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Transfer and ramified coverings

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Abstract

In this note we introduce a general class of finite ramified coverings $\pi:\tilde{X}\downarrow X$. Examples of ramified covers in our sense include: finite covering spaces, branched covering spaces and the orbit map $Y\downarrow Y/G$ where G is a finite group and Y an arbitrary G-space. For any d-fold ramified covering $\pi:\tilde{X}\downarrow X$ we construct a transfer homomorphism

$$\pi_{\natural}:H_*(X;\mathbb{Z})\rightarrow H_*(\tilde{X};\mathbb{Z}),$$

with the expected property that

$$\pi_*.\pi_{\natural}:H_*(X;\mathbb{Z})\circlearrowleft$$

is multiplication by d. As a consequence we obtain a simple proof of the Conner conjecture; viz. the orbit space of an arbitrary finite group action on a $\mathbb{Q}$-acyclic space is again $\mathbb{Q}$ acyclic.

1. *Ramified coverings*

Fix a natural number $d\in\mathbb{N}$ and a topological space Y. Introduce the following notations:

$$\begin{aligned}P^d(Y)&:=\overbrace{Y\times\ldots\times Y}^{d},\\ UP^d(Y)&:=P^d(Y)\times_{\Sigma_d}\{1,\ldots,d\},\\ SP^d(Y)&:=P^d(Y)/\Sigma_d,\end{aligned}$$

where Σ_d is the symmetric group on d elements acting on $[d]:=\{1,\ldots,d\}$ via permutations and on $P^d(Y)$ via permutation of the coordinates. The space $SP^d(Y)$ is called the d-fold symmetric product of Y (2). There is the projection map

$$p:UP^d(Y)\rightarrow SP^d(Y)=P^d(Y)\times_{\Sigma d}[1]$$

which serves as the generic example of a d-fold ramified covering. Specifically we introduce:

Definition. A surjective finite to one map $\pi:\tilde{X}\downarrow X$ is called a *d-fold ramified covering* iff there is a map

$$\mu:\tilde{X}\rightarrow\mathbb{N},$$

called the *multiplicity map* (*N.B.* μ is part of the structure) such that

(1) $\forall x\in X \sum_{\tilde{x}\in\pi^{-1}(x)}\mu(\tilde{x})=d,$

(2) the map

$$f_\pi:X\rightarrow SP^d(\tilde{X})$$

given by sending x into $\pi^{-1}(x)$, where each $\tilde{x}\in\pi^{-1}(x)$ occurs $\mu(\tilde{x})$ times, is continuous.

PROPOSITION 1·1. *For any space Y and $d \in \mathbb{N}$ the map $p: UP^d(Y) \downarrow SP^d(Y)$ is a d-fold ramified covering.*

Before taking up the proof we need to make explicit the multiplicity map

$$\mu: UP^d(Y) \to \mathbb{N}.$$

To this end we regard $P^d(Y)$ as the space of functions

$$Y^{[d]} := \{y: [d] \to Y\}.$$

The symmetric group Σ_d acts on $P^d(Y)$ via

$$\sigma \cdot y(j) = y(\sigma^{-1}(j)): \forall y \in P^d(Y), \quad \sigma \in \Sigma_d.$$

$UP^d(Y)$ is the orbit space of $P^d(Y) \times [d]$ under the diagonal action. To each point $(y,j) \in P^d(Y) \times [d]$ we associate the subset

$$A(y,j) := y^{-1}(y(j)) \subseteq [d].$$

Note that $A(\sigma \cdot (y,j)) = \sigma A(y,j)$ so that $|A(y,j)|$ depends only on the orbit of (y,j) in $UP^d(Y) = P^d(Y) \times_{\Sigma_d} [d]$ and thus we may define the multiplicity function μ by

$$\mu([y,j]) := |A(y,j)|,$$

where $|\quad|$ denotes cardinality. This defines

$$\mu: UP^d(Y) \to \mathbb{N},$$

completing the requisite structure for a ramified cover.

Proof of (1·1). We begin by analysing the map $p: UP^d(Y) \downarrow SP^d(Y)$.

