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HOMEOMORPHISM AND DIFFEOMORPHISM CLASSIFICATION OF MANIFOLDS

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I want to compare the classification problems for four different kinds of manifolds.

(i) *Smooth* (or differential) *manifolds*. These are comparatively familiar mathematical objects: they are Hausdorff topological spaces with additional structure which can be specified by an atlas of local coordinate systems (homeomorphisms of open sets in the manifold onto open sets in Euclidean space) in which transformations between different coordinate systems are always given by smooth (i. e. C^∞) functions. Here we will only consider compact manifolds.

(ii) *PL* (or piecewise linear, or combinatorial) *manifolds*. These are defined analogously, with the modification that coordinate transformations must now be piecewise linear—i.e. linear on each simplex for some locally finite decomposition in simplices of the open set in question.

(iii) *Topological manifolds*: here no restriction is made on the coordinate transformations beyond their being homeomorphisms.

(iv) It is also convenient to have a homotopy-theoretic analogue of the above. The clue here is provided by the Poincaré duality theorem, which holds for all manifolds. We define a *Poincaré complex* to be a *CW* complex which satisfies a suitably strong form of the Poincaré duality theorem (the detailed definition is somewhat technical; see Wall [1] or [2]).

A manifold of any of these types determines one of each subsequent type, in an essentially unique manner:

(i) → (ii) by smooth triangulation (due to Whitehead [1], see also Munkres [1]),

(ii) → (iii) by just ignoring the *PL*-structure,

(iii) → (iv) by ignoring all but the homotopy type of our manifold (this step ignores the local nice properties of a manifold to concentrate on the global structure). The problem I want to discuss is that of going in the opposite direction — i. e. imposing stronger structures. First, we must construct invariants of the various types of structure.

(i). As is well-known from differential geometry, a smooth manifold has tangent vectors, which are assembled in a vector bundle over M, the *tangent bundle*. We can describe this by saying that the vectors which form the fibre over a point $P \in M$ correspond diffeomorphically (by the exponential map) with a neighbourhood of P in M. This bundle has structure group the orthogonal group, O_m, and hence is classified by a (homotopy class of) maps from M to the classifying space, BO_m (see e. g. Milnor [1]).

(ii) and (iii). At the last international congress, J. Milnor introduced the theory of microbundles, which gives analogous results in cases (ii) and (iii) (Milnor [4], see also [3], [5]). There are several refinements and variants of his original definition (see Kister [1], Mazur [2], [3], Hirsch [1] and Kuiper and Lashof [1]): I choose the simplest, a bundle with fibre a Euclidean space of dimension m, and structure group known as PL_m or Top_m in the two cases: to be thought of as a group of homeomorphisms leaving the origin fixed. (The former has to be defined as a semi-simplicial group.) It is shown in the papers cited that tangent bundles exist and are unique. Milnor also obtained corresponding spaces BPL_m, $BTop_m$, maps into which classify such bundles.

(iv). It has been shown recently by Spivak [1] that an analogous theory exists for this case also. The appropriate objects turn out to be fibre spaces (not bundles)

$$\begin{array}{ccc} \Sigma & \longrightarrow & E \\ & & \downarrow \pi \\ & & M \end{array}$$

in which Σ has the homotopy type of a sphere of some (usually large) dimension. Such fibrations are called spherical. The stable normal fibration of a Poincaré complex M is characterised by the requirement that its Thom space (the mapping cone of π) be reducible. It turns out also that given two such fibrations, and supposing (as we may, by suspension) that the fibres are homotopy equivalent to spheres of the same dimension $k-1$, and that $f_i : S^{m+k} \to M \cup_{\pi_i} CE_i$ $(i = 1, 2)$ have degree 1, then there exists a fibre homotopy equivalence (unique up to fibre homotopy) of π_1 on π_2 which carries f_1 to f_2. This result will be important below: it reduces complicated problems about homotopy groups of Thom spaces, which arose in the pioneering work of Novikov [1], to much simpler questions concerning equivalence of fibrations.

Instead of a structure group for spherical fibrations, one has a structure monoid G_k, the space of homotopy equivalences of S^{k-1} on itself: it too possesses a classifying space BG_k (Dold and Lashof [1], Stasheff [1].)

