Vladimir Abramovich Rokhlin (obituary)

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Vladimir Abramovich Rokhlin

On 3 December 1984 Professor Vladimir Abramovich Rokhlin died. Outstanding achievements in several fields of mathematics—topology, real algebraic geometry, and the theory of dynamical systems—are due to him. In Leningrad, where he worked for the last 24 years of his life, he founded a scientific school, which obtained wide recognition in the mathematical world.

Rokhlin was born in Baku on 23 August 1919. After graduating from secondary school at Alma Ata he enrolled, at the age of not quite 16, in the Faculty of Mathematics and Mechanics at the Moscow State University. His teacher during his undergraduate years was Plesner, the well-known functional analyst. In 1940 Rokhlin graduated from University and became a postgraduate student at the Institute of Mathematics of the Moscow State University. At that time his interests were measure theory, dynamical systems and topology.

At the beginning of July 1941 Rokhlin went as a volunteer to the front. At the end of 1946 he returned to scientific work. In December 1947 he brilliantly defended his candidate's thesis on the subject "Lesbesgue spaces and their automorphisms". From 1947 to 1952 Rokhlin was a junior member of the research staff at the Institute of Mathematics of the Academy of Sciences of the USSR, and in 1951 he defended his doctoral dissertation "On the most important metric classes of dynamical systems". In 1960 Rokhlin accepted the proposal of A.D. Aleksandrov, the Rector of the University of Leningrad, that he move to Leningrad and become professor of geometry in the Faculty of Mathematics and Mechanics at the Leningrad State University, where he worked right up to his retirement in 1981. In Leningrad Rokhlin organized an ergodic seminar (which functioned until 1972) and a topological seminar, which he ran until his last days, and which became one of the most influential topology seminars in the USSR. He gave many compulsory and optional lecture courses and supervized postgraduate and diploma students. His pedagogical work had great influence on mathematics instruction at the Leningrad State University.

Rokhlin's scientific work was basically devoted to three subjects: topology, real algebraic geometry, and ergodic theory.

In Rokhlin's work on topology a special place is occupied by four-dimensional manifolds. They first appeared in his research in the early fifties in connection with enumerations of homotopy groups of spheres—the groups $\pi_{n+3}(S^n)$. The series of papers [17], [18], [20] is devoted to the computation of these groups. The papers of Pontryagin, which showed the deep connection with geometrical problems of the topology of smooth manifolds, were the starting point. The problems arising in the computation of the groups $\pi_{n+3}(S^n)$ by these means were related to three- and four-dimensional manifolds. Their solution was later to play an incomparably more fundamental role than the actual computation of the group $\pi_{n+3}(S^n)$.

Indeed, the origins of the so-called cobordism theory, whose role in the later development of topology is well known, appeared in Pontryagin's and Rokhlin's papers. Rokhlin worked out the deep theory of three- and four-dimensional manifolds, and discovered the remarkable properties of the signature of four-dimensional manifolds: its divisibility and relationship to Pontryagin's characteristic classes. The Rokhlin-Thom formula $p_1 = 3\tau$ for four-dimensional manifolds, where p_1 and τ are respectively the Pontryagin number and the signature of the four-dimensional manifold, was the intellectual basis of Hirzebruch's multidimensional theorems. A special place in the later development of topology is held by Rokhlin's theorem that the signature of the spinor four-dimensional manifold is divisible by 16. Its multidimensional generalizations, discovered by Kervaire and Milnor, have played an important role in the theory of smooth structures. This theorem of Rokhlin is a very important component of Kirby's and Siebenmann's proof of the existence of topological manifolds that do not admit a combinatorial triangulation. Rokhlin's theorem showed that not every unimodular quadratic form can be realized as an intersection form of a smooth simply-connected closed four-dimensional manifold. Rokhlin's researches served as the starting point for the well-known recent papers by Casson and Friedman on the topology of four-dimensional manifolds. The geometrical techniques developed by Rokhlin in the theory of cobordisms of manifolds played a decisive role in the calculation of the torsion of cobordism groups (Rokhlin and Wall, late 50's). The special features of the signature of manifolds later proved to be extraordinarily important in various problems of topology. In 1957 Rokhlin, Schwarz, and Thom proved, on the basis of the above, the invariance with respect to piecewise-linear (PL) homeomorphisms of integrals of Pontryagin classes over cycles. Rokhlin's work played an important role in the proof of Novikov's definitive theorem about the topological invariance of these integrals. Later, in 1966, Rokhlin and Novikov (jointly) discovered the important property of "additivity" of the signature of manifolds with a boundary when they are glued along a whole component of the boundary, isolating the signature together with the Euler characteristic from a number of other topological invariants.

