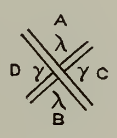
XVIII.—*Non-Alternate ± Knots, of Orders Eight and Nine.* By C. N. Little, of Nebraska State University. (With a Plate.)

(Read 15th July 1889.)

1. To complete the census of knots of any given order, that is, minimum number of crossings, it is necessary to include not only those in which the crossings are taken alternately over and under (*Alternate* ±), but also the *Non-Alternate* ±, those in which two or more consecutive crossings are alike over or alike under. Professor Tait has figured the forms of the alternate ± knots, of orders three to nine inclusive, on Pl. XLIV. vol. xxxii., *Trans. Roy. Soc. Edin.;* the object of this paper is to describe the non-alternate knots of these orders.

2. The projection of a non-alternate knot as a single closed line with double points only must be found in the complete series of forms of the alternate knots. The converse is not true, and the first operation is to exclude from consideration all forms which are not projections of non-alternate knots.

I find it convenient to use* λ and γ, as shown in the figure, to give the character of a crossing. The crossing shown looked at as belonging to the compartment, or, briefly, part A or B is a λ crossing. Looked at as belonging to C or D, it is a γ crossing.



3. It is to be remembered that in an alternate knot, or in any portion of a knot where the law of over and under is preserved, the crossings looked at as belonging to the parts of either *partition* (group of compartments) are alike.

It is evident that in a *coil* (succession of 2-gons), all the 2-gon crossings are either lambda or gamma.

If two parts are opposite at a crossing, and have besides a connection—as a coil—in which all crossings are, say, λ, then will the first be a λ crossing. It follows that all or none of the forms of an alternate will be included among the forms of a non-alternate knot.

4. It is now easy to decide whether a given alternate form can be the projection of a non-alternate knot. For example, in the first form of VI., of the nine folds as shown on Professor Tait's plate, the 6-gon amplexum is joined to a 4-gon by a 2-gon, twice by a 3-gon and by a single crossing, λ say. Each 3-gon connection has its three crossings alike, and λ by § 3 above. If now the single crossing be shifted beyond a 3-gon, the 2-gon also is seen to have λ crossings, and the law of over and under must hold throughout the form. No form then of this knot VI. can be the projection of a non-alternate knot.

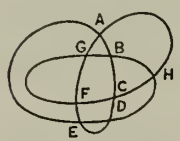
In like manner it is easily seen that the only forms of Pl. XLIV., which are the projections of non-alternate knots, are the following:—

Eightfold: I, III, IV, VII, IX, and XIV.

Ninefold: I, III, IV, V, VII, IX, X, XI, XIII, XIX, XXIII, XXIX, XXXV, and XXXVI.

* Listing called a lambda crossing δ and used λ for a gamma crossing (*Vorstudien zur Topologie*, p. 52, Gottingen, 1848).

5. It now becomes necessary to obtain the complete series of forms of non-alternate knots of these orders by assigning to the crossings all possible combinations of λ and γ. In 8III, 8IX, 8XIV, 9IV, 9XIII, 9XIX, 9XXIII, 9XXIV, and 9XXXVI, two parts are joined by three connections, in each of which all crossings are alike, and the one, two, or three forms can be drawn at once. In each form of 8III, for example, two parts are connected by a 2-gon and twice by a 3-gon. The two forms have γ 2-gon, λ 3-gon, γ 3-gon; and λ 2-gon, γ 3-gon, γ 3-gon. Perversion doubles all numbers where amphicheiralism is not found, and will not be again referred to until the final summing up.



6. Inspection shows that three consecutive overs cannot exist in these orders without degradation of the form; and this consideration greatly shortens the labour of treating the remaining forms. For illustration, 8I may be taken. Lettering the form as shown, a table is made out where + and − are used for over and under crossings respectively. A portion of the knot in which the crossings are alike is enclosed in a parenthesis.

