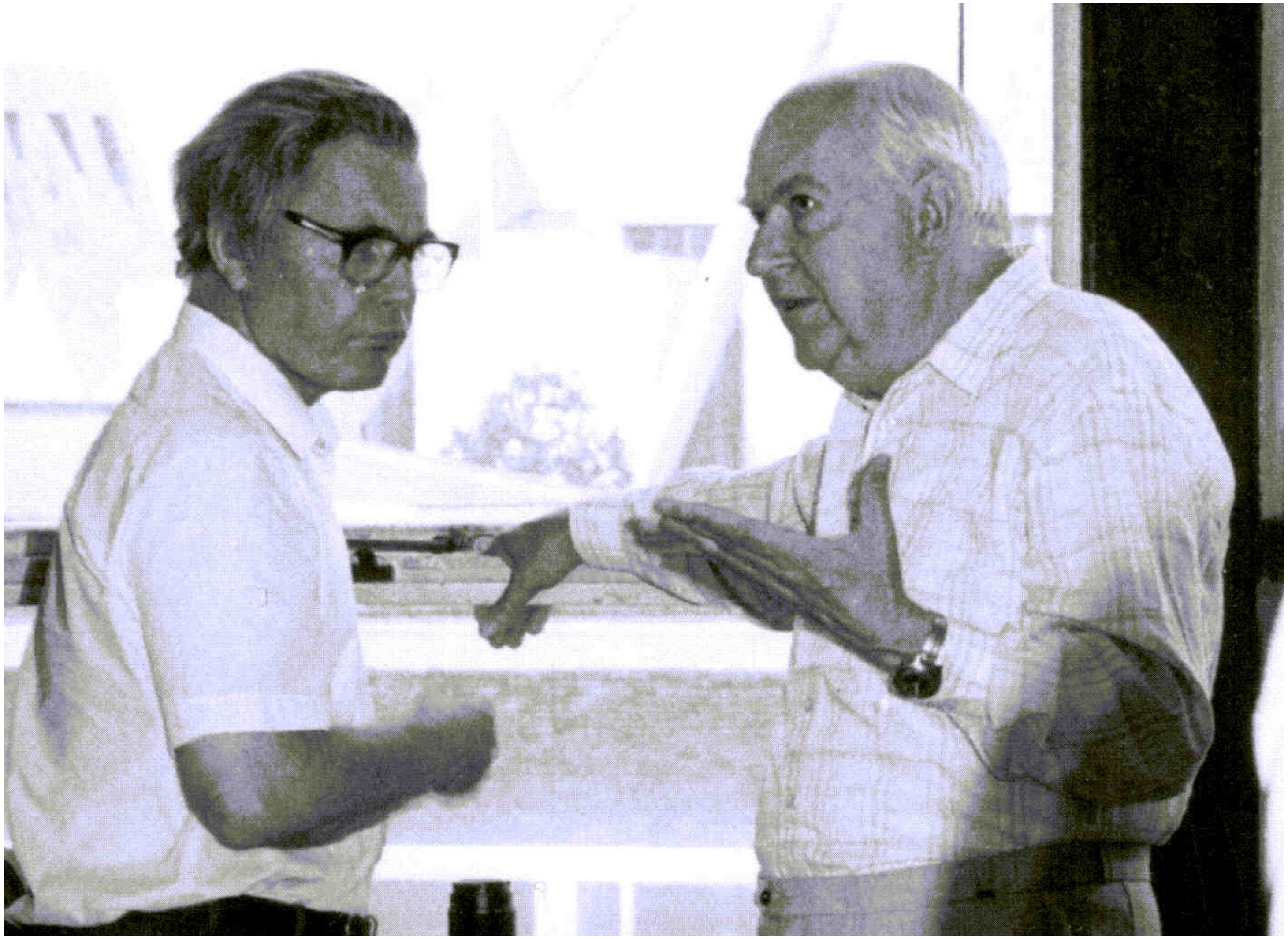
In Memoriam

Ian R. Porteous 9 October 1930 - 30 January 2011

A tribute by
Peter Giblin
(University of Liverpool)
Będlewo, Poland
16 May 2011

Caustics 1998



with Christopher Longuet-Higgins at the Rank Prize Funds symposium on computer vision in Liverpool, summer 1987, jointly organized by Ian, Joachim Rieger and myself

After a first degree at Edinburgh and National Service, Ian worked at Trinity College, Cambridge, for a BA then a PhD under

first William Hodge, but he was about to become Secretary of the Royal Society and Master of Pembroke College Cambridge

so when Michael Atiyah returned from Princeton in January 1957 he took on Ian and also Rolph Schwarzenberger (6 years younger than Ian) as PhD students

Ian’s PhD was in algebraic geometry, the effect of blowing up on Chern Classes, published in Proceedings of the Cambridge Philosophical Society in 1960:

The behaviour of the Chern classes or of the canonical classes of an algebraic variety under a dilatation has been studied by several authors (Todd, Segre, van de Ven). This problem is of interest since a dilatation is the simplest form of birational transformation which does not preserve the underlying topological structure of the algebraic variety. **A relation between the Chern classes of the variety obtained by dilatation of a subvariety and the Chern classes of the original variety has been conjectured by the authors cited above but a complete proof of this relation is not in the literature**.

