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★The topological classification of stratified spaces. (English summary)
Chicago Lectures in Mathematics.
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The wide scope of the book is indicated by the chapter titles: Part I. The theory of manifolds. 1. Algebraic K-theory and topology. 2. Surgery theory. 3. Spacification and functoriality. 4. Applications. Part II. The general theory of stratified spaces. 5. Definitions and examples. 6. Classification of stratified spaces. 7. Transverse stratified classification. 8. PT category. 9. Controlled topology. 10. Proof of the main theorems in Top. Part III. Applications. 11. Manifolds and embedding theory revisited. 12. Supernormal spaces and varieties. 13. Group actions. 14. Rigidity conjectures.

The book is an expanded version of the notes of a graduate course given by the author at the University of Chicago, with the aim of introducing "students as quickly as possible to techniques and problems that are at the forefront of research in the topological theory of stratified spaces". It addresses (at least) two separate audiences: (a) mathematicians who want to learn about surgery theory; (b) topologists already familiar with surgery theory who want to find out about the applications to stratified spaces.

About half the book is occupied by Part I, which is a non-technical account of the Browder-Novikov-Sullivan-Wall surgery theory of high-dimensional manifolds and some of its applications. This should be particularly useful for readers in class (a) who do not want to be bothered with details such as the definition of the surgery obstruction groups $L_*(\pi)$. For this purpose they can consult Wall's classic book *Surgery on compact manifolds* [Academic Press, London, 1970; MR0431216 (55 #4217)]. Readers in class (b) will also find this part useful, especially if they are giving a course on surgery theory, and can supply the missing details themselves.

Part II develops the surgery classification theory of stratified spaces. Part III describes the applications of stratified surgery, notably to the classification of spaces with certain types of singularities, finite group actions on manifolds and equivariant rigidity. Stratified surgery theory is still not as highly evolved as the classical one-stratum case of manifolds, so readers in both classes will have to face some difficulties in dealing with Parts II and III. For readers in class (a) there is a problem of motivation: why should they be interested in stratified spaces? Readers in class (b) may wish to know the extent to which the theory presented in the book gives new classifications of stratified sets.

Roughly speaking, a stratified set is a space which is a union of manifolds (the "strata") fitting together nicely. Whitney and Thom invented stratified spaces in order to better understand the topological properties of algebraic varieties. Milnor used stratified spaces and algebraic K-theory to disprove the Hauptvermutung, constructing examples of homeomorphic polyhedra which are combinatorially distinct. The main result of surgery theory for high-dimensional manifolds is the Sullivan-Wall exact sequence, which relates the manifold structure set to homotopy theory and algebra. W. Browder and F. Quinn [in *Manifolds—Tokyo 1973 (Tokyo, 1973)*, 27–36, Univ. Tokyo Press, Toyko, 1975; MR0375348 (51 #11543)] proceeded by an induction from on the manifold strata, starting with the case of a single stratum. The theory was made more effective by the application of controlled topology in a paper by Quinn [J. Amer. Math. Soc. **1** (1988), no. 2, 441–499; MR0928266 (89g:57050)], which also provided a workable topological category of stratified spaces. The main new results presented in the book are the stratified surgery exact sequences of Section II.6, providing a global formulation of Quinn's work. In the first instance, these exact sequences are defined geometrically, and it is only possible to use them for classification in the rather special circumstances of the examples treated in Part III.

Overall, the style of the book is quite informal. The author's enthusiasm for the subject is much in evidence. However, there is also much sloppiness, which may either infuriate readers or drive them to work things out for themselves. Here is a typical example which occurs near the beginning. The Siebenmann end obstruction of a tame manifold end ε is defined on page 33 as the Wall finiteness obstruction $w(N)$ of a finitely dominated neighbourhood N of ε. Unfortunately, the terminology $w(N)$ is used here for the first time, and in any case the introduction of the invariant in $\S1.1$ is somewhat long-winded—a short sentence such as: "The Wall finiteness obstruction of a finitely dominated space X is an element $w(X) \in \widetilde{K}_0(\mathbf{Z}[\pi_1(X)])$ which vanishes if and only if X is homotopy equivalent to a finite CW complex" would certainly have helped the reader (in either class) unfamiliar with the invariant.

Readers prepared to put in a lot of work will get much out of the book. This is true of most books in mathematics, but particularly so of this one. On the positive side, the book covers much ground, bringing together many of the ideas and results currently fashionable in high-dimensional manifold theory. At the very least, this book is an excellent syllabus for a lecture course! On the negative side, it must be noted that much of the coverage is sketchy, and that an advanced reader seeking a work of reference is likely to be disappointed by the lack of precision.

Reviewed by *A. A. Ranicki*