In 1972 Rokhlin turned to the study of topological properties of real algebraic manifolds. Until the 70's this field, in spite of individual significant achievements, had remained isolated from the general progress of topology. The change was due to the work of Arnol'd and Rokhlin. The starting point was a conjecture made by Gudkov about the arrangement of ovals of a real plane algebraic curve and the connection discovered by Arnol'd between the problem of the arrangement of ovals and the topology of four-dimensional manifolds. Rokhlin in [54] introduced more refined methods for the topology of manifolds. This enabled him to prove Gudkov's conjecture concerning manifolds of arbitrary dimension. In later papers Rokhlin undertook the systematic study of the complex topology of real algebraic curves, leading to new statements of basic problems of the topology of real algebraic manifolds. The first of these was the paper [58], published in 1974, where he extended the definition of complex orientations and the relation connecting them to arbitrary curves of type 1 (that is, to all curves that decompose their complexification) [60], [61]. In [61] Rokhlin undertook the systematic study of the connections between the real topology of a curve and its arrangement in the complexification. In particular, he clarified how the real topology of a curve of degree 6 depends on its type. In the same paper he turned to the study of rigid isotopies (now, following Rokhlin, called isotopies composed of non-singular curves of the same degree). In it he gave a complete survey of the state of the topology of real algebraic curves up to 1978 and posed numerous problems stimulating subsequent development.

In [61] Rokhlin proved new inequalities linking the numerical characteristics of the real topology of curves of type 1. This paper pointed out the reserves in the application of the theory of ramified coverings to the study of algebraic manifolds.

The contribution of Rokhlin to the topology of real algebraic manifolds has greatly influenced the modern aspect of this area.

Twenty five of Rokhlin's publications are devoted to ergodic theory. His interest in it had developed even before the war as a result of collaboration with Plesner and the influence of Kolmogorov, who at that time was already proselytizing this field of mathematics. From the very beginning Rokhlin set himself the task of working out the fundamental problems of this theory. Ergodic theory was established as an independent part of the general theory of dynamical systems in the early 30's, following work by Birkhoff and von Neumann. The first period of its development (up to 1958, that is, before Kolmogorov's introduction of the concept of the entropy of dynamical systems) was a period of increasing precision of the foundations and of research into various specific examples (from geodesic flows to number-theoretical systems). The most important work of this period is due to von Neumann, Kolmogorov, Khinchin, Halmos, Hopf, Kakutani, and Rokhlin. The paper [9], prepared during Rokhlin's undergraduate years, later formed the basis of his candidate's dissertation. In it he gave the axiomatics of Lebesgue spaces, that is, a certain category of measure spaces. The very term "Lebesgue space", proposed by Rokhlin, is now generally used. It denotes a complete separable (in the sense of [9]) space with a complete probability measure. The axiomatics of such spaces became very successful and fruitful. Lebesgue spaces are free from the well-known pathologies of general measure theory and fully cover the needs of analysis and ergodic theory. The most important achievement of [9] was the complete classification of measurable partitions in Lebesgue spaces. Only long afterwards, following the introduction of entropy and the development of combinational methods, did it become clear how important and useful was the concept of a measurable partition. Rokhlin applied the result about the classification of measurable partitions to the study of the decomposition of an automorphism into ergodic components. von Neumann had proved the existence of such a decomposition, but it remained unclear to what extent the structure of the components determines the automorphism as a whole. This was done by Rokhlin on the basis of the theory of measurable partitions, and was its first application. In [10] and [14], expounding the problem of the decomposition of an automorphism into ergodic components, Rokhlin essentially proved the theorem of measurable choice, now used in the most diverse problems of mathematics. The canonical system of measures (more often called the system of conditional measures), which he introduced as an invariant of a measurable partition, enabled him in [24] to give a remarkable metric classification of measurable functions.