If $[y] \in SP^d(Y) = P^d(Y)/\Sigma_d$ is represented by $y \in P^d(Y)$, then we claim

$$p^{-1}([y]) = \{[y,1], \dots, [y,d]\} \subset UP^d(Y).$$

To see this suppose (y',j'), $(y'',j'') \in P^d(Y) \times [d]$ represent the same point in $UP^d(Y)$. Then there exists $\sigma \in \Sigma_d$ such that

$$\begin{aligned} j' &= \sigma(j'') \\ y'(i) &= y''(\sigma(i)): \forall i \in [d]. \end{aligned}$$

In particular the elements of $p^{-1}([y])$ are representable by some $(y,j) \in P^d(Y) \times [d]$. Therefore unravelling the definitions we see that the sets

$$A(y,1), \dots, A(y,d) \subseteq [d],$$

define a partition $\pi[y]$ of $[d]$ and hence

$$\sum_{[y',j'] \in p^{-1}(y,j)} \mu([y',j']) = \sum_{A \in \pi[y]} |A| = d,$$

as required of the multiplicity function.

The preceding analysis shows that the map

$$f_p: SP^d(Y) \to SP^d(UP^d(Y)),$$

arises as follows. Let

$$l: P^d(Y) \to P^d(UP^d(Y)),$$

be defined by

$$l(y)(j) = [y,j]: j \in [d].$$

Then l is Σ_d equivariant and so induces a continuous map on quotient spaces, which upon unravelling the definitions is seen to be f_p, so f_p is continuous. ∎

Proposition 1·2. *If $(\tilde{X} \downarrow^{\pi} X, \mu)$ is a d-fold ramified covering then there is a cartesian square*

$$\begin{tikzcd} \tilde{X} \arrow[r, "\tilde{f}"] \arrow[d, "\pi"'] & UP^d(\tilde{X}) \arrow[d, "p"] \\ X \arrow[r, "f_\pi"'] & SP^d(\tilde{X}) \end{tikzcd}$$

Proof. To define $\tilde{f}$ let $\tilde{x} \in \tilde{X}$ and set $x = \pi(\tilde{x})$. Choose an arrangement of the points of $\pi^{-1}(x)$ counted according to multiplicity so that all $\mu(\tilde{x})$ copies of $\tilde{x}$ occur at the beginning; say for example

$$(\underbrace{\tilde{x}, \ldots, \tilde{x}}_{\mu(\tilde{x})}, \tilde{x}', \ldots) \in P^d(\tilde{X}).$$

Define

$$(*) \qquad \tilde{f}(\tilde{x}) := [(\tilde{x}, \ldots, \tilde{x}, \tilde{x}', \ldots), 1] \in UP^d(\tilde{X}),$$

where the square brackets denote equivalence class. To see that this is well defined let $\Sigma_{d-1} \hookrightarrow \Sigma_d$ as the isotropy group of $1 \in [d]$. Then for any $\sigma \in \Sigma_{d-1}$ and $(\tilde{x}_1, \ldots, \tilde{x}_d)$ belonging to $P^d(\tilde{X})$ we have

$$[(\tilde{x}_1, \ldots, \tilde{x}_d), 1] = [(\tilde{x}_1, \tilde{x}_{\sigma(2)}, \ldots, \tilde{x}_{\sigma(d)}), 1],$$

in $UP^d(X)$. Thus

$$[(\tilde{x}, \ldots, \tilde{x}, \tilde{x}', \ldots), 1] \in UP^d(\tilde{X}),$$

does not depend on the choice of the ordering of $(\tilde{x}', \ldots)$ and so $\tilde{f}(\tilde{x}) \in UP^d(\tilde{X})$ is well defined, and makes the diagram

$$\begin{tikzcd} \tilde{X} \arrow[r, "\tilde{f}"] \arrow[d, "\pi"'] & UP^d(\tilde{X}) \arrow[d, "p"] \\ X \arrow[r, "f_\pi"'] & SP^d(\tilde{X}) \end{tikzcd}$$

commute.

