Scottish Physics and Knot Theory's Odd Origins

Daniel S. Silver

My mother, a social worker and teacher, encouraged my interest in the mysteries of thought. My father, a physical chemist, fostered my appreciation of the history of science. The former chair of my department, prone to unguarded comment, once accused me of being too young to think about such things. As the years pass, I realize how wrong he is.

Introduction. Knot theory is one of the most active research areas of mathematics today. Thousands of refereed articles about knots have been published during just the past ten years. One publication, *Journal of Knot Theory and its Ramifications*, published by World Scientific since 1992, is devoted to new results in the subject.

It is perhaps an oversimplification, but not a grave one, to say that two mildly eccentric nineteenth century Scottish physicists, William Thomson (a.k.a. Lord Kelvin) and Peter Guthrie Tait were responsible for modern knot theory. Our purpose is not to debate the point. Rather it is to try to understand the beliefs and bold expectations that motivated Thomson and Tait.

James Clerk Maxwell's influence on his friend Tait is well known. However, Maxwell's keen interest in the developing theory of knots and links has become clear only in recent years, thanks in great part to the efforts of M. Epple. Often Maxwell was a source of information and humorous encouragement. At times he attempted to restrain Tait's flights from the path of science. Any discussion of the origin of knot theory requires consideration of the colorful trefoil of Scottish physicists: Thomsom, Tait and Maxwell.

The reductionist nature of modern science exposes its researchers to accusations of being excessively narrow. Tait and Thomson would have been free from any such charge. The questions that they wrestled were huge: What is the ultimate nature of matter? Where do thoughts go? What happens to us after we die?

This is a story of magnificent failure.

Early History of Knots. Imagine a length of rope or, better yet, a highly elastic cord looped and weaved about in any manner. Now connect its ends. The result is called a knot. Mathematicians usually regard two knots as the same if one can be deformed – stretched or twisted perhaps, but never broken – so that it looks exactly like the other. A knot is a simple circle, but how the circle is situated in space can be far from simple.

It is relatively easy to convince someone that two knots are the same: deform one until it appears identical to the other. But showing that two knots are different is difficult: One cannot rule out all possible contortions, since the possibilities are endless. To borrow a phrase of mathematician Tobias Dantzig, we encounter "the *dilemma of infinity*, like the legendary dragon guarding the entrance to the enchanted garden." Novel and cunning weapons are needed to subdue the beast.

Links are also the subject of knot theory. A *link* consists of several knots (called *components*) intertwined in any manner. A popular motif in ancient Rome, links were often added to mosaics that adorned homes and temples. Celtic knot and link patterns, the best examples being found in the magnificent Book of Kells (800 AD), appeared in Ireland in the seventh century, and then spread to Scotland, the scene of our story.

Mathematicians before Thomson, Tait and Maxwell played with knots. Johann Carl Frederich Gauss (1777-1855), the son of a Brunswick bricklayer and the greatest mathematician of his day, was the first to discover a nontrivial fact about links. In 1833, Gauss showed that the "number of intertwinings," what we call today the *linking number* of two knots, can be computed by an integral. Figure 1 below illustrates the idea: the two components of the so-called Hopf link on the left have linking number 1. Each pair of components of the Borromean rings on the right have linking number zero.) It is likely that Gauss was motivated by the problem of determining the smallest region of the celestial sphere, the *zodiacus*, onto which the orbits of two heavenly bodies can be projected.

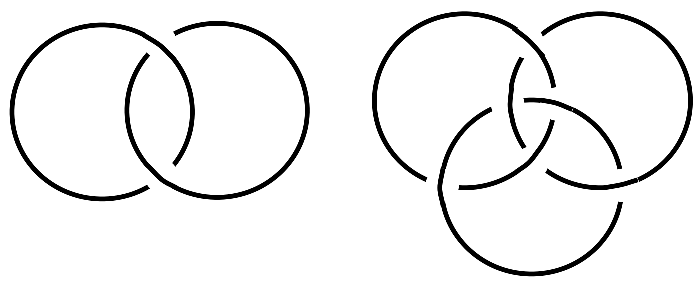


Figure 1: Hopf Link (left) and Borromean Rings (right)

Gauss discussed knots with his doctoral student, Johann Benedict Listing (1808-1882). Listing later coined the word "topology," a combination of the two Greek words topos (form) and logos (reason), to describe the new geometry of position. (He is responsible for many other scientific words and phrases commonly employed, "nodal points," "telescopic system" and "micron" being just a few.) Knots appear in Listing's first monograph, appropriately entitled *Topologie* (1847), but there is a lack of significant mathematical theorems about them. Listing had no effective tools at his command. A second monograph, *Der Census räumlicher Complexe oder Verallgemeinerung des Euler'schen Satzes von den Polyedern* (Census of spatial aggregates, or generalization of Euler's theorem on polyhedra) (1862), introduced ideas about connectivity that with sufficient effort might have been brought to bear on problems about knots. *Census* contains the first appearance what today is called the Möbius strip. Had Listing recognized the one-sided nature of the half-twisted band, posterity might have been kinder to his memory.

Gauss and Listing were curious about knots, but it does not appear that either was driven by the subject. Furher progress required someone completely obsessed with the idea.

