[8] W. L. Chow, "Abelian Varieties over Function Fields," *Trans. Am. Math. Soc.* (to appear); the proof of the result quoted in the text is contained in a separate note, entitled "On Abelian Varieties over Function Fields," which will soon appear in print.

[9] Koizumi, *op. cit.*, **2**, 267.

[10] O. Zariski, "The Reduction of the Singularities of an Algebraic Surface," *Ann. Math.*, **40**, 639–689, 1939; "A Simplified Proof for the Resolution of Singularities of an Algebraic Surface," *ibid.*, **43**, 583–593, 1942; S. Abhyankar, "The Theorem of Local Uniformization on Algebraic Surfaces over Modular Fields," *Ann. Math.* (to appear).

[11] O. Zariski, "Pencils on an Algebraic Variety and a New Proof of a Theorem of Bertini," *Trans. Am. Math. Soc.*, **50**, 48–70, 1941; T. Matsusaka, "The Theorem of Bertini on Linear Systems in Modular Fields," *Kyoto Math. Mem.*, **26**, 51–62, 1950.

[12] Cf. Chow, "Abstract Theory of Picard Varieties," to appear soon in print.

A PATH SPACE AND THE STIEFEL-WHITNEY CLASSES

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The path space described here provides a very simple demonstration of the topological invariance of the Stiefel-Whitney classes for a differentiable manifold M. This invariance has been proved by other means by R. Thom.[1]

Actually, Thom proves a more general result, which implies the topological invariance of these classes. He shows that the fiber homotopy type of the tangent bundle of M depends only on the topological structure of M. Because the Stiefel-Whitney classes are dependent only on the fiber homotopy type of the tangent bundle, their topological invariance follows.

The path space we introduce is regarded as a fiber space[2] over M, and it turns out to have the same fiber homotopy type as the tangent bundle. Since the definition of the path space is purely topological, the general result of Thom follows immediately. Also, one sees directly that the Stiefel-Whitney classes have an analogue for manifolds without differentiability structure.

The paths considered are all continuous parametrized paths in M (parametrized by t, $0 \leq t \leq 1$), which do not recross the starting point (where $t = 0$). So, if $x(t)$ is the point with parameter t,

$$x(t) \neq x(0) \qquad \text{for} \qquad t > 0$$

is the requirement. These paths form a fiber space over M if we define the projection mapping to M by mapping each path into its starting point, $x(0)$.

We can regard M as provided with a smooth Riemannian metric. It is convenient[2] to assume that this metric is such that the geodesic distance between any pair of conjugate points is always more than one. Then, if two points are not more than one unit apart, there is a unique shortest geodesic segment joining them, and this segment varies continuously with the points.

The tangent bundle can now be regarded as formed by the geodesic paths of length 1, parametrized by arc length. This makes it a subspace of our fiber space of paths. What we prove is that the tangent bundle (thus represented) is a fiber deformation retract of the path space.

The retraction is easy to construct. The essential point is to find a parameter value t^* for each path which varies continuously with the path and has the property that for $0 \leq t \leq t^*$ the points $x(t)$ and $x(0)$ are less than one unit of distance apart. Then a unique shortest geodesic runs from $x(0)$ to $x(t)$.

To define t^*, let $d(t)$ be the distance from $x(t)$ to $x(0)$. Then we define a continuous monotone decreasing function $\delta(t)$ by

$$\delta(t) = \min_{\tau \leq t} |1 - d(\tau)|.$$

This function $\delta(t)$ varies continuously with the path; hence we can define t^* by the equation

$$t^* = \delta(t^*).$$

Using t^*, the deformation of the paths into geodesics of unit length can be defined as a three-stage process. First, contract the path along itself until it becomes the subpath ending at the parameter value t^*. We may now deform it into the geodesic segment from $x(0)$ to $x(t^*)$, with the use of the geodesic segments from $x(0)$ to $x(t), 0 \leq t \leq t^*$. Finally, the geodesic segment from $x(0)$ to $x(t^*)$ is gradually extended until it has unit length.

[1] Réné Thom, *Ann. sci. École norm. supér.* (ser. 3), **69**, 109–182, 1952.

[2] The path space is a true fiber space in the sense that the covering homotopy theorem holds. This fact is actually not needed to justify any of the results we obtain by use of the path space, but it deserves a remark.

One can show by construction that covering homotopy holds locally, even if there is no global differentiability structure on M, so we at least have a fiber space in Hu's sense. But then a recent (unpublished) theorem of Hurewicz shows that this local property implies the global covering homotopy property under very general conditions. Hence the name "fiber space" is justified.

[3] This assumption can be avoided, if desired, by changing the later construction. It is obviously realizable for compact manifolds by scale change and is fairly easily justified in general, so we include no proof of its realizability.

CHANGES IN THE NUCLEI OF DIFFERENTIATING GASTRULA CELLS, AS DEMONSTRATED BY NUCLEAR TRANSPLANTATION

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The hypothesis that the nucleus controls the differentiation of embryonic cells dates back to the time of Roux and Weismann. Originally it involved the assumption of a segregation of nuclear determiners of differentiation during cleavage. When put to the test, the hypothesis was found wanting.[1] Embryological experimentation showed that the distribution of nuclei during the early cleavages could be altered without producing a corresponding alteration of the developmental pattern. Furthermore, cytological evidence of the equational nature of mitosis was