The next important remark is as follows: there exist maps

$$BO_m \longrightarrow BPL_m \longrightarrow BTop_m \longrightarrow BG_m$$

corresponding to natural transformations of bundle functors. Thus an m-vector bundle over a simplicial complex X is classified by a map $X \to BO_m$; we form the composite $X \to BO_m \to BPL_m$, and this induces a PL-bundle over X, which "triangulates" the vector bundle (see Lashof and Rothenberg [1], also Hirsch and Mazur [1]). Similarly $BPL_m \to BTop_m$ corresponds to forgetting the PL-structure: this is essentially due to Milnor [3, 4]. Finally given a bundle with fibre $\mathbf{R}^m$, by removing the section corresponding to 0 we change the fibre to $\mathbf{R}^n - 0$, with the homotopy type of S^{m-1}, and thus obtain a spherical fibration.

We have described these transformations in geometrical terms: it is now not difficult to see that if we take a smooth triangulation of a smooth manifold M, its tangent PL-bundle is given by triangulating the tangent vector bundle of M (proof in Lashof and Rothenberg [1]). Even more clearly, the tangent bundle of a PL-manifold is unchanged (as a bundle) by regarding the manifold as a topological manifold. The final step to BG_m is more complicated as we have only defined a normal bundle for that situation: it is an open problem whether one can characterise a tangent spherical fibration ξ for a Poincaré complex X, e. g. by requiring a map of degree 1 from $X \times X$ to the Thom space of ξ to satisfy some natural extra conditions. (With some restrictions on X, this has been solved by W. Sutherland.)

For this and other reasons, we now stabilise. On increasing m by 1 we obtain a commutative diagram

$$\begin{array}{ccccccc} BO_m & \longrightarrow & BPL_m & \longrightarrow & BTop_m & \longrightarrow & BG_m \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ BO_{m+1} & \longrightarrow & BPL_{m+1} & \longrightarrow & BTop_{m+1} & \longrightarrow & BG_{m+1}; \end{array}$$

we denote the direct limit as $m \to \infty$ by

$$BO \longrightarrow BPL \longrightarrow BTop \longrightarrow BG.$$

Then the tangent bundle of a manifold M induces a map $M \to BTop \to$ $\to BG$ which is homotopic to that induced by an inverse to the stable normal fibration of M regarded as Poincaré complex. This follows from Spivak's work, and is also closely related to the earlier paper of Milnor and Spanier [1].

The following may be regarded as the fundamental question in the subject.

P r o b l e m. Suppose given a manifold of one of our four types, and a reduction of the structural group of its stable tangent bundle to an earlier type: does the manifold similarly admit extra structure? And is this extra structure unique up to some natural equivalence relation? We can reformulate this using the classifying spaces above. For example: suppose M^m to be a topological manifold, with stable tangent bundle classified by $\tau : M \to BTop$. Suppose given a map $f : M \to BO$, and a homotopy of the composite map $M \xrightarrow{f} BO \longrightarrow BTop$ to τ. Then can M be given a corresponding differentiable structure? Is this unique up to equivalence? (Note the explicit homotopy: this is an important part of the data.)

Before we go on to consider answers to this problem, I will mention some generalisations which can be treated by similar methods, but detailed consideration of which is outside the scope of this talk. First, we may consider manifolds M with boundary ∂M (M still compact): for case (iv) we insist that ∂M be a Poincaré complex, and that the CW pair $(M, \partial M)$ satisfy the Lefschetz duality theorem. We do not include the noncompact case, which can also be formulated, but probably not yet in the right form, and seems (except for the transition (i)$\to$(ii)) to be apreciably harder. Next, we might consider submanifolds of a fixed larger manifold: e.g. the problem of smoothing PL-submanifolds of a smooth manifold. The normal bundle plays a dominant role here. For a discussion of the case (i) $\to$ (iv) c. f. the talk immediately preceding this one (W. Browder [2]) and for the case (i) → (ii) compare the talk of A. Haefliger [1] at this congress, as also recent work of Morlet [1] and Rourke and Sanderson [1]. Finally, one can also classify automorphisms instead of objects, and even investigate the homotopy type of groups of homeomorphisms or spaces of embeddings. This problem is complicated by technical considerations of concordance and isotopy: see the talk of Cerf [1] at this congress, also a recent paper of Hudson [1].