Poincaré's aesthetic sieve. "The genesis of mathematical creation is a problem which should intensely interest the psychologist." The distinguished French mathematician Henri Poincaré opened an address to l'Institute General Psychologique in Paris in 1908 with these words. While it might have been presumptious for a mathematician to lecture psychologists about their subject, Poincaré was no ordinary mathematician. In the same year he became a member of the French Academy, one of 40 *immortels* who hold lifetime seats. Besides, Poincaré had something urgent to discuss.

Poincaré was struck by the way that thoughts arrive. On several occasions, he had worked hard but unsuccessfully on a difficult problem, only to have its solution later pop into his head during a moment of relaxation. Poincaré assumed that our brains continue to work after we have put down the pencil. But with so many unpromising combinations of thoughts in play, how could the unconscious mind recognize a worthy idea before intruding our peace? Poincaré offered an answer:

> *The useful combinations are precisely the most beautiful, I mean those best able to charm this special sensibility that all mathematicians know, but of which the profane are so ignorant as often to be tempted to smile at it... It is this special aesthetic sensibility which plays the role of [a] delicate sieve.*

Of course, the wonderful ideas that come to us in such moments can turn out to be wrong. Perhaps it is evidence for Poincaré's theory that in such situations, mathematicians are often heard to mutter words to the effect of "Too bad. It would have been beautiful."

A colleague of mine once remarked, "Sometimes it is not a new idea that occurs to me when I am resting. Instead I suddenly realize that something that I thought was true is actually false!" An unpleasant experience such as this is common among mathematicians, and in fact it is another instance of the aesthetic sieve operating. The negative subjective value that we place on such publication-wrecking insights are our own, but insights they are nevertheless. Too bad, we say once more, it would have been beautiful.

The *aesthetic sieve* operates in all areas of creative thought. The beautiful combinations that it catches are "bisociations," to borrow a word coined by Arthur Koestler, "two self-consistent but habitually incompatible frames of reference." According to Koestler, such combinations are the basis of scientific humor, discovery and poetic image.

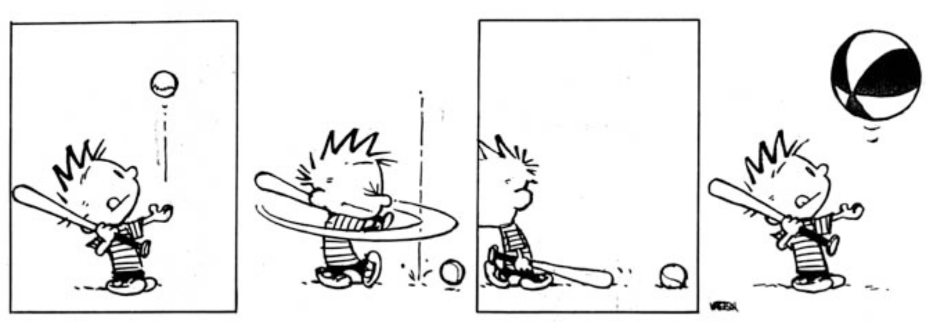


Figure 2: An example of bisociation in humor.

Combining disparate ideas gives us pleasure. It gives us a sense of a unity in the world. The scientist and poet Jacob Bronowski argued that a desire for such a sense is the driving force for all creative endeavor. Again quoting Koestler:

Newton saw that the moon behaved like an apple. Pasteur saw the analogy between a spoilt culture and a cow-pox vaccine; Fleming saw the analogy between the action of a mould and the action of a drip from his nose. Freud, by his own account, conceived the idea of sublimation of instincts by looking at a funny cartoon in the Fliegende Blätter – the one-time German equivalent of Punch.

The role of analogy. It is possible that Poincaré's aesthetic sieve operates in individuals with differing force. I am convinced that one can train the sieve to function, much as one can develop memory. Learning to form and appreciate analogies, especially at a young age, is an effective exercise.

Victorian scientists revered analogy, and honed their ability to use it. Possibly the most famous example is offered by Darwin's "Origin of Species." In his 1859 classic, Darwin compared the mechanism of language development and evolution.

Another example of Victorian zeal for analogy, less familiar but relevant for us, is Maxwell's 1856 essay "Are there Real Analogies in Nature?" read to the Apostles Club at Cambridge. Maxwell had a keen sense of humor, and it is fascinating to see him anticipate Koestler in his opening paragraph:

Now, as in a pun two truths lie hid under one expression, so in an analogy one truth is discovered under two expressions.

Exploring the subject of analogy by means of an analogy, one involving arithmetic, Maxwell reasoned:

Every question concerning analogies is therefore a question concerning the reciprocal of puns, and the solutions can be transposed by reciprocation.

During the preceding year, Maxwell had published his first monumental paper, "On Faraday's Lines of Force," developing a novel analogy between electric charge and heat flow. He had learned about the analogy from a paper of 1842. The author was William Thomson.

Kelvin. William Thomson (later Lord Kelvin) (1824-1907), was a brilliant mathematician and physicist. He was also an unreasonably confident man. As a Cambridge undergraduate, he was so sure that he would be Senior Wrangler, the student who scored highest on the grueling mathematical tripos examination, that after taking the exam he said to his servant, "Oh, just run down to the Senate House, will you, and see who is *Second* Wrangler." The servant returned with a devastating answer, "You are, sir!"

