

ZEROS OF THE ALEXANDER POLYNOMIAL OF KNOT

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ABSTRACT. The leading coefficient of the Alexander polynomial of a knot is the most informative element in this invariant, and the growth of orders of the first homology of cyclic branched covering spaces is also a familiar subject. Accordingly, there are a lot of investigations into each subject. However, there is no study which deal with the both subjects in a same context. In this paper, we show that the two subjects are closely related in p -adic number theory and dynamical systems.

1. INTRODUCTION

The leading coefficient of the Alexander polynomial $\Delta_K(t)$ of a knot K is a well-known invariant for detecting fibered knots. The Alexander polynomial of a fibered knot is always monic [19]. The converse is not always true, but it holds for many knots, for example, alternating knots [18]. Moreover the monic condition characterizes fibered knots in a sense of realization [2, 21].

The leading coefficient of the Alexander polynomial of a knot is also related to the commutator subgroup G'_K of the knot group $G_K = \pi_1(S^3 \setminus K)$. The abelianization of G'_K is finitely generated if and only if the leading coefficient is ± 1 [4, 22].

The r -fold cyclic covering $X_r(K)$ branched over a knot K is a fundamental object in the knot theory because topological invariants of it are also invariants of the knot. In [8], Gordon studied the growth of the order of $H_1(X_r(K); \mathbb{Z})$ with respect to r and asked whether the growth is exponential in case some zeros of $\Delta_K(t)$ are not a root of unity. More than 15 years later, this question was affirmatively answered by Riley [23] and González-Acuña and Short [7] independently.

As Gordon commented, the difficulty in computing the growth of orders arises from the case in which all zeros belong to the unit circle but some are not a root of unity (e.g. $\Delta_{5_2}(t) = 2t^2 - 3t + 2$). In this case, the standard norm is useless. Reliy [23] managed the difficulty by the p -adic analysis. On the other hand, González-Acuña and Short [7] managed it by showing that the growth is equal to the Mahler measure of the Alexander polynomial. Moreover, in their introduction, González-Acuña and Short remarked possibility of interpretation of the growth as entropy of a dynamical system.

In this paper, we clear the dynamical system which was predicted by González-Acuña and Short and also explain that the p -adic approach of Reliy is useful in essence.

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Solenoidal dynamical systems are natural generalizations of toral dynamical systems. The entropy of solenoidal dynamical systems was first calculated by Juzvinskiĭ [9] and latterly re-calculated by Lind and Ward [15] with p -adic number theory. Furthermore Einsiedler and Ward [5] also investigate the relation between the Mahler measure and the entropy of a solenoidal dynamical system associated with a Fitting ideal of a module.

Applying the above results of solenoidal dynamical systems to knot theory, we refine some topics of the Alexander polynomial of a knot. Here is an instance of results, in which the apparently different topics are really close. The reason of this similarity are explained in this paper.

Let $\Delta_K(t) = \sum_{i=0}^n a_i t^i$ ($a_0 a_n \neq 0$) be the Alexander polynomial of a knot K and α_i the zeros (counted with multiplicity) of $\Delta_K(t)$. Then,

- (1) the leading coefficient of $\Delta_K(t)$ is

$$\log |a_n| = \sum_{p < \infty} \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p$$

(Corollary 4), and

- (2) the growth of order of the first homology of the r -fold cyclic covering branched over K is

$$\lim_{\substack{r \rightarrow \infty \\ |H_1(\cdot)| \neq 0}} \frac{\log |H_1(X_r(K); \mathbb{Z})|}{r} = \sum_{p \leq \infty} \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p$$

(Corollary 1).

Here, $|\cdot|_p$ are the p -adic norms and $|\cdot|_\infty$ is the standard norm. (we assume that the embedding $\overline{\mathbb{Q}} \rightarrow \overline{\mathbb{Q}_p}$ are fixed.)

In our study, we also establish the followings.

- The distribution of the zeros measures a “distance” of the Alexander module from being finitely generated as \mathbb{Z} -module. (Section 3.2)
- In [8], the primary interest in investigating the growth (2) might be to study the periodicity of $H_1(X_r(K); \mathbb{Z})$. However, the growth also measures complexity of the Alexander module naturally in a sense of solenoidal dynamical systems. This means, this growth can be interpreted as volume growth in an adèle ring. (Section 3.1)

Here, we have a few comments on this study. It might make our study a little more attractive.