To establish the continuity of $\tilde{f}$ regard $P^d(\tilde{X})$ as the function space $\tilde{X}^{[d]}$. The evaluation map

$$e: P^d(\tilde{X}) \times [d] \to \tilde{X} : e(\tilde{x}, j) = \tilde{x}(j),$$

is Σ_d equivariant and so defines

$$\bar{e}: UP^d(\tilde{X}) \to \tilde{X}.$$

The composite φ defined by the commutative square

$$\begin{tikzcd}
X \times_{SP^d(X)} UP^d(X) \arrow[r, hook] \arrow[d, "\varphi"] & X \times UP^d(X) \arrow[d, "\text{proj.}"] \\
X & UP^d(X) \arrow[l, "\bar{e}"']
\end{tikzcd}$$

is closed since f is continuous. Moreover the map

$$\psi: \tilde{X} \to X \times_{SP^d(\tilde{X})} UP^d(\tilde{X}): \psi(\tilde{x}) = (\pi(\tilde{x}), \tilde{f}(\tilde{x})),$$

is inverse to φ so φ is bijective and hence a homeomorphism. Thus

$$\begin{tikzcd}
\tilde{X} \arrow[dr, "\varphi"', "\simeq"] \arrow[drr, "\tilde{f}"] \arrow[ddr, "\pi"'] & & \\
& X \times_{SP^d(X)} UP^d(\tilde{X}) \arrow[r, hook] \arrow[d, "\text{proj.}"] & UP^d(\tilde{X}) \arrow[d, "p"] \\
& X \arrow[r] & SP^d(\tilde{X})
\end{tikzcd}$$

establishes the continuity of $\tilde{f}$ and the cartesian nature of the square. ∎

PROPOSITION 1·3. *Suppose $(\tilde{Y} \downarrow^{\pi} Y, \mu)$ is a d-fold ramified covering and $\varphi: X \to Y$ is a continuous map. Then the pullback $\pi_\varphi: \tilde{X} \downarrow X$ is a ramified covering.*

Proof. To begin note

$$\pi_\varphi: \tilde{X} := X \times_Y \tilde{Y} \downarrow X: \pi_\varphi(x, \tilde{y}) := x.$$

We define

$$\mu: \tilde{X} \to \mathbb{N}: \mu(x, \tilde{y}) = \mu(\tilde{y}).$$

Then for any $x \in X$ we have

$$\sum_{\tilde{x} \in \pi_\varphi^{-1}(x)} \mu(\tilde{x}) = \sum_{\tilde{y} \in \pi_\varphi^{-1}(x)} \mu(\tilde{y}) = d,$$

as required. To obtain the continuity of

$$f_{\pi_\varphi}: X \to SP^d(\tilde{X}),$$

consider the inclusion $\tilde{X} \hookrightarrow X \times \tilde{Y}$. This induces an inclusion

$$SP^d(\tilde{X}) \hookrightarrow SP^d(X \times \tilde{Y}).$$

There is the commutative diagram

$$\begin{tikzcd}
X \arrow[rr, "f_{\pi_\varphi}"] \arrow[d, "{(1,f)}"] & & SP^d(\tilde{X}) \arrow[dr, hook] & \\
& & & SP^d(X \times \tilde{Y}) \\
X \times Y \arrow[rr, "{(1,f_\pi)}"'] & & X \times SP^d(\tilde{Y}) \arrow[ur, "D"'] &
\end{tikzcd}$$

where D is induced by the diagonal $X \hookrightarrow P^d(X)$. Since the composition $D \cdot (1 \times f_\pi) \cdot (1,f)$ is continuous and $SP^d(\tilde{X}) \hookrightarrow SP^d(X \times \tilde{Y})$ has the subspace topology it follows that f_{π_φ} is continuous. ∎

By combining the preceding results we obtain

COROLLARY 1·4. *$\pi: \tilde{X} \downarrow^{\pi} X$ is a d-fold ramified covering iff there is a cartesian square*

$$\begin{tikzcd}
\tilde{X} \arrow[r, "\tilde{f}"] \arrow[d, "\pi"] & UP^d(\tilde{X}) \arrow[d, "p"] \\
X \arrow[r, "f_\pi"'] & SP^d(\tilde{X})
\end{tikzcd}$$ ∎

PROPOSITION 1·5. *Let G be a finite group and Y a G-space. Then the orbit map*

$$\pi: Y \to Y/G,$$

admits the structure of a $d := |G|$ fold ramified covering.