We may also note some important results of this cycle: the special representation of flows (a more precise version of the Ambrose-Kakutani construction for Lebesgue spaces), a measurable realization of flows with a discrete spectrum [14], and a lemma obtained in the same paper, popular in ergodic theory and now called in the world's literature the "Rokhlin lemma" or sometimes the "Rokhlin-Halmos lemma". Later (together with Hurewicz) Rokhlin generalized it to measurable flows [12], [16].

With his characteristic breadth Rokhlin studied examples of dynamical systems of very various fields: algebraic automorphisms of compact commutative groups [13], [40] and number-theoretic endomorphisms, including the Gauss endomorphism. Starting from the last example, in [35] Rokhlin constructed the theory of exact endomorphisms and natural extensions of endomorphisms. These constructions are now used in the theory of one-dimensional maps, trajectory theory, and so on.

Kolmogorov's work on the entropy of dynamical systems gave a new impetus to the pursuit of ergodic theory, opening up a new period in the research into dynamical systems as a whole. Entropy as a metric invariant of dynamical systems permitted further advances in the problem of isomorphism and in addition stimulated the application of the methods of probability and information theory to the theory of dynamical systems. In this connection it turned out that the language created by Rokhlin in the theory of measurable partitions of a Lebesgue space is completely adequate; it had already been used in Kolmogorov's first papers on this subject.

Rokhlin quickly turned to the study and application of entropy. He established a number of important properties of it [30] and gave an axiomatic definition [37]. In a joint paper with Sinai he proved a theorem about the structure of automorphisms with positive entropy and established deep connections with the theory of stationary random processes; he constructed a theory of invariant partitions, gave methods for their construction, and so on. Later this was presented in the survey [47] which, together with the first survey [32] on entropy theory, became the main source of information before the publication of detailed monographs. Two papers by Rokhlin were devoted to generators in ergodic theory [39], [44]. He proved that every automorphism of a Lebesgue space has a countable generator, that is, it may be realized as a displacement in the space of trajectories of a stationary random process with a countable set of states. If the entropy of an automorphism is finite, then a generator may be chosen with finite entropy. Later Krieger proved the existence of a finite generator for an automorphism with finite entropy.

Rokhlin was very seriously concerned with questions of the teaching of mathematics in school, college, and university. His article "Area and volume" [48] may undoubtedly be counted among the classical papers on the methodology of mathematics teaching. He put forward a number of ideas about the state of mathematical education in our country. These were the subject of a series of his lectures to students and teachers at meetings of the Moscow and Leningrad Mathematical Societies, and these lectures were memorable to those who heard them. Until the end of his life Rokhlin willingly and with interest shared his thoughts and plans, many of which, unfortunately, were never carried out. His ideas had a strong influence on the teaching of mathematics in the Faculty of Mathematics and Mechanics at the Leningrad State University. For some years he headed the faculty's methodology committee and under his leadership nearly all the main mathematics courses were discussed and modernized; he strove to make the education of mathematics students modern both in content and in form. One of the plans that Rokhlin carried out was the inclusion of a compulsory topology course in the geometry course; his life showed how valuable this innovation was.

Rokhlin steadfastly developed a school of topology. He stimulated the activities of his students in very different fields of geometry, topology, and the theory of dynamical systems. Under his supervision over 20 people defended candidate and doctoral dissertations.

Rokhlin's varied interests included literature, the natural sciences, history, languages—conversations with him on these subjects were fascinating.