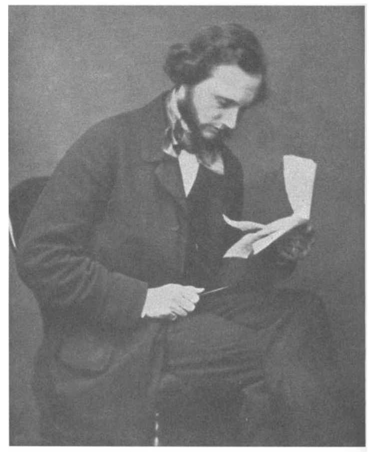


Figure 3: William Thomson in 1859

Thomson helped lay the first trans-Atlantic telegraph cable, and for his efforts he became Sir William Thomson, at the age of 42. (He became Lord Kelvin, or more precisely, Baron Kelvin of Largs, 26 years later.) His invention of the mirror galvanometer, which allowed faint signals to be detected, made him a wealthy man. The idea for the device came while Thomson was thinking about light reflecting off his monocle, another instance of bisociation.

Thomson is remembered by physics students for the absolute temperature scale that bears his name, a by-product of his most important research, which was in the study of heat, or thermodynamics. (The Kelvinator refrigerator was also named in his honor, certainly a lesser tribute.) After reading Carnot's "*Reflexions sur la Puissance Motrice du Feu*" (Relections on the Motive Power of Heat), he proposed two laws to express the indestructibility of energy. All physics, he concluded, should be based on the primacy of the energy concept.

Forming sweeping conclusions was a characteristic behavior, and one that caused him trouble. In 1866, without an awareness of radioactivity, discovered later, Thomson used Newton's Law of Cooling to estimate the age of the earth, and concluded that it was far younger than geologists proposed. Contradicted by Darwin's theories, Thomson embraced an attitude that today would be termed creationism.

In a superb biographical work, "Degrees Kelvin," David Lindley contends that Thomson became a crank in his later years, "a living fossil, a holdover from a forgotten era." Thomson rejected Maxwell's universally embraced theory of electricity and magnetism. He rejected radioactivity, still insisting that the earth was only 100 million years old. And above all he rejected the quickly developing atomic theory. A list of his many regrettable proclamations would include: *Radio has no future. Heavier-than-air flying machines are impossible. X-rays will prove to be a hoax.*

Tait. Peter Guthrie Tait (1831-1901), another Scottish student at Cambridge and a Senior Wrangler, was aggressive, argumentative and fiercely loyal to Thomson. "We never agreed to differ," Thomson recalled about his lifelong friend and collaborator, "[we] always fought it out. But it was almost as great a pleasure to fight with Tait as to agree with him."

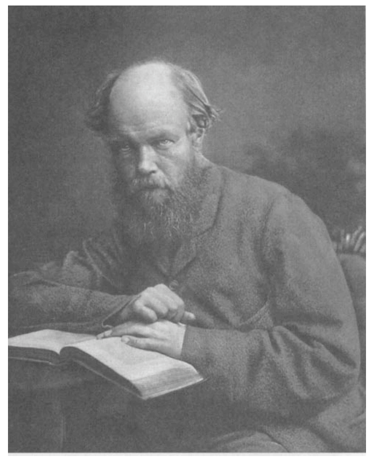


Figure 4: Peter Guthrie Tait in 1870

Tait joined the fray over the age of the earth. In 1885, he was still battling on behalf of Thomson. In a lecture for the general public, Tait summarized his position: "We cannot give more scope for [geologists'] speculation than about ten or (say at most) fifteen million years." If this upsets geologists, then "so much the worse for geology."

In 1860, the Curators of University of Edinburgh offered the vacant Chair of Natural Philosophy to Tait, passing over the more scientifically accomplished Maxwell. Tait's stronger teaching ability decided the issue. One of his students, J.M. Barrie, the author of *Peter Pan*, recalled Tait's fierce manner of instruction:

Never, I think, can there have been a more superb demonstrator. I have his burly figure before me. The small twinkling eyes had a fascinating gleam in them; he could concentrate them until they held the object looked at; when they flashed back around the room he seemed to have drawn a rapier. I have seen a man fall back in alarm under Tait's eyes, though there were a dozen benches between them.

Barrie added, "Tait's science weighed him to the earth." This was a grossly unfair judgment, and one that suggests that the two cultures of science and the arts decried a century later by C.P. Snow were already in place. In fact, Tait was an accomplished flutist. He knew by heart many Greek and Latin verses.

But above all, Tait loved golf. His habit of playing at half past six on summer mornings usually necessitated that he play alone. Often, as he chased after his guttapercha and feather golf ball, he recited Horace and Homer aloud. He composed an ode, entitled *The Morning Round*, that he would sing when classics were not enough. Based on the same popular song of the time, *Star of the Evening*, that Lewis Carroll had adopted for his *Soup of the Evening, Beautiful Soup*, Tait's verse began:

Beautiful Round! Superbly played –

Round where never mistake is made:
Who with enchantment would not bound
For the round of the morning, Beautiful Round?

Tait and Thomson were perfectly matched. Energetic, confident, and profoundly playful.