Proof. Choose an ordering $g_1, \ldots, g_d$ of the elements of G. There are the continuous maps

$$\tilde{f}: Y \to UP^d(Y) \,|\, \tilde{f}(y) = [g_1 y, \ldots, g_d y, 1]$$
$$f: Y/G \to SP^d(Y) \,|\, f[y] = [g_1 y, \ldots, g_d y],$$

and the square

$$\begin{tikzcd}
Y \arrow[r, "f"] \arrow[d] & UP^d(Y) \arrow[d, "p"] \\
Y/G \arrow[r, "f"'] & SP^d(Y)
\end{tikzcd}$$

commutes. To see it is cartesian note that we can reinterpret Y as follows. A point y of Y is the pointed G-orbit $\pi^{-1}\pi(y)$ with basepoint y. But by definition this is

$$(Y/G) \times_{SP^d(Y)} UP^d(Y).$$

N.B. Since the group G is finite the orbit map $\pi: Y \to Y/G$ is clopen. For if A is open then $\pi^{-1}\pi(A) = \bigcup_{g \in G} gA$ is a union of open sets in Y, so open, whereas if A is closed the finiteness of the union implies $\pi^{-1}\pi(A)$ is closed. This assures the continuity of f and $\tilde{f}$. ▮

Remark 1. A 2-fold ramified covering $\pi: \tilde{X} \downarrow X$ is nothing but an involution $T: \tilde{X} \to \tilde{X}$.

To see this note that by (1·5) an involution defines a ramified double covering. On the other hand switching the points in each fibre of a ramified double covering $\tilde{X} \downarrow^{\pi} X$ defines an involution on $\tilde{X}$.

Remark 2. By (1·1) the quotient map $p: UP^d(Y) \downarrow SP^d(Y)$ is a d-fold ramified covering. Note that for any $k \in \mathbb{N}$ the k-fold diagonal map $\Delta_k: Y \to P^k(Y)$ induces a cartesian square:

$$\begin{tikzcd}
UP^d(Y) \arrow[r] \arrow[d] & UP^{kd}(Y) \arrow[d] \\
SP^d(Y) \arrow[r] & SP^{kd}(Y)
\end{tikzcd}$$

so that $UP^d(Y) \downarrow SP^d(Y)$ may be regarded as a kd-fold ramified covering. Since $p: UP^d(Y) \downarrow SP^d(Y)$ is the 'universal example' of a d-fold ramified covering, this says that by 'multiplying' all multiplicities by k we can regard any d-fold covering as a kd-fold ramified covering.

One can also examine the converse, namely when can one suitably redefine the multiplicity function of a kd-fold ramified cover so as to obtain a d-fold ramified covering. In this connection we have the useful:

Observation. Suppose given the diagram of solid arrows

$$\begin{tikzcd}
\tilde{X} \arrow[r, dashed] \arrow[d] \arrow[rr, bend left] & UP^d(\tilde{X}) \arrow[r] \arrow[d] & UP^{kd}(\tilde{X}) \arrow[d] \\
X \arrow[r, dashed] \arrow[rr, bend right] & SP^d(\tilde{X}) \arrow[r] & SP^{kd}(\tilde{X})
\end{tikzcd}$$

defining $\tilde{X} \downarrow X$ as a kd-fold ramified covering. If the dotted arrows exist as functions, then they are continuous.

Proof. $SP^d(\tilde{X}) \hookrightarrow SP^{kd}(\tilde{X})$ and $UP^d(\tilde{X}) \hookrightarrow UP^{kd}(\tilde{X})$ have the subspace topology. ▮

This implies:

COROLLARY 1·6. *Let $G \times Y \to Y$ be an action of a finite group G. If $d' \in \mathbb{N}$ is such that $d' \mid [G; Gy]$ for all $y \in Y$, where Gy is the isotropy group of y, then it is possible to define multiplicities so as to make the orbit map $Y \to Y/G$ into a d'-fold ramified covering.* ▮

For completeness we note:

PROPOSITION 1·7. *If $\tilde{X} \downarrow^{\pi} X$ is a finite covering then assigning the multiplicity* 1 *to every point of X makes it a ramified covering.*

