.2) and (4.4), we eventually see that the Pontrjagin-Thom construction applied to the map h yields the "Q-fold" suspension of the map with which we began. Equivalently, we have shown that $\sigma(Q)\circ\Phi^S\circ\Psi(\alpha)=\sigma(Q)(\alpha)$; since $\sigma(Q)$ is an isomorphism this completes the proof. □

The way transversality was circumvented here may have many more applications in equivariant bordism theory.

Theorem (4.1) was proved in [5] for the special case of the group $\mathbb{Z}_2$, using very different methods. A proof of the present version was later discovered and shown to tom Dieck, among others. He and the first-named author raised objections to this proof, concerned with assumed stability properties of homotopy theory which led to false consequences in the case $G=\{1\}$. Fortunately, the first named author was able to circumvent the need for these assumptions.

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