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Corrections for "Patch Spaces"

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In Corollary 7.7 of [1],

$$\text{``}u^{4*+1} + u^{4*+3} \in H^{4*+1}(BSF,\ Z_8)H^{4*+3}(BSF,\ Z_2)\text{''}$$

should read

$$\text{``}u^{4*+1} + u^{4*+3} \in H^{4*+1}(BSF,\ Z_{(2)}) + H^{4*+3}(BSF,\ Z_2)\text{''}\ .$$

The proof given for 7.7 shows that u^{4*+1} *is an element of order* 8 *in* $H^{4*+1}(BSF,\ Z_{(2)})$.

The surgery composition formula, 7.0 in [1], is wrong. I have been too vague with the definition of a "surgery problem". In the language of p. 311 [1], we need, in addition to the degree one mapping of spherical fibration

$$\begin{array}{ccc} \gamma & \xrightarrow{\quad f^* \quad} & \tau \\ \downarrow & & \downarrow \\ (P,\ \partial P) & \xrightarrow{\ f\ } & (X,\ \partial X) \end{array}\ ,$$

a framing for the Whitney sum of the "tangent bundle" of P with the disc bundle associated to γ. In the Poincaré category this framing may be given by the choice of a spherical class representative for the top dimensional homology generator of the Thom Spectrum for γ. Now, the mistake in the proof of 7.0 of [1] occurs on p. 322 where it is assumed that composing a surgery problem

$$\begin{array}{ccc} \gamma & \xrightarrow{\quad f^* \quad} & \tau \\ \downarrow & & \downarrow \\ (P,\ \partial P) & \xrightarrow{\ f\ } & (X,\ \partial X) \end{array}$$

with another surgery problem

$$\begin{array}{ccc} \gamma' & \xrightarrow{\quad h^* \quad} & \gamma \\ \downarrow & & \downarrow \\ (P',\ \partial P') & \xrightarrow{\ h\ } & (P,\ \partial P) \end{array}\ ,$$

for which h is a homotopy equivalence, leaves the surgery obstruction unchanged. *But this can be inferred only if* h^* *preserves the framings associated to the two surgery problems.* There are a number of useful cases where this framing problem doesn't arise:

1. For *rational surgery* ([1, 7.11]) no problem arises because, mod torsion groups, all BF framings are determined by their fiber-degree. So 7.0 is true in this case.

2. For *splitting problems** no framing problems arise, because *the framings needed for surgery are constructed from the map on which surgery is done.* So 7.0 is true in this case.

3. If

$$\begin{array}{ccc} \gamma & \xrightarrow{\quad f^* \quad} & \tau \\ \downarrow & & \downarrow \\ (X, \partial X) & \xrightarrow{f} & (Y, \partial Y) \end{array}$$

is a surgery problem (modulo boundaries) on which surgery has been completed up to the middle dimensions, and for which $(X, \partial X)$ has homological dimension different from 2, 6, 14, then 7.0 applies to any composition,

$$\begin{array}{ccccc} \gamma' & \xrightarrow{\quad g^* \quad} & \gamma & \xrightarrow{\quad f^* \quad} & \tau \\ \downarrow & & \downarrow & & \downarrow \\ (X', \partial X') & \xrightarrow{g} & (X, \partial X) & \xrightarrow{f} & (Y, \partial Y) \end{array} .$$

To see this, it suffices (by 3.18 in [1]) to consider only those g which are homotopy equivalences. This is the special case discussed in 3.16 of [1].

4. For the two surgery problems

$$\begin{array}{ccc} \gamma & \xrightarrow{\quad g_1^* \quad} & \tau \\ \downarrow & & \downarrow \\ (X, \partial X) & \xrightarrow{g_1} & (Y, \partial Y) \end{array}$$

with framing information $f_1 \in \pi_N(T(\gamma), T(\gamma|_{\partial X}))$, and

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* In the Poincaré category a splitting problem consists of a Poincaré space Y, with Poincaré subspace Y', and a homotopy equivalence $h: X \to Y$ in Poincaré-transverse position to $Y' \subset Y$. This splitting problem can be solved if there is an H-cobordism of $h: X \to Y$, $H: (W, \partial W) \to Y \times (I, \partial I)$, with H in Poincaré-transverse position to $Y' \times (I, \partial I) \subset Y \times (I, \partial I)$ so that

$$H: H^{-1}(Y' \times I) \cap \partial_+ W \to Y' \times 1 \quad \text{and} \quad H: \partial + W - H^{-1}(Y' \times I) \to (Y - Y') \times 1$$

are both homotopy equivalences. There are of course blocked versions (see [1, 7.0]) of splitting problems, and 7.0 holds for these as well.

$$\begin{array}{ccc} \tau & \xrightarrow{g_2^*} & \mathcal{H} \\ \downarrow & & \downarrow \\ (Y, \partial Y) & \xrightarrow{g_2} & (Z, \partial Z) \end{array}$$

with framing information $f_2 \in \pi_N(T(\tau), T(\tau|_{\partial Y}))$, if f_1 is sent to f_2 by mapping of Thom spaces $T(g_1^*)$, then the surgery composition formula holds for the composition of these two surgery problems. In this case it is part of our hypothesis that no framing problems arise.

There are only two places that 7.0 (or 3.18) is used in [1]: in Section 5 where it is repeatedly used for the composition of splitting problems; and in 7.1 where it used in its general and incorrect form. To make the proof of 7.1 correct, note that any surgery problem can be represented by a splitting problem.* Thus in the language of 7.1, each of the normal maps $1: S^8 \to S^8$, $g: M \to S^8$, $1: B_i \to B_i$, $f: B_1 \to B_2$ may be replaced by a splitting problem. Now 7.0 can be correctly applied to the composition diagram in 7.1

I'm indebted to G. Brumfiel for the correction of 7.7, and to I. Madsen for pointing out the error in 7.0.

BIBLIOGRAPHY

[1] L. E. JONES, Patch Spaces, Ann. of Math. **97** (1973), 306–343.

* But the two segments in the composition of surgery problems cannot necessarily be represented *simultaneously* by splitting problems.