We return to our own problem: the answer depends, not surprisingly, on the case investigated. The result is simplest for the case (i) → (ii): the problem of smoothing PL-manifolds. This was discussed in M. W. Hirsch's talk [3] (following Hirsch and Mazur [1]): the answer to our problem is yes. Furthermore, equivalence classes of smoothings of the PL-manifold M correspond bijectively to homotopy classes of homotopy factorisations

$$\begin{array}{ccc} & M & \\ \swarrow & & \searrow \\ BO & \longrightarrow & BPL. \end{array}$$

We next consider the case (ii) → (iv): the problem here is to characterise the homotopy types of PL-manifolds. It is tackled as follows. We are given a Poincaré complex X, and a homotopy factorisation of the classifying map of its Spivak fibration

$$\begin{array}{ccc} & X & \\ \swarrow & & \searrow \\ BPL & \longrightarrow & BG; \end{array}$$

or equivalently, a PL-bundle ν over X, and a fibre homotopy equivalence with the Spivak fibration; or equivalently again, the PL-bundle ν, and a homotopy class of maps of degree 1 from a sphere to its Thom space. A transversality argument due essentially to Williamson [1] and Browder [1] now shows that we can find at least a closed PL-manifold M, and a map $\varphi\colon M \to X$ of degree 1 such that $\varphi^*\nu$ is the stable normal bundle of M. Indeed, our data determine a bordism class of such maps (M, φ): we seek an (M_0, φ_0) in this class with φ_0 a homotopy equivalence, and also want to know about uniqueness of M_0 when it exists. The problem is now amenable to the method of surgery (initiated by Milnor [2] and first applied to this situation by Novikov [1]).

For technical convenience, it is more convenient to replace (iv) by a new class (iv)′ of finite Poincaré complexes (the definition, even more technical, involves Whitehead's theory of simple homotopy types, for which see Whitehead [2]). This is related to the other classes of "manifolds" by functors

$$\begin{array}{ccccc} (i) & \to & (ii) & \to & (iii) \\ & & \downarrow & & \downarrow \\ & & (iv)' & \to & (iv). \end{array}$$

(An outstanding problem is whether one can define a transformation of structures in the direction (iii) $\to$ (iv)'; this contains the problem whether a compact manifold has the homotopy type of a finite complex, also that of topological invariance of simple homotopy type.) The map (iv)' $\to$ (iv) is related to the projective class group and the Whitehead group of the fundamental group $\pi_1(X)$ of the Poincaré complex X. We define w to be the homomorphism $w : \pi_1(X) \to \{\pm 1\}$ which takes the value -1 on orientation-reversing loops. Then surgery leads to the following result.

T h e o r e m. *There exist functors L_m depending only on the integer m modulo 4, and defining abelian groups $L_m(\pi_1(X), w)$. Given a bordism class of maps (M, φ) of degree 1, as above, with $m \geqslant 5$, there is an obstruction in $L_m(\pi_1(X), w)$ to the existence of an element (M_0, φ_0) of it with φ_0 a simple homotopy equivalence, and an obstruction in $L_{m+1}(\pi_1(X), w)$ to its uniqueness, when it exists.*

We can formulate this as an exact sequence (of based sets), which can be extended in one direction

$$\ldots \to L_{m+1}(\pi_1(X), w) \to PL\text{-homeomorphism classes of } (M_0, \varphi_0) \to$$
$$\to \text{Bordism classes of } (M, \varphi) \to L_m(\pi_1(X), w).$$

Example 1 : $X = S^m$. The transversality argument mentioned above allows an easy proof that the bordism classes form the group $\pi_m(G, PL)$. A result of Smale [1] shows that the PL-homeomorphism class is unique. The sequence (continued to the left) then provides an isomorphism

$$\pi_m(G, PL) \to L_m(1)$$

for $m \geqslant 5$ (in fact this holds for $m \geqslant 1$ except for $m = 4$, when the image has index 2). The group $L_m(1)$ can be computed algebraically (see also Kervaire and Milnor [1], Levine [1], where it is called P_m): it is trivial for m odd, infinite cyclic for $m \equiv 0 \pmod 4$, and cyclic of order 2 for $m = 2 \pmod 4$.