Tait's smoke ring box. On a day in 1867, Tait showed Thomson how to make smoke rings do tricks. Nine years ealier, Tait had learned from a paper of Helmholtz that a vortex-ring in an ideal incompressible fluid would be stable, and remain for all time. Air is not an ideal fluid, but Tait was content with an approximate model. He put a large hole in one end of a wooden box, removed the other end, substituting a towel tightly stretched. Inside the box, he sprinkled a strong solution of ammonia, and then placed inside a dish containing common salt and sulfuric acid. As Tait explained during a series seven years later:

These two gases combine to form solid sal ammoniac, so that anything visible which escapes from the box is simply particles of sal ammoniac, which are so very small that they remain suspended by fluid-friction, like smoke in the air. Now notice the effect of a sudden blow applied to the end of the box opposite the hole.

The air in the room would have been pungent. As Tait whacked the towel, vortex-rings emerged, vibrating violently, "just as if they were solid rings of india-rubber." Tait marveled at their stablility. If an elliptical or square hole was used instead of a round hole, the vortex shape would shake and vibrate until it assumed the shape of the circle, which Tait regarded as a "position of equilibrium."

Tait worked hard at perfecting his smoke ring experiment. In a letter to Thomson in 1867, he wrote:

Have you ever tried plain air in one of your boxes? The effect is very surprising. But eschew NO_5 *and* Zn. *The true thing is* $SO_3 + NaCl$. *Have* NH_3 *rather in excess– and the fumes are very dense+ not unpleasant.* NO_5 *is DANGEROUS. Put your head into a ring and feel the draught.*

Victorian scientists believed in the existence of an invisible, perfect fluid, called the *ether*, a notion that originated with Aristotle. Such a mechanical medium seemed essential if, say, the sun were to exert its gravitational pull on the earth. Thomson could not accept the idea of atoms, at least not as they were imagined at the time, as small, hard bodies. How could atoms, so conceived, account for the great variety of chemical elements? How could they vibrate and emit visible light?

Thomson had read Helmholtz's paper in 1858, and several reasons have been offered for Thomson's delayed response. But when he saw Tait's smoke rings gliding silently across the classroom, Thomson formed a remarkable bisociation. The basic blocks of matter are vortex-rings in the ether. Vortex motion imparted by a Creator had forever distinguished the otherwise homogeneous ether into its chemical parts. It was simple and beautiful. It had to be true.

Tait lectured with only the barest of notes. We are fortunate that his 1874 lectures were transcribed and published as “Recent Advances in Physical Science.” His audience consisted of “a number of my friends, mainly professional men, who wished to obtain in this way a notion of the chief advances made in Natural Philosophy since their student days.” From the transcription we have a faithful idea of Tait’s lecture style.

I shall try now to show you the effect of one vortex-ring upon another, just as I showed it to Thomson, when he at once formed his theory... You must all have seen that when you draw a teaspoon along the surface of a cup of tea, and lift it up from the surface, there are a couple of little eddies or whirlpools going around in the tea rotating in opposite directions... These two little eddies are simply the ends of a half vortex-ring. There can be ends in such a case as that, because these two ends are in the free surface of the fluid. A vortex-ring, then, cannot have ends...and if we adopt Thomson’s notion of a perfect fluid filling infinite space, of course, there can be no ends. All vortex-rings– and therefore, according to Sir William Thomson, all atoms of matter– must necessarily be endless, that is to say, must have their ends finally united together after any number of convolutions or knots.

In Thomson’s vortex-atom theory, chemical elements were knotted tubes of ether. Simplicity was part of its appeal. No cumbersome hypotheses would be needed to explain chemical properties. They were were a result of topology.

Elements exhibit characteristic colors or *spectra* when brought to a sufficiently high temperature. Sodium exhibits two “spectral lines.” Thomson concluded that sodium consists of two vortex-rings linked in the simplest fashion (Figure 1). In 1867, Thomson wrote to Helmholtz:

... every variety of combintations might exist. Thus a long chain of vortex rings, or three rings, each running through each other, would give each very characteristic reactions upon other such kinetic atoms.

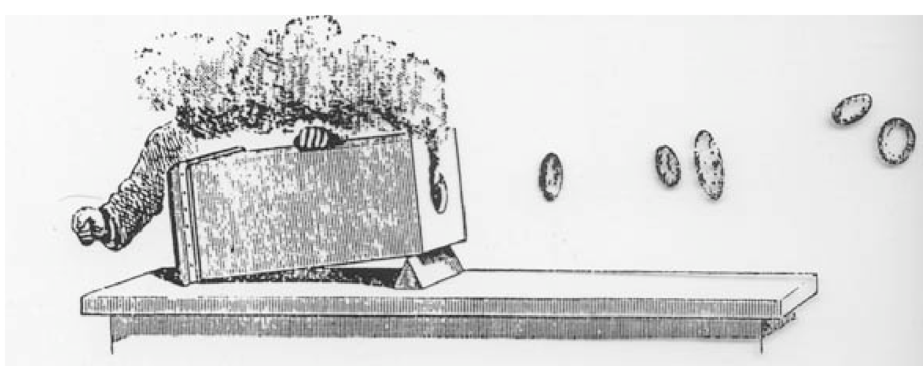


Figure 5: Tait’s smoke ring box

“Recent Advances in Physical Science” contains a detailed account of Tait’s smoke ring experiments with measurements of the boxes and description of chemicals used. However, the text and accompanying woodcut illustration give the reader only a limited idea of what Tait and Thomson experienced. Determined to learn the feel – and the smell – of the experiment, I made my own smoke ring boxes and spent a bizarre afternoon poisoning myself. Aided by colleagues Drs. Andrzej Wierzbicki and Susan Williams, we whacked at boxes filled with the combined fumes of acid and ammonia. As the rings emerge they wobble and shake. However, as they grow in diameter, often as much as 2 feet, they become stable. The sight of the rings sailing silently and gracefully across the room must have been deeply satisfying to Tait and Thomson. The corrosive fumes must have been less so.