The Alexander polynomial of a knot is defined as a greatest common divisor of the initial Fitting ideal (elementary ideal) of the Alexander module $H_1(X_\infty(K); \mathbb{Z})$ as $\mathbb{Z}[t^\pm]$ -module. Here, the indeterminate t is identified with the meridian action on $H_1(X_\infty(K); \mathbb{Z})$ and $X_\infty(K)$ is the infinite cyclic cover of $X(K) = S^3 \setminus K$. Then, by tensoring with the rational numbers \mathbb{Q} , the Alexander polynomial is also generator of the Fitting ideal of the module $H_1(X_\infty(K); \mathbb{Q})$ as $\mathbb{Q}[t^\pm]$ -module, and hence it is the characteristic polynomial of the meridian action on $H_1(X_\infty(K); \mathbb{Q})$, up to units (see Theorem 6.17 in [13]).

While the rational homology $H_1(X_\infty(K); \mathbb{Q})$ gives a nice explanation of the Alexander polynomial, the leading coefficient a_n is lost in $H_1(X_\infty(K); \mathbb{Q})$ because a_n is a unit in $\mathbb{Q}[t^\pm]$. On the other hand, the entropy has an advantage over the Fitting ideal because we can replace $H_1(X_\infty(K); \mathbb{Z})$ with $H_1(X_\infty(K); \mathbb{Q})$ with preserving the entropy (cf. Step 1 in the proof of Proposition 1).

In [23], Riley also proved a result on the p -part of $|H_1(X_r(K); \mathbb{Z})|$. He obtained the upper bounds for the p -parts: $|H_1(X_r(K); \mathbb{Z})|^{(p)} < AH^{pE}r^n$, where A, H, E, n are constants depending on a knot and this upper bounds is best possible (up to constant multiplier). That is, the every p -parts growth at most polynomially with respect to r nevertheless the entire part $|H_1(X_r(K); \mathbb{Z})|$ can growth exponentially. (Silver and Williams [25] also studied this topic. Unfortunately, this part of their results had been already established by Riley, however they discussed this topic under mild hypotheses.)

In this paper, the p -adic coefficient homology group $H_1(X_\infty(K); \mathbb{Q}_p)$ plays an important role in studying the distribution of zeros (Theorem 1). This approach was motivated by the previous paper [20], in which we discussed that an analogy between the reciprocity of Δ_K and the functional equation in the Weil conjecture. Then, the p -adic (co)homology theory is also investigated to approach the Weil conjecture.

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2. SOLENOIDAL ENTROPY AND ALEXANDER POLYNOMIAL

A solenoid Σ^n is, by definition, a compact connected finite-dimensional abelian group, which was arose from a generalization of the torus \mathbb{T}^n . The following theorem, which was given by Lind and Ward [15], plays a key role for our results.

Proposition 1 (Lind and Ward [15]). *Let T be an automorphism of n -dimensional solenoid Σ^n . Then,*

- (1) *the entropy of T is the sum of the entropies of the automorphisms of \mathbb{Q}_p^n induced by T*

$$h(T; \Sigma^n) = \sum_{p \leq \infty} h(T; \mathbb{Q}_p^n),$$

and

- (2) the p -adic entropy is computed by the eigenvalues $\lambda_1, \dots, \lambda_n$ of the induced automorphism in $GL(d, \mathbb{Q}_n)$ as follows:

$$h(T; \mathbb{Q}_p^n) = \sum_{|\lambda_i|_p > 1} \log |\lambda_i|_p.$$

We shall confuse the topological entropy and the measure theoretic entropy by the formula of Bowen [1]:

$$h_d(T) = \lim_{\varepsilon \rightarrow 0} \limsup_{n \rightarrow \infty} \left[-\frac{1}{n} \log \mu(\cap_{k=0}^{n-1} T^{-k} B(e, \varepsilon)) \right],$$

where $B(e, \varepsilon)$ is a open ε -ball of the identity element with respect to an invariant metric d , μ is a Haar measure and T a surjective endomorphism on a locally compact abelian group. In other word, we regard the entropy as the topological entropy and also the measure theoretic entropy with respect to a Haar measure.

In [15], Lind and Ward computed the entropy of solenoidal automorphisms with intrinsic arguments. The proof is really helpful for our applications later. Although we review an outline of the proof, referring to the original paper is strongly recommend.

Outline of the proof.