Proof. Only the continuity of f_π is at stake. But this is a local question, so it suffices to look at the situation

$$V \times [d] \downarrow V: (v,j) \downarrow v,$$

where d is the number of sheets. However in this case continuity is clear. ∎

In the usual definition of a *branched* covering (4) one can compute the multiplicity of points from 'local data'. This suggests that for a d-fold ramified cover $\tilde{X} \downarrow X$ one introduce the set

$$X_d := \{x \in X \mid |\pi^{-1}(x)| = d\}.$$

N.B. It can well happen that $X_d = \varnothing$. A d-fold ramified cover is called *nice* iff $X_d \subset X$ is a dense subset. In this case we see that

$$f_\pi: X \to SP^d(\tilde{X}),$$

is determined by its restriction to X_d. So to compute the multiplicity of a point $\tilde{x} \in \tilde{X}$ we choose a net of points $x_\lambda \in X$ converging to $\pi(\tilde{x})$. Then $f_\pi(x_\lambda) \in SP^d(\tilde{X})$ converges to $f_\pi(\pi(\tilde{x}))$ which is a d-tuple of points of $\tilde{X}$ containing $\tilde{x}$ exactly $\mu(\tilde{x})$ times.

The following theorem of Chernowski (1) shows that the above discussion applies in the manifold situation.

THEOREM. *If a finite group G acts effectively on a smooth manifold M^n such that M^n/G is again a manifold, then the branching set $B \subset M^n/G$ has codimension $\leqslant n-2$. In particular the set $M^n/G - B = (M^n/G)_d$; $d := |G|$ is open and dense.*

2. *Transfer for ramified coverings: applications*

Let $\pi: \tilde{X} \downarrow X$ be a d-fold ramified covering. Then there is the map

$$f_\pi: X \to SP^d(\tilde{X}).$$

Consider the composite

$$\pi_\natural: X \xrightarrow[f_\pi]{} SP^d(X) \xhookrightarrow[i]{} SP^\infty(X)$$

where i is the standard inclusion induced by a choice of basepoint $\tilde{x} \in \tilde{X}$. By the theorem of Dold and Thom (2) there is a weak homotopy equivalence

$$SP^\infty(\tilde{X}) \to K(\tilde{H}_*(\tilde{X}; \mathbb{Z})),$$

where $K(A_*)$ denotes the Eilenberg–Mac Lane space for the graded abelian group A_*. Thus the based homotopy class of $\pi_\natural$ defines an element

$$[\pi_\natural] \in \tilde{H}^*(X; \tilde{H}_*(\tilde{X}; \mathbb{Z})),$$

and composing with the universal coefficient map

$$\begin{array}{c} \tilde{H}^*(X; \tilde{H}_*(\tilde{X}; \mathbb{Z})) \\ \downarrow \\ \operatorname{Hom}(\tilde{H}_*(X; \mathbb{Z}), \tilde{H}_*(\tilde{X}; \mathbb{Z})), \end{array}$$

gives a map

$$\pi_\natural: \tilde{H}_*(X; \mathbb{Z}) \to \tilde{H}_*(\tilde{X}; \mathbb{Z}),$$

which we call the *transfer homomorphism* for the ramified covering $\pi: \tilde{X} \downarrow X$.

PROPOSITION 2·1. *The transfer construction is natural with respect to pull backs and compositions of ramified coverings.* |

PROPOSITION 2·2. *If $\tilde{X} \downarrow^{\pi} X$ is a d-fold ramified covering then*

$$\pi_* . \pi_\natural : \tilde{H}_*(X;\mathbb{Z})\circlearrowleft$$

is multiplication by d.

Proof. One simply notes that

$$\begin{tikzcd}
X \arrow[r, "f_\pi"] \arrow[dr, "\Delta_d"'] & SP^d(\tilde{X}) \arrow[r, "i"] \arrow[d, "SP^d(\pi)"] & SP^\infty(\tilde{X}) \arrow[d, "SP^\infty(\pi)"] \\
& SP^d(X) \arrow[r, "i"] & SP^\infty(X)
\end{tikzcd}$$

commutes, where Δ_d is the d-fold diagonal map. The composition

$$SP^\infty(\pi)\cdot i\cdot f_\pi : X \to SP^\infty(X)$$

represents, by definition,

$$\pi_*([\pi_\natural]) \in \tilde{H}^*(X;\tilde{H}_*(X;\mathbb{Z}))$$

whereas

$$i\Delta_d \in \tilde{H}^*(X;\tilde{H}_*(X;\mathbb{Z}))$$

under the universal coefficient map goes to multiplication by $d: \tilde{H}_*(X;\mathbb{Z}) \to \tilde{H}_*(X;\mathbb{Z})$, which completes the proof. |