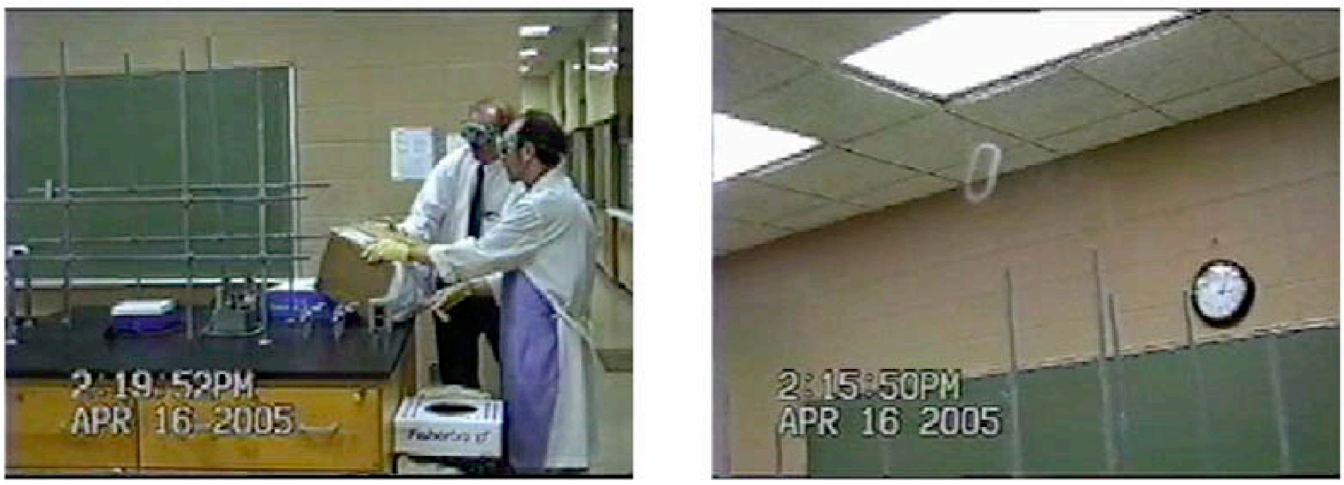


Figure 6: The author and Dr. Wierzbicki

We had limited success trying to make one ring bounce off another, as Tait described in his book. We had less success attempting to make one ring catch up and pass through another, a phenomenon that Tait also describes. We concluded that Tait practiced a great deal.

Victorian science was imbued with Newton’s mechanical philosophy. Pierre-Simon Laplace brought the philosophy to a natural conclusion when he asserted that if one could somehow know the forces at every point of the universe at some instant, then one could predict the future and describe the past. God, it seemed, was a billiards player resting after a particularly good shot. The vortex atom theory fit wonderfully into this dynamical vision of the universe.

dp/dt. James Clerk Maxwell and Tait were life-long friends. They met first at Edinburgh Academy. Maxwell was cheerful, humorous and athletic. However, he was also very private. His mind raced ahead of conversations, leaving others baffled in its wake.

Maxwell and Tait corresponded by the new half-penny postcards almost daily. Mail delivery was prompt, and it was possible to send a card and receive a reply in a single day. Fortunately, many of the cards that Maxwell sent have been preserved. Maxwell often used some amusing short-hand: Thomson was T while Tait was T′. John Tyndall, a highly successful popularizer of science who both Maxwell and Tait regarded as mediocre, was T″, a private joke suggesting that Tyndall was a "second-order quantity." Maxwell signed himself dp/dt, an abbreviation of an equation from thermodynamics:

$$\frac{dp}{dt} = JCM$$

(p and t denote pressure and temperature; J Joule's mechanical equivalent of heat; M is a coefficient of proportionality, the heat absorbed per unit volume change in an isothermal expansion; C is the universal Carnot function.)

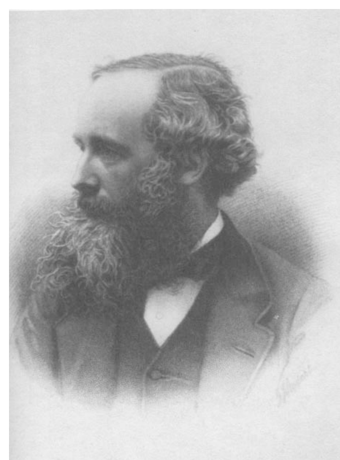


Figure 7: James Clerk Maxwell

It is likely that Maxwell became deeply interested in knots and topology generally because of Thomson's vortex atom theory and the influence of Tait. A card sent to Tait on November 13, 1867 suggests that he had been thinking about Helmholtz's papers on vortex motion and knots, most likely for implications about electricity and magnetism.

