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TORUS PHILOSOPHY (of R. D. Edwards)

> A result which is true for a 'homeomorphism' of tori up to homotopy, is true for a 'homeomorphism' of manifolds up to isotopy.

Examples (T^n = n-torus ; replace with $B^k \times T^n$ as necessary.)

I <u>Local Contractibility</u> (Černavskii, Edwards-Kirby)

Known : a small homeomorphism of T^n is homotopic to the identity.

Conclude: a small homeomorphism of a manifold is isotopic to the identity.

II <u>Triangulation-Hauptvermutung</u> (Kirby-Siebenmann)

Known : a topological homeomorphism $h:(B^k \times T^n, \partial B^k \times T^n) \to (Q^{PL}, \partial Q)$ which is PL on boundary, is homotopic rel boundary to a PL homeomorphism (after lifting ; $k \neq 3$).

Conclude: a topological homeomorphism $h: W \to Q$ of PL manifolds is isotopic to a PL homeomorphism, if $H^3(W, \mathbb{Z}_2) = 0$.

III <u>Approximating Cellular Maps by Homeomorphisms</u>

(Siebenmann, Topology 11 (1972)).

Known: a cellular map $h: W \to T^n$ is homotopic to a homeomorphism.

Conclude: a cellular map $h: W \to Q$ of topological manifolds is pseudoisotopic to a homeomorphism.

IV <u>Locally Homotopically Unknotted</u> $\Rightarrow$ <u>Locally Flat</u> in codim 1

(Daverman BAMS 1972, using Price-Seebeck)

Known: if $h: T^{n-1} \to T^n$ is a locally homotopically unknotted embedding which is homotopic to standard inclusion, then $T^n - h(T^{n-1}) \approx T^{n-1} \times R^1$.

Conclude: if $h: W^{n-1} \to Q^n$ is a locally homotopically unknotted, codimension 1 embedding of topological manifolds, then h is a locally flat embedding.

V. Topological Regular Neighborhoods.

Known : any two mapping cylinder neighborhoods of a standard T^m in R^q are homeomorphic fixing core.

Conclude (TOP regular neighborhood uniqueness): Any two mapping cylinder neighborhoods of M in Q are homeomorphic fixing core, by a well-controlled ambient isotopy.