	A	B	C	(D	E)	(F	G)	A	H	C	(F	G)	B	H	(D	E)	
1	+	+	−	−	+	+	−	−	+	+	−	+	−	−	+	−	A + B + F +
2	+	+	−	−	+	−	+	−	−	+	+	−	−	+	+	−	" " F −
3	+	−	+	+	−	+	−	−	+	−	−	+	+	−	−	+	A + B − C + D + F +
4	+	−	+	+	−	−	+	−	+	−	+	−	+	−	−	+	" " " " " F −
5	+	−	+	−	+	+	−	−	+	−	−	+	+	−	+	−	" " " D − F +
6	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	" " " D − F −
7	+	−	−	+	−	+	−	−	+	+	−	+	+	−	−	+	" " C − F +
8	+	−	−	+	−	−	+	−	−	+	+	−	+	+	−	+	" " " F −

From this table the crossings are marked on the forms. No. 6 is the alternate form. No. 3 is an amphicheiral form, but it degrades. No. 7 is the perversion of No. 2, and No. 5 of No. 4. This leaves the four distinct forms: 1, 2, 4, and 8. I find 19 eightfold forms, and for the ninefolds 88 distinct forms.

7. Thus far, the work has been straightforward and a matter of routine. In deriving the knots from the knot-forms the conditions to be observed are so many that a single worker cannot be *absolutely* certain that all have been observed, and that all the groups of forms obtained are really distinct knots.

I find 3 eightfold non-alternate knots, none of which has an amphicheiral form; so that to the 31 alternate knots already known must be added six new non-alternate, making 37 distinct eightfold knots. Professor Tait has shown at N on Pl. LXXIX. vol. xxxii., *Trans. Roy. Soc. Edin.*, five forms of knot I.

In the ninefolds I find 8 non-alternates, and their 8 perversions, making with the 82 alternates, 98 ninefold knots.

These new knots are shown on the Plate.

8. All of the new knots, with the exception of VIII. of the ninefolds, have single degree of beknottedness; for in every case it is possible to remove in some form of the knot at least two crossings by changing the sign of one, and without making the form alternate throughout. But there is no non-alternate knot of fewer than eight crossings. Knot VIII. has twofold beknottedness.