Example 2 : $X = S^m \times S^1$. A similar argument can be used here, using results of Browder and Levine [1]. One finds that in the orientable case, $L_m(\mathsf{Z}) \cong [S^{m-1} \vee S^m : G/PL]$ $(m \geqslant 6)$. Similarly, using the nontrivial bundle over S^1 with fibre S^m, in the nonorientable case, $L_m(\mathsf{Z}) \simeq [S^{m-1} \cup_2 e^m : G/PL]$ $(m \geqslant 6)$.

There are analogous results also for bounded manifolds, and in one important case, the corresponding result is simpler. Suppose (Y, X) a Poincaré pair, with X and Y connected, and such that inclusion induces an isomorphism of $\pi_1(X)$ on $\pi_1(Y)$. Then the group corresponding to L_m vanishes: provided $m \geqslant 6$, the corresponding M_0 exists and is unique. (So the answer to our problem is yes in this case also.)

Armed with this, we can give a complete answer in the closed, simply-connected case. Given a closed PL-manifold M, we write M' for the manifold obtained by deleting the interior of an embedded disc D^n. One can invent a corresponding decomposition $X = X' \cup_f e^m$ for the case of Poincaré complexes: if $m \neq 2$, it is essentially unique (see Wall [2]). Now suppose the Spivak fibration for X' reduced to a PL-bundle:

$$\begin{array}{ccc} & X' & \\ \swarrow & & \searrow \\ BPL & \longrightarrow & BG. \end{array}$$

As (X', S^{m-1}) is a Poincaré pair, and both are simply connected, we get a unique corresponding manifold M' $(m \geqslant 6)$: moreover, $\partial M'$ is homotopy equivalent to S^{m-1}, hence is PL-homeomorphic to it. We then attach a disc D^m along S^{m-1} (this process is unique) to give a closed PL-manifold M. Thus in the closed, simply-connected case the answer to our problem is: yes, provided we consider reductions over X' instead of ones for all of X. Our proof assumed $m \geqslant 6$: the result holds also for $m = 5$ and (trivially) for $m \leqslant 2$. In dimension 3, X is homotopy unique and S^3 proves existence: uniqueness is equivalent to the Poincaré conjecture. The statement given above is due to D. Sullivan [1].

In the non-simply-connected case, the results are more complicated. Let me cite by way of example that there are infinitely many PL-manifolds homotopy equivalent to $P_7(\mathbf{R})$, all corresponding to the same reduction from G to PL, but no two homeomorphic. This comes from the fact that $L_8(\mathbf{Z}_2) \cong \mathbf{Z} \oplus \mathbf{Z}$ (orientable case). We can describe explicitly an invariant that distinguishes our manifolds Q: note first that both they and their double covers are rational homology spheres, so if $\partial W = Q \cup Q'$ (W orientable) we can speak unambiguously of the signature of W. Now it is easy to show that for any of our manifolds, say Q_7, we can find $\partial W = Q_7 - P_7(\mathbf{R})$ with W oriented, of signature 0, and $\pi_1(Q) \cong \pi_1(W) \cong \pi_1(P_7(\mathbf{R}))$ by inclusion. The required invariant of Q is then the signature of the double cover of W.

Sullivan has used the methods of surgery to gain insight into the homotopy type of the quotient space G/PL. We have already seen how to calculate its homotopy groups. Now suppose given a closed, 1-connected PL-manifold M, and homotopy class of maps $f: M \to G/PL$.

Now the tangent bundle of M is classified by a map $\tau : M \to BPL$. Also, we have a principal fibration

$$\begin{array}{ccc} G/PL & \longrightarrow & BPL \\ & & \downarrow \\ & & BG: \end{array}$$

multiplication is induced by Whitney sum of bundles (see e. g. Lashof and Rothenberg [1]). Thus we can operate fibrewise on τ by f, obtaining a new map $f \circ \tau : M \to BPL$ whose projection on BG is the same as that of τ. Proceeding as above, we then reach a surgery obstruction in $L_m(1)$. This gives a retraction of the oriented PL-bordism group

$$\Omega_m^{SPL}(G/PL) \to \pi_m(G/PL).$$

In the case $m = 4k+2$ we need not suppose M 1-connected or even oriented, and can work with the unoriented bordism group. From these retractions, Sullivan deduces that all k-invariants of G/PL vanish mod 2 (though not modulo the class of finite groups of odd order: the first k-invariant already has order 2). Equivalently, there is a map

$$G/PL \to \prod_{k \geqslant 0} K(\mathbf{Z}_2, 4k+2)$$

inducing epimorphisms of homotopy groups. A slightly stronger result can in fact be obtained.