Step 1: Because the dual group of Σ^d can be embed into $\mathbb{Q}^d \varinjlim \Gamma_n \cong \mathbb{Q}^d$ for $\Gamma_n = \frac{1}{n!} \widehat{\Sigma}^d$. That is, $\widehat{\mathbb{Q}}^d \cong \varinjlim \widehat{\Gamma}_n$. Hence there are K_n such that $\widehat{\Gamma}_n \cong \widehat{\mathbb{Q}}^d / K_n$ and consequently $h(T; \widehat{\mathbb{Q}}^d) = h(T; \widehat{\Gamma}_n) + h(T; K_n)$. Because $h(T; \widehat{\Gamma}_n) = h(T; \Sigma^d)$ for any n and $h(T; K_n) \rightarrow 0$ as $n \rightarrow \infty$, we obtain that

$$h(T; \Sigma^d) = h(T; \widehat{\mathbb{Q}}^d).$$

Step 2: The entropy on the full solenoid $\widehat{\mathbb{Q}}^d$ can be lifted to the entropy on the adele ring $\mathbb{A}_{\mathbb{Q}}^d$ because $\mathbb{A}_{\mathbb{Q}}/\mathbb{Q} \cong \widehat{\mathbb{Q}}$. So, we have

$$h(T; \widehat{\mathbb{Q}}^d) = h(T; \mathbb{A}_{\mathbb{Q}}^d).$$

Step 3: Since the adele ring is a restricted direct product space, the entropy on it can be decomposed into the entropies of each direction, i.e.

$$h(T; \mathbb{A}_{\mathbb{Q}}^d) = \sum_{p \leq \infty} h(T; \mathbb{Q}_p^d).$$

Step 4: Finally, the p -adic norm is the module on \mathbb{Q}_p (i.e. $\mu(a \cdot) = |a|_p \mu(\cdot)$ for any $a \in \mathbb{Q}_p$, where μ is a Haar measure), we have

$$h(T; \mathbb{Q}_p^d) = \sum_{|\lambda_i|_p > 1} \log |\lambda_i|_p.$$

□

To connect the Alexander polynomial with a solenoidal automorphism, we need the following lemma.

Lemma 1. *For any knot, the dual group of the first homology group of an infinite cyclic cover $H_1(X_{\infty}(K); \mathbb{Z})$ is a n -dimensional solenoid. Here, n is the degree of the Alexander polynomial of the knot.*

Proof. It is sufficient to prove that $H_1(X_\infty(K); \mathbb{Z})$ is a discrete torsion-free abelian group which has finite-rank n . Rapaport [22] and Crowell [4] proved that $H_1(X_\infty(K); \mathbb{Z})$ is torsion-free and has finite rank n . (Here, the rank of A means the cardinality of any maximal set of \mathbb{Z} -linearly independent elements of A .) \square

Because the Alexander polynomial is equal to the characteristic polynomial of the meridian action on $H_1(X_\infty(K); \mathbb{Q})$, up to multiplication by a unit, the following theorem follows immediately from Proposition 1 and Lemma 1.

Theorem 1. *Let α_i be the zeros (counted with multiplicity) of the Alexander polynomial of a knot. Then,*

- (1) *the entropy of the meridian action on the p -adic Alexander module; $t_p : H_1(X_\infty(K); \mathbb{Q}_p) \rightarrow H_1(X_\infty(K); \mathbb{Q}_p)$ is*

$$h(t_p) = \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p,$$

where $|\cdot|_p$ is the p -adic norms, and

- (2) *the entropy of dual action of meridian $\hat{t} : H_1(\widehat{X_\infty(K)}; \mathbb{Z}) \rightarrow H_1(\widehat{X_\infty(K)}; \mathbb{Z})$ is $h(\hat{t}) = \sum_{p \leq \infty} h(t_p)$, that is*

$$h(\hat{t}) = \sum_{p \leq \infty} \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p.$$

Here $\mathbb{Q}_\infty = \mathbb{R}$ by convention.

3. APPLICATIONS

3.1. Growth of order of homology of branched cyclic covering space. In this section, we study the relation between the p -adic norm of the zeros of Δ_K and the growth of orders of the first homology groups of the r -fold cyclic covering of S^3 branched over K . Roughly speaking, the r -fold cyclic covering branched over K is a compact space which associated with a homomorphism $G = \pi_1(S^3 \setminus K) \rightarrow \mathbb{Z}/r\mathbb{Z}$ (for precise definition, see [3, 13]).

The order of the first homology group of this space can be computed by the following formula.

Proposition 2 (Fox [6], Weber [26]). *Let $X_r(K)$ be the r -fold cyclic covering of S^3 branched over K . Then, the order of the first homology group of $X_r(K)$ is given by*

$$|H_1(X_r(K); \mathbb{Z})| = \left| \prod_{d=1}^{r-1} \Delta_K(\exp(2d\pi\sqrt{-1}/r)) \right|.$$

By convention, $|H_1(X_r(K); \mathbb{Z})| = 0$ means that $H_1(X_r(K); \mathbb{Z})$ is an infinite group.