In letter of December 4, 1867, Maxwell wrote: "I have amused myself with knotted curves for a day or two." He proceeded to explained that the linking number of the two knots has physical significiance. Running electrical current through one knot produces a magnetic field. The linking number is essentially the work done by a charged particle moving along the path of the second knot. Maxwell expressed the linking number as a double integral that Gauss had discovered earlier.

The Borromean rings (Figure 1) have the curious property that any two of its three components have linking number zero. Still it is "interlocked," to use Maxwell's term. That is to say, it holds together. In his monumental *Treatise on Magnetism and Electricity* we find a similar example. Maxwell expressed his amazement in verse:

It's monstrous, horrid, shocking

Beyond the power of thinking,

Not to know, interlocking

Is no mere form of linking.

Listing's topological ideas and terminology play a significant role in the *Treatise.* Indeed Maxwell reported on Listing's *Cenus* to the London Mathematical Society in 1869. Tait was initially unaware of Listing's efforts in knot theory. Maxwell's postcard of January 22, 1877 to Tait contains the first known utterance of what has now become a very tired topological pun:

You are reading a paper in 4° in Gött. Comm. The knots are in an older paper in Gött serial (name forgotten) which lasted from '45 to '50 or so & died. There is good stuff in 4° paper but knot nots.

Maxwell's deep interest in knots and links is revealed in letters and notes published recently by Cambridge University Press. There one finds stereoscopic slides of knots, mentioned in Campell and Garnet's biography, intended to be viewed with a stereoscope of Maxwell's own improved design. There is also a photograph of a zoetrope, or wheel of life. Campell and Garnet describe Maxwell's design:

Another extremely pretty optical toy of his construction, at present in the possession of Mrs. Maxwell, is a Zoetrope, or "Wheel of Life." In the ordinary instrument, on looking through the slits in the revolving cylinder the figures are seen moving on the opposite side of the cylinder. Maxwell inserted concave lenses in place of the slits, the lenses being of such focal length that the virtual image of the object at the opposite extremity of the diameter of the cylinder was formed on the axis of the cylinder, and consequently appeared stationary as the cylinder revolved.

A close look at the hand-drawn figures in the photograph of Maxwell's zoetrope reveals that they consist of three simple rings. They represent smoke rings. In a paper of 1867, Helmholtz described how two vortex rings traveling in the same direction would affect one another:

If they have the same direction of rotation, they travel in the same direction; the foremost widens and travels more slowly, the pursuer shrinks and travels faster, till, finally, if their velocities are not too different, it overtakes the first and penetrates it. Then the same goes on in the opposite order, so that the rings pass through each other alternatively.

Having appreciated the *pas de deux*, Maxwell wished to understand the three ring dance. In a letter to Kelvin of October 6, 1868, he announced:

H^2's [Helmholtz's] 3 rings do as the 2 rings in his own paper that is those in front expand and go slower those behind contract and when small go faster and thread through the others. I drew 3 to make the motion more slow and visible not that I have solved the case of 3 rings more than to get a rough notion about this case...

Figure 8: Maxwell's zoetrope

Despite his racing mind and brilliant wit, Maxwell maintained a careful and objective voice in all of his writing. No one ever offers such words about Tait.

Unseen Universe. While the successes of Newton's mechanics was a source of national pride for Victorians, the materialistic philosophy it spawned was a source of unease. If all phenomena are explainable, then so are miracles. The certainty of revelation would be at risk.

Spiritualism became widely and wildly popular in Britain by 1870. A large number of prominent Victorians devoted their energy to its investigation. The Society for Psychical Research, founded in 1886, included William Gladstone, Lewis Carroll, John Ruskin and Alfred Lord Tennyson. The desire to buttress religious beliefs with science was at the core. Some hoped to find a rational argument that proved the existence of life after death.

A few scientists such as William Kingdon Clifford proclaimed their skepticism — even atheism — openly. Tyndall, champion of materialism, provoked many listeners with his address before the British Association meeting at Belfast in 1868. He argued that scientific arguments must be divorced from religious doctrine:

All religious theories, schemes and systems, which embrace notions of cosmogony, or which otherwise reach into the domain of science, must, in so far as they do this, submit to the control of science, and relinquish all thought of controlling it.

Tyndall's argument that scientific arguments were superior to religious faith outraged Tait. Tait and Belfast physicist Stewart Balfour responded with *The Unseen Universe*, a hastily written, rambling effort to justify miracles, spirits and, above all, an afterlife. Despite its literary and scientific shortcomings, *The Unseen Universe* was a commercial success. Tait and Balfour published the first two editions anonymously. A sequel, *Paradoxical Philosophy*, followed shortly afterwards.

The basic thesis of *The Unseen Universe* is that while the ether around us is imperfect, there are interspersed, parallel universes with more and more perfect ether:

Our meaning will be made clear by the following diagram. Here (0) denotes the evanescent smoke-ring, (1) the visible universe, (2) the invisible universe immediately anterior to the present, (3) that of the next order, and so on.

If we go infinitely far back, the authors contend, we reach "a universe possessing infinite energy, and of which the intelligent developing agency possesses infinite energy." Our thoughts cause molecular disturbances that ripple into the next world. Each vibration in our world dissipates and fades, but it transmits to the perfect ethereal realm a motion that is eternal. Technical arguments of thermodynamics mingle with the words of St Paul. The reader should not be surprised to learn that a trefoil knot adorns the spine and title page of the book.