Work currently in progress is aimed at studying the behaviour of odd primes: it is conjectured that, modulo finite 2-groups, G/PL has the homotopy type of BO (though, of course, the natural maps $G/PL \to BPL \leftarrow BO$ do not correspond to a homotopy equivalence modulo 2-groups).

We come finally to the case (iii) of topological manifolds. Some information may be gathered here by observing that (iii) lies between (ii) and (iv) (but not—at present—(iv)′). In addition to this, we have Novikov's recent proof [2] of topological invariance of rational Pontrjagin classes. It is easy deduction from this that if M is compact, the kernel of the homomorphism

$$[M: BPL] \to [M: BTop]$$

is finite. We can sharpen the method to prove that for any n, $\pi_n(G, PL) \to \pi_n(G, Top)$ is a split monomorphism. What does this tell us about our problem? Unfortunately, it gives no method (and I know none) for constructing topological manifolds (other than PL ones), or for proving given topological manifolds homeomorphic. Newman's recent solution [1] of the Poincaré conjecture for topological manifolds is a pointer in this direction, however. Thus we must abandon the case (iii) $\to$ (iv) of our problem.

For the case (ii) $\to$ (iii), one can obtain "stable" theorems by using the results of Milnor: see Mazur [2, 3] and Hirsch [2]. Thus, given a (not necessarily compact) topological manifold M, and a homotopy factorisation

$$\begin{array}{ccc} & M & \\ \swarrow & & \searrow \\ BPL & \longrightarrow & BTop \end{array}$$

then for some n, $M \times R^n$ admits a PL-structure, and two such (both inducing the above factorisation) determine equivalent PL-structures on $M \times R^{n+n'}$ for large enough n'. More interesting, however, are the recent "unstable" theorems of Sullivan and Wagoner. These concern only the uniqueness (Hauptvermutung) problem, and not the existence (Triangulation). Suppose given a homeomorphism h of compact PL-manifolds M and M'. Then we have a homotopy equivalence, and the two reductions of structural group from G to PL are equivalent in Top. Assume

(A) The structural groups are already equivalent in PL. Then we can apply the method of surgery to attempt to prove the manifolds PL-homeomorphic (by constructing an s-cobordism, and applying the s-cobordism theorem (Mazur [1], Kervaire [1])). We need to assume that h is a simple homotopy equivalence $M \to M'$; also (in the bounded case) $\partial M \to \partial M'$. If also

(B) Surgery can be performed, it follows that M and M' are PL-homeomorphic, as desired.

It follows from the discussion above that (B) is all right in the closed, simply-connected case, or if $\pi_1(\partial M) \cong \pi_1(M)$ by inclusion, or in certain other cases (e.g. closed, nonorientable, odd-dimensional, with fundamental group of order 2). This assumes that the dimension of the manifold M (and, if ∂M is nonempty, of ∂M) exceeds 4: if dim $M \leqslant 3$ the Hauptvermutung and triangulation have also been proved, without further assumptions, by Moise [1] and Bing [1].

For (A) we encounter obstructions θ_i in $H^i(M; \pi_i(G, PL))$. For $i \equiv 0 \pmod 4$, their image in rational cohomology can be identified with the difference of the rational Pontrjagin classes (or rather, L-classes) of M and M'. By Novikov's result, this difference is zero. Thus θ_{4k} is a torsion element. Next consider θ_{4k+2} which, by the result on k-invariants of G/PL, is a well-defined obstruction. If all lower θ_i vanish, this is annihilated by the homomorphism induced by $G/PL \to G/Top$; on the other hand, we have seen that this homomorphism is injective. Thus (A) can be justified on the sole assumption that for all k, $H^{4k}(M; Z)$ is torsion free. I believe that the best known result (due, again, to Sullivan) is slightly stronger—but in any case, the main obstacles to further progress are now presented by the Whitehead group (simple homotopy equivalence) and by the surgery obstructions.

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