Definition 1 (logarithmic Mahler measure [16]). For non-zero Laurent polynomial $f(x)$ with integral coefficients, the logarithmic Mahler measure of f is defined by

$$m(f) = \int_0^1 \log |f(\exp(2\pi t\sqrt{-1}))| dt.$$

The growth of orders $|H_1(X_r(K); \mathbb{Z})|$ is expressed by the logarithmic Mahler measure of the Alexander polynomial, which was proved by González-Acuña and Short [7].

Proposition 3 (González-Acuña and Short [7]).

$$\lim_{\substack{r \rightarrow \infty \\ |H_1(\cdot)| \neq 0}} \frac{\log |H_1(X_r(K); \mathbb{Z})|}{r} = m(\Delta_K).$$

Remark 1. In [24], Silver and Williams generalized the result of González-Acuña and Short [7] to links and also pointed out that Proposition 3 holds though the order $|H_1(X_r(K); \mathbb{Z})|$ is replaced with the order of the torsion subgroup $|TH_1(X_r(K); \mathbb{Z})|$.

By the way, the Mahler measure is deeply related to the entropy of an algebraic dynamical system, which is found in [14] for example. In this paper, we use a more suitable result which was proved by Einsiedler and Ward [5].

Proposition 4 (Einsiedler and Ward [5]). *Let $0 \rightarrow F_n \rightarrow \dots \rightarrow F_1 \xrightarrow{\phi_1} F_0 \rightarrow M \rightarrow 0$ be a finite free resolution of the $\mathbb{Z}[t^{\pm}]$ -module M and $J(\phi_1)$ the initial Fitting ideal. Let α_t a natural automorphism which is induced by the shift of the indeterminate t . Then the entropy of α_t is*

$$h(\hat{\alpha}_t) = m(\gcd(J(\phi_1))).$$

By combining Proposition 2 and 4, we can see that the growth of orders gives another method to compute the entropy in Theorem 1. Therefore, we can obtain the following corollary.

Corollary 1.

$$\lim_{\substack{r \rightarrow \infty \\ |H_1(\cdot)| \neq 0}} \frac{\log |H_1(X_r(K); \mathbb{Z})|}{r} = \sum_{p \leq \infty} \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p,$$

where α_i are the zeros of the Alexander polynomial $\Delta_K(t)$.

Proposition 3 and Corollary 1 are similar. However, the every terms of the right hand side of Corollary 1 make sense in a dynamical system. Actually, Corollary 1 can be rewritten into the following form:

$$\lim_{\substack{r \rightarrow \infty \\ |H_1(\cdot)| \neq 0}} \frac{\log |H_1(X_r(K); \mathbb{Z})|}{r} = \sum_{p \leq \infty} h(t_p),$$

where $h(t_p)$ is the entropy of the meridian action $t_p : H_1(X_{\infty}(K); \mathbb{Q}_p) \rightarrow H_1(X_{\infty}(K); \mathbb{Q}_p)$. In other words, we resolved the growth into fine factors of the p -adic entropy.

Needless to say, we also obtain the following property as the spacial case of Corollary 1.

Corollary 2 (Riley [23], González-Acuña and Short [7]). *Let $X_r(K)$ be the r -fold cyclic covering branched over K . Then, if the Alexander polynomial $\Delta_K(t)$ have zeros which are not roots of unity, the finite values of the order of the first homology group $|H_1(X_r(K); \mathbb{Z})|$ grows exponentially with respect to r .*

Proof. (Indirect proof) Because all α_i belong to the valuation ring $\mathcal{O}_p = \{x \in \overline{\mathbb{Q}_p} \mid |x|_p \leq 1\}$, $f(t) = \prod_i (t - \alpha_i)$, which is $\Delta(t)$ up to constant multiples, belongs to $\mathbb{Z}_p[t] \cap \mathbb{Q}[t]$, where $\mathbb{Z}_p = \{x \in \mathbb{Q}_p \mid |x|_p \leq 1\}$. This holds for any prime p . Hence $f(t) \in \mathbb{Z}[t]$. (Another way to see this is to prove Corollary 4 at the first. But in this proof, the condition $\Delta(1) = \pm 1$ is not necessary.) Consequently, the zeros of the Alexander polynomial must be roots of unity from $|\alpha_i| \leq 1$ and the Kronecker's theorem [12]. \square

3.2. Leading coefficient of Alexander polynomial. In this section, we apply Theorem 1 to a criterion for being finitely generated as \mathbb{Z} -module. By the corollary below, we can regard the entropies $h(t_p)$ for all primes $p < \infty$ as obstructions for being finitely generated.