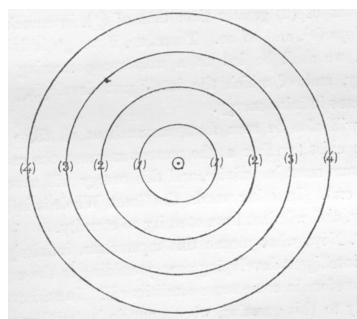


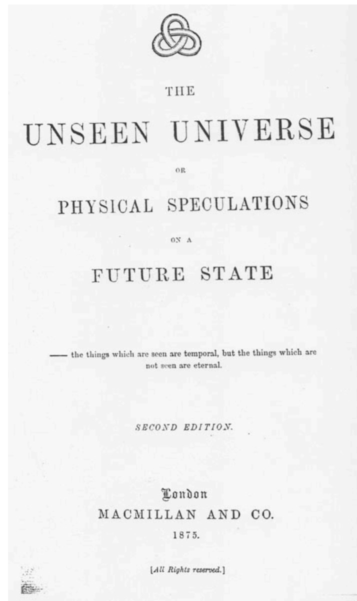
Figure 9: Illustration from Unseen Universe

From *The Unseen Universe* we see that Tait was not interested merely in mathematical questions about knots. He was hoping to answer the most intractable philosophical questions about consciousness, the soul and afterlife:

No doubt religion informs us... that there are other beings above man, but these do not live in the visible universe, but in that which is unseen and eternal.

Later we read:

We have now reached the stage from which we can very easily dispose of any scientific difficulty regarding miracles.


THE

UNSEEN UNIVERSE

OR

PHYSICAL SPECULATIONS

ON A

FUTURE STATE

— the things which are seen are temporal, but the things which are not seen are eternal.

SECOND EDITION.

London

MACMILLAN AND CO.

1875.

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Figure 10: Title page of Unseen Universe

Clifford's review in the Fortnightly Review was predictably sharp:

Rather let us contemplate the reposeful picture of the universal divan, where these intelligent beings while away the tedium of eternity by blowing smoke rings from sixty-three kinds of mouths . . . How fertile of resource is the theologic method, when once it has clay for its wheel!

Maxwell regarded Thomson's theory with characteristic caution. Explaining mass and gravitation would be formidable hurdles. In an article entitled *Atom*, written for Encyclopaedia Britannica, he wrote, "The difficulties of this method are enormous, but the glory of surmounting them would be unique."

Maxwell was deeply religious, but his beliefs were personal. While he admired the vortex atom theory, he had only sarcasm for Tait's attempts to mix scientific and religious arguments. In a letter of 1878, he wrote:

It is said in Nature *that U.U. [*Unseen Universe*] is germinating into some higher form. If you think of extending the collection of hymns given in the original work, do not forget to insert 'How happy could I be with Ether.' "*

A few months later, Maxwell reviewed the sequel, *Paradoxical Philosophy*, in Nature:

To exercise the mind in speculations on [ether] may be a most delightful employment for those who are intellectually fitted to indulge in it, though we cannot see why they should on that account appropriate the words of St Paul...No new discoveries can make the argument against the personal existence of man after death any stronger than it has appeared to be ever since men began to die.

Maxwell then sent Tait a 3-stanza poem, taking a more playful poke at his friend's speculations. The original draft was pasted into Tait's *Scrap Book*. The first stanza begins:

My soul is an entangled knot,
Upon a liquid vortex wrought
By Intellect in the Unseen residing
And thine doth like a convict sit,
With marlinespike untwisting it,
Only to find its knottiness abiding;
Since all the tools for its untying
In four-dimensional space are lying

As Maxwell apparently knew, knots can be undone if we allow space an extra dimension. Victorian spirituality nourished a popular interest in a 4th space dimension. Edwin Abbott's *Flatland: A Romance of Many Dimensions*, a satire on Victorian satire that explores worlds of different dimensions, was an immediate success when it was published in 1884, and it has remained popular in successive editions.

Maxwell informed and inspired Tait, as the two explored basic mathematical questions about knots and links. It does not appear that he was able to restrain his friend's spiritual impulses.

Tait's Program. Nine years after Kelvin proposed the vortex atom theory, Tait began an ambitious program of cataloging knots. He believed he was in fact building a table of chemical elements. Tait had no rigorous techniques for showing that his pictures represented different knots. What he had was good geometric intuition and courage. (In the 1920's, new techniques of Poincaré and J. Alexander verified that Tait's tables were essentially correct.)

After building tables for knots that can be drawn with 6 or fewer crossings, Tait was still dissatisfied.

The enormous numbers of lines in the spectra of certain elementary substances show that the form of the corresponding Vortex Atoms cannot be regarded as very simple.

Tait produced a table of "the first seven orders of knottiness" (knots that require 7 crossings). Even that was insufficient for Thomson's theory.

"Eight and higher numbers are not likely to be attacked by a rigorous process until the methods are immensely simplified," Tait wrote in 1877. In 1883, he reflected on his efforts:

We find that it becomes a mere question of skilled labour to draw all the possible knots having any assigned number of crossings. The requisite labour increases with extreme rapidity as the number of crossings is increased... It is greatly to be desired that some one, with the requisite leisure, should try to extend this list...