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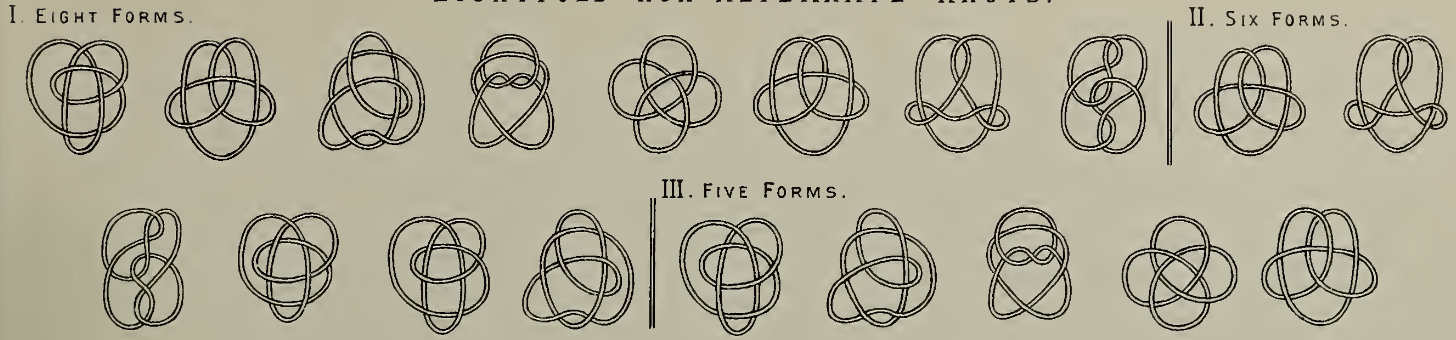
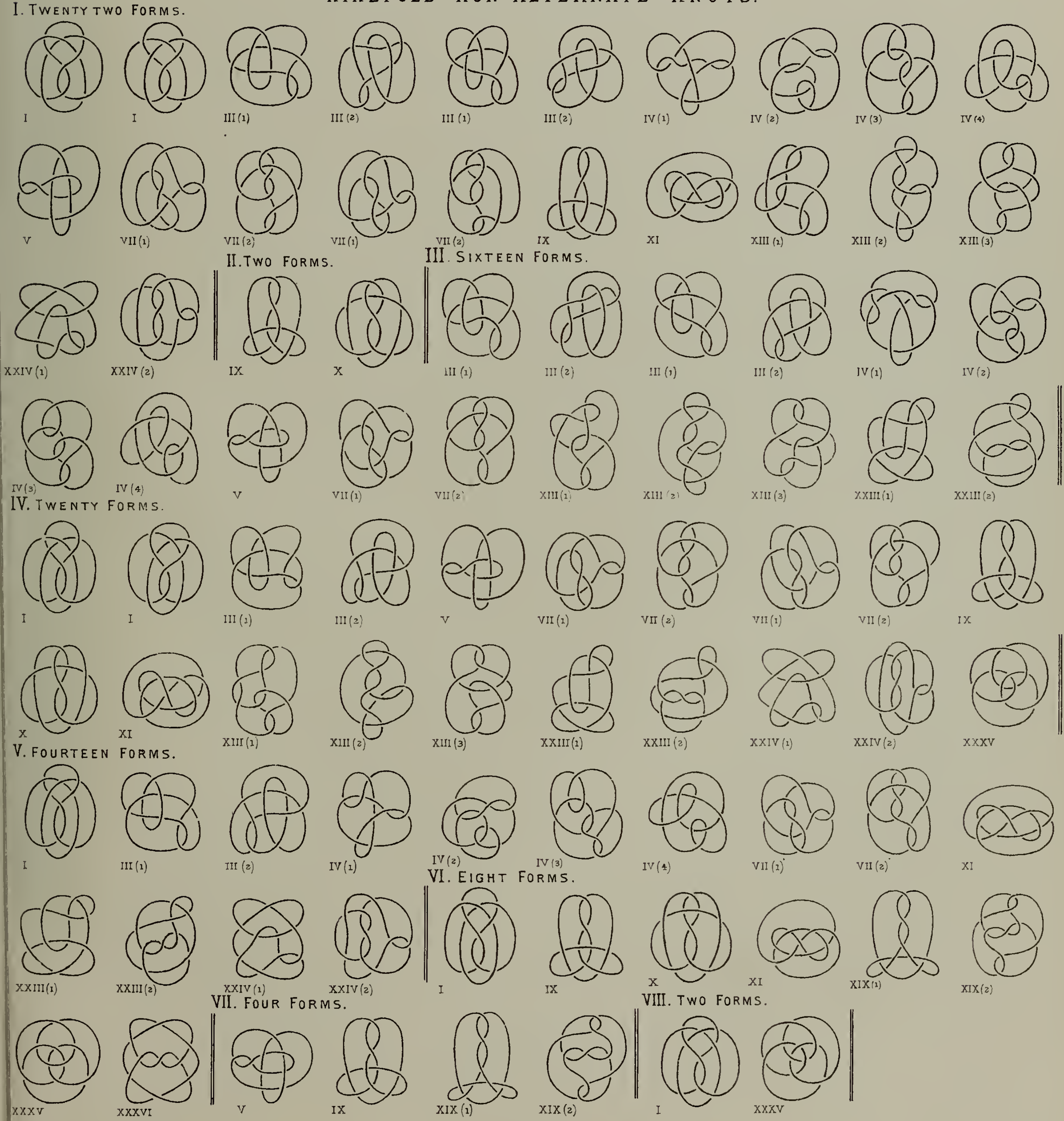
PROFESSOR LITTLE ON NON-ALTERNATE KNOTS.

EIGHTFOLD NON-ALTERNATE KNOTS.



NINEFOLD NON-ALTERNATE KNOTS.



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