Corollary 3. *Let $h(t_p)$ be the entropy of the meridian action on the p -adic Alexander module $H_1(X_\infty(K); \mathbb{Q}_p)$. If the Alexander module $H_1(X_\infty(K); \mathbb{Z})$ is finitely generated as \mathbb{Z} -module, then all the entropies $h(t_p)$ are equal to zero for all finite primes $p < \infty$.*

Proof. The Alexander module $H_1(X_\infty(K); \mathbb{Z})$ is finitely generated if and only if $H_1(\widehat{X_\infty(K)}; \mathbb{Z})$ is isomorphic to the n dimensional torus. Then, it is covered by $H_1(X_\infty(K); \mathbb{R})$. By the well-known result for toral automorphisms, the entropy of the meridian action on $H_1(\widehat{X_\infty(K)}; \mathbb{Z})$ is

$$h(t) = \sum_{|\alpha_i| > 1} \log |\alpha_i|,$$

where α_i are eigenvalues of the meridian action on $H_1(X_\infty(K); \mathbb{R})$.

These entropy must be equal to the entropy in Theorem 1-(2). Therefore, the entropies of the meridian action on $H_1(X_\infty(K); \mathbb{Q}_p)$ are zero for any $p < \infty$, that is

$$h(t_p) = \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p = 0 \quad \text{for } p < \infty.$$

\square

This obstructions give a new viewpoint of the leading coefficient of the Alexander polynomial. The following Corollary means that the entropies $h(t_p)$ are fine factors of the leading coefficient of the Alexander polynomial.

Corollary 4. *Let α_i be the zeros of $\Delta_K(t) = \sum_{i=0}^n a_i t^i$. Then the leading coefficient of $\Delta_K(t)$ is the sum of the entropies of the meridian action on the p -adic Alexander module $H_1(X_\infty(K); \mathbb{Q}_p)$ for the finite primes $p < \infty$. In other words, the following equation holds:*

$$\log |a_n| = \sum_{p < \infty} \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p.$$

Proof. Let $f(t) = \Delta_K(t)/a_n = \prod (t - \alpha_i)$ and s the least common multiple of the denominators of the coefficients of $f(t)$. Then,

$$\sum_{p < \infty} h(t_p) = \sum_{p < \infty} \sum_{|\alpha_i|_p > 1} \log |\alpha_i|_p = \log s.$$

Because $\Delta_K(1) = \pm 1$, the coefficients are relatively prime $(a_n, \dots, a_1) = 1$. Hence $s = |a_n|$. (The above argument is essentially found in the proofs of Theorem 3 in [15] and Theorem 2 in [23].) \square

Since $\Delta_K(1) = \pm 1$ for any knot, the Alexander polynomial can be completely determined (up to ± 1) by the zeros. Hence the leading coefficient is also determined. On the other hand, Corollary 4 shows that the leading coefficient is resolved into the p -adic entropy and consequently can be recovered from the zeros.

3.3. Final remarks.

3.3.1. Determining knots by cyclic branched covers. In [11], Kojima showed that prime knots are determined by their cyclic branched covers. So, there might be a method to compute the Alexander module by the data of cyclic branched covers.

The growth of orders $|H_1(X_r(K); \mathbb{Z})|$ does not determine completely the Alexander module $H_1(X_\infty(K); \mathbb{Z})$. However the growth measures complexity of the Alexander module from dynamical viewpoint, and also it is a similar invariant with the leading coefficient of the Alexander polynomial.

In addition, it is still remain open whether infinitely many branched covers are necessary for determining the knot.

3.3.2. Volume conjecture. The volume (or Kashaev) conjecture [10, 17] expects that the asymptotic behavior of the Kashaev invariant (= the specialization of the colored Jones polynomial) implies the hyperbolic volume of the complement of the knot (hyperbolic case).

In general, the topological entropy picks up natural measures by the variational principle. In other words, the topological entropy is related with natural measure theory behind it.

In this paper, the asymptotic behavior of the special values of the Alexander polynomial is related with the entropy of an action on the adèle ring of \mathbb{Q} . Consequently, the asymptotic behavior is related with the volume growth with respect to the Haar measure on the adèle ring.

Our point is that, when an asymptotic behavior is interpreted as an entropy, it can be related with a natural measure theory. Does this strategy work out for the volume conjecture?

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