Inspired by Tait, others become involved. The Reverend Thomas P. Kirkman sent Tait a redundant list of 10-crossing knots. Tait worked hard to weed out duplications. When Charles Little, a mathematician and civil engineer at Nebraska State University sent Tait his own list, the two lists disagreed. Tait successfully located his single error before publication.

Kirkman also sent Tait a list of 1581 knots with 11 crossings. Tait decided that he could spare no more time for the project.

Others would take up Tait's labor. Tabulation, while no longer the main focus of knot theory, continues. Current tables extend through 16 crossings and contain 1.7×10^6 entries. The methods of compilation depend on a coding scheme found in Gauss's notes.

Conclusion. Thomson's confidence and aesthetic judgement combined with Tait's enthusiasm and philosophical nature to affect the first sustained research program in knot theory. Maxwell provided Tait with a steady supply of ideas and encouragement.

No evidence for the ether has ever been found. By the end of the 19th century, Michelson and Morley had conducted their now famous experiment on two occasions, failing to detect the expected "ether drag." The vortex atom theory faded like Tait's smoke rings. Thomson gradually abandoned his theory. In his 1889 Presidential Address to the Institution of Electrical Engineers, he remained hopeful. One can well imagine him thinking, "Too bad. It would have been beautiful":

And here, I am afraid I must end by saying that the difficulties are so great in the way of forming anything like a comprehensive theory that we cannot even imagine a finger-post pointing to a way that lead us towards the explanation. That is not putting it too strongly. I can only say we cannot now imagine it. But this time next year,– this time ten years, – this time one hundred years, – probably it will be just as easy as we think it is to understand that glass of water, which seems now so plain and simple. I cannot doubt but that these things, which now seem to us so mysterious, will be no mysteries at all; that the scales will fall from our eyes; that we shall learn to look on things in a different way– when that which is now a difficulty will be the only common-sense and intelligible way of looking at the subject.

The vortex atom theory represented what we might call the first attempt at a physical application of knot theory. Since then, knot theory techniques have made their way into fluid dynamics, solar physics and DNA research, and quantum computation. However, the physical significance of knotting remains elusive.

It is possible that a knot, like its simpler geometric cousin, the circle, represents a fundamental relationship of quantities. Abstract applications of Euclid's geometry are common today after two thousand years of reflection. It would be understandable a few more years are of thought will be needed before we can break free from an overly literal view of knots and links. When we finally understand their deepest nature, profound physical applications will blossom. And it will be beautiful.

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References.

There are several accounts of the history of knot theory, concentrating on technical aspects of the subject. The reader is particularly encouraged to consult the articles by M. Epple, listed in the references. Przytycki and A. Ranicki maintain a web page at http://www.maths.ed.ac.uk∼aar/knots/ with numerous articles and primary source materials.

E.A. Abbott, Flatland, reprinted, Dover Books, 1952.

J. Bronowski, Science and Human Values, Hutchinson, London, 1961.

L. Campell and W. Garnet, Life of James Clerk Maxwell, Macmillan and Co., London, 1882.

T. Dantzig, Number: The Language of Science, 4th edition, The Free Press, New York, 1954.

M. Epple, Topology, matter, and space, I: topological notions in 19th-century natural philosophy, Arch. Hist. Exact Sci. **52** (1998), 297–392.

M. Epple, Geometric aspects in the development of knot theory, in History of Topology (I.M. James, ed.), North-Holland, Amsterdam, 1999, 301–358.

C.F. Gauss, Werke, 6 Volumes, Köigliche Gesellschaft der Wissenshaften, Göttingen, 1863–1933, Volume VI, 106–118.

J. Hadamard, The Psychology of Invention in the Mathematical Field, Princeton University Press, 1945.

H. Helmholtz, On the integrals of the hydrodynamical equations, which express vortex motion, Phil. Mag., ser. 4, **33** (1867), 485–512.

J. Hoste, M. Thistlethwaite and J. Weeks, "The first 1,701,936 knots," *Math. Intelligencer* **20** (1998), 33–48.

C.G. Knott, Life and Scientific Work of P.G. Tait, Cambridge University Press, 1911.

A. Koestler, The Act of Creation, Hutchinson, London, 1964.

P. Laplace, Essai Philosophique sur les Probabilits (1820) forming the introduction to his Thorie Analytique des Probabilits, Paris: V Courcier; repr. F.W. Truscott and F.L. Emory (trans.), A Philosophical Essay on Probabilities, New York: Dover, 1951.

D. Lindley, Degrees Kelvin: A Tale of Genius, Invention and Tragedy, Joseph Henry Press, Washington DC, 2004.

J.C. Maxwell, The Scientific Letters and Papers of James Clerk Maxwell, 3 volumes, edited by P.M. Harman, Cambridge University Press, 1990, 1995, 2002.

J.C. Maxwell, The Scientific Papers of James Clerk Maxwell, (W.D. Niven, ed.), Dover, 1965.

H. Poincaré, Mathematical creation, in The World of Mathematics (J.R. Newman, ed.), Volume IV, 2041–2050, Simon and Schuster, 1956.

W. T. Thomson, Mathematical and Physical Papers, III, Cambridge University Press, 1890.

John Tyndall, Address Delivered Before the British Association Assembled at Belfast, With Additions.London: Longmans, Green, and Co., 1874.

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