Remark 1. The above construction defines an integral homology transfer, and hence by tensoring a rational homology transfer. To define a transfer for homology with finite coefficients $\mathbb{Z}/n$ we replace the map

$$i: SP^d(\tilde{X}) \hookrightarrow SP^\infty(\tilde{X}),$$

with the map

$$i_n: SP^d(\tilde{X}) \longrightarrow \varinjlim\{SP^d(\tilde{X}) \xrightarrow{\Delta_n} SP^{nd}(\tilde{X}) \xrightarrow{\Delta_n} \ldots\}.$$

A moment's reflection shows that

$$\varinjlim\{SP^d(\tilde{X}) \xrightarrow{\Delta_n} SP^{nd}(\tilde{X}) \longrightarrow \ldots\} \sim K(\tilde{H}_*(\tilde{X};\mathbb{Z}/n)),$$

which leads to a transfer for $\mathbb{Z}/n$ homology.

Remark 2. If G is a group acting on $\tilde{X}$ and X such that the ramified covering map $\pi: \tilde{X} \downarrow X$ is a G-map, then

$$\pi_\natural : \tilde{H}_*(X;\mathbb{Z}) \to \tilde{H}_*(\tilde{X};\mathbb{Z}),$$

is a $\mathbb{Z}(G)$-module homomorphism. This follows by simply noting that the map

$$f_\pi : X \to SP^d(\tilde{X}),$$

is G-equivariant.

COROLLARY 2·3. *If $\tilde{X} \downarrow^{\pi} X$ is a d-fold ramified cover and $\tilde{X}$ is $\mathbb{Q}$, resp. $\mathbb{Z}/n$ acyclic, then X is $\mathbb{Q}$ acyclic, resp. $\mathbb{Z}/n$ acyclic provided $d \in \mathbb{Z}/n^*$.*

Proof. One has

$$\begin{tikzcd} \tilde{H}_*(X;k) \arrow[r] \arrow[rr, bend right, "\text{multiplication by } d"'] & \tilde{H}_*(\tilde{X};k) \arrow[r] & H_*(X;k) \end{tikzcd}$$

where k is either $\mathbb{Q}$ or $\mathbb{Z}/n$. But $\tilde{H}_*(\tilde{X}; k) = 0$, and the result follows. ∎

COROLLARY 2·4. (*Conner Conjecture for finite groups*): *If a finite group G acts on a rationally acyclic manifold M, then M/G is rationally acyclic.* ∎

In a similar vein we have:

PROPOSITION 2·4. *Let G be a finite group acting on the space Y with orbit map π: $Y \to Y/G$ structured as a $d := |G|$ fold ramified covering. Then*

$$\pi_\natural . \pi_* : \tilde{H}_*(Y) \circlearrowleft$$

is given by

$$\pi_\natural . \pi_*(u) = \sum_{g \in G} g_*(u).$$

Proof. The composite

$$Y \xrightarrow{\pi} Y/G \xrightarrow{f_\pi} SP^d(Y)$$

is given by

$$y \mapsto \{gy \mid g \in G\},$$

regarded as an unordered d-tuple with repeats. Since

$$Y \to Y/G \to SP^d(Y) \to SP^\infty(Y)$$

represents $\pi_\natural . \pi_*$ the result follows. ∎

COROLLARY 2·5. *Let G be a finite group acting on a space Y with orbit map π: $Y \to Y/G$ structured as a $d := |G|$ fold ramified covering. Let $k = \mathbb{Q}$ or $\mathbb{Z}/n$ where $(n, d) = 1$. Then*

$$\pi_* : \tilde{H}_*(Y;k)^G \to \tilde{H}_*(Y/G;k),$$

is an isomorphism.

Proof. By (2·2) $\pi_* \pi_\natural : \tilde{H}_*(Y/G;k) \circlearrowleft$ is multiplication by d while by (2·4) $\pi_\natural \pi_*$: $H(Y;k)^G \circlearrowleft$ is also multiplication by d. Since $d \in k^*$ the result follows. ∎

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