In his last years he fought manfully against serious illness, not leaving off work for a minute. His self-command and steadfastness inspired admiration in all who saw him during those years.

The bright memory of Vladimir Abramovich Rokhlin will for ever be cherished in the hearts of all who knew him.

V.I. Arnol'd, A.M. Vershik, O.Ya. Viro,
A.N. Kolmogorov, S.P. Novikov,
Ya.G. Sinai, D.B. Fuks

List of V.A. Rokhlin's published works(1)

[48] Area and volume, in: *Entsiklopediya elementarnoi matematiki, Kniga* 5 (Encyclopaedia of elementary mathematics, book 5), Nauka, Moscow 1966, 5-87.

[49] Embeddings and immersions in Riemannian geometry, Uspekhi Mat. Nauk **23**:4 (1968), 245.

[50] The normal Euler numbers of the projective plane and the Klein bottle in four-dimensional Euclidean space, Dokl. Akad. Nauk SSSR **191** (1970), 27-29. MR **44** # 2246.
= Soviet Math. Dokl. **11** (1970), 316-319.

[51] Two-dimensional homology and two-dimensional submanifolds of four-dimensional manifolds, Uspekhi Mat. Nauk **25**:3 (1970), 258.

[52] Embeddings and immersions in Riemannian geometry, Uspekhi Mat. Nauk **25**:5 (1950), 3-62 (with M.L. Gromov). MR **44** # 7571.

[53] Two-dimensional submanifolds of four-dimensional manifolds, Funktsional. Anal. i Prilozhen. **5**:1 (1971), 48-60. MR **45** # 7733.
= Functional Anal. Appl. **5** (1971), 39-48.

[54] Proof of a conjecture of Gudkov, Funktsional. Anal. i Prilozhen. **6**:2 (1972), 62-64. MR **45** # 5131.
= Functional Anal. Appl. **6** (1972), 136-138.

[55] Congruences modulo 16 in Hilbert's 16th problem, Funktsional. Anal. i Prilozhen. **6**:4 (1972), 58-64. MR **47** # 232.
= Functional Anal. Appl. **6** (1972), 301-306.

[56] Congruences modulo 16 in Hilbert's 16th problem. II, Funktsional. Anal. i Prilozhen. **7**:2 (1973), 91-92. MR **48** # 9734.
= Functional Anal. Appl. **7** (1973), 98-102.

[57] Recent work in the topology of real algebraic manifolds, Uspekhi Mat. Nauk **29**:3 (1974), 180.

[58] Complex orientations of real algebraic curves, Funktsional. Anal. i Prilozhen. **8**:4 (1974), 71-75. MR **51** # 4286.
= Functional Anal. Appl. **8** (1974), 331-334.

[59] *Nachalnyi kurs topologii. Geometricheskie glavy* (A beginning course in topology: geometrical chapters), Nauka, Moscow 1977 (with D.B. Fuks). MR **58** # 31080.
French translation: Mir, Moscow 1981.
English translation: Springer-Verlag, Berlin 1984.

[60] Type and signature of a real plane algebraic curve, Uspekhi Mat. Nauk **33**:3 (1978), 145.

(1) The begining of this list is published in the book *Matematika v SSSR za sorok let* 1917-1957 (Forty years of mathematics in the USSR, 1917-1957), Vol. 2, Fizmatgiz, Moscow 1959, 603-604, and in *Matematika v SSSR* 1958-1967 (Mathematics in the USSR, 1958-1967), Vol. 2, Nauka, Moscow 1970, 1142-1143.

[61] Complex topological characteristics of real algebraic curves, Uspekhi Mat. Nauk **33**:5 (1978), 77–89. MR **81m**:14024.
= Russian Math. Surveys **33**:5 (1978), 85–98.

[62] New inequalities in the topology of real plane algebraic curves, Funktsional. Anal. i Prilozhen. **14**:1 (1980), 37–43. MR **82e**:14066.
= Functional Anal. Appl. **14** (1980), 29–33.

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