Ian’s work uses the then recent Grothendieck-Riemann-Roch theorem

According to Terry Wall, Ian gave a seminar in which a secretary misread

Blowing Up Chern Classes

as

Blowing Up Chem Classes

and it appeared on the noticeboard as

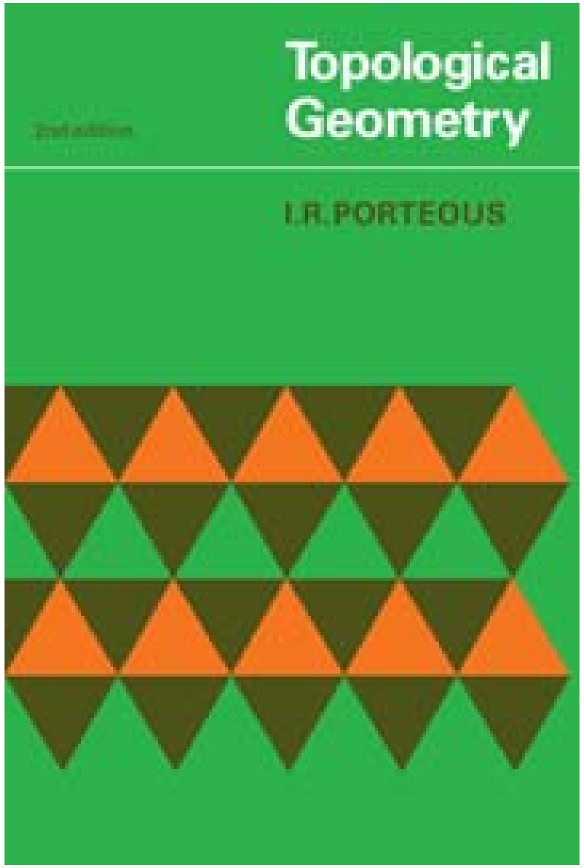
Blowing Up Chemistry Classes

Ian took up a lectureship at Liverpool in 1959 and spent the academic year 1961-2 at Columbia University where he came under the spell of

Serge Lang (1927-2005), the French-born American mathematician and member of *Bourbaki*

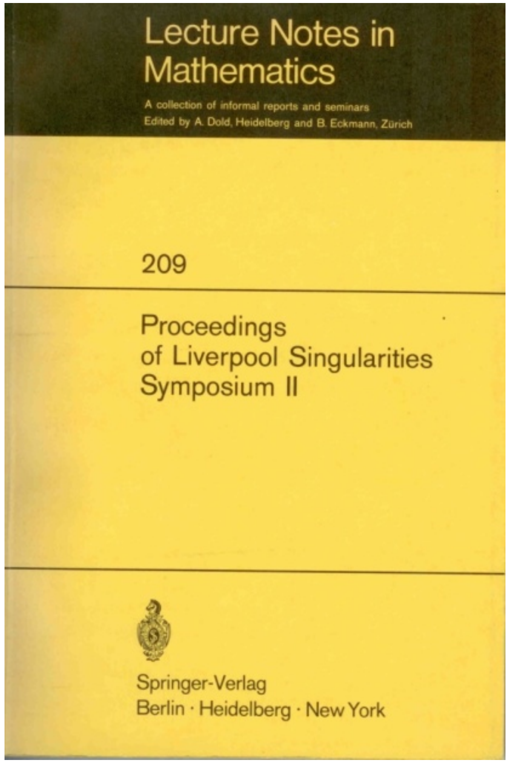
There is Lang, typing one of his more than 50 books and talking on the phone at the same time

Apart from this Ian spent his whole career at Liverpool, becoming Senior Lecturer in 1972 and 'retiring' in 1998.



On returning to Liverpool, Ian started to give lectures on modern analysis, in the Dieudonné style, and influenced by his time at Columbia.

He wrote a book about this (1969 and 1981), which starts from linear algebra, moves through quaternions, Lie groups and Clifford algebras, to differential calculus on (real and complex) euclidean and Banach spaces, and on manifolds.



A defining moment in Singularity Theory came in 1969-70 with the *Liverpool Singularities Symposium*.

The proceedings of that symposium occupy two *Lecture Notes in Mathematics* volumes, edited by Terry Wall, and contain, for example

• a long introductory article by Harold Levine

• new proofs of the Malgrange Preparation Theorem (Glaeser, Nirenberg, Mather)

• one of Mather's key papers on the 'nice dimensions'

• important survey articles by Wall

• articles by Thom, Sullivan, Łojaciewicz, Haefliger, Hirzebruch and even Stephen Hawking on singularities in general relativity.

Ian published three articles in the Liverpool Singularities Symposium 1969-70 volumes:

Simple singularities of maps

Todd's canonical classes

Geometric differentiation—a Thomist view of differential geometry

The singularity set Σ^i of a smooth map $f : V \to W$ between manifolds is the set of points of V for which the Jacobian matrix of f has kernel rank i. If f satisfies appropriate transversality conditions then Σ^i is a smooth submanifold.

Σ^i is not closed but the frontier is of much lower dimension and there is an associated cohomology class in $H^*(V)$.

In principle one can iterate the procedure: $\Sigma^{i,j} = \Sigma^j(f|\Sigma^i)$ and so on.

The $\Sigma^i, \Sigma^{i,j}, \ldots$ are often called Thom-Boardman singularity sets of f and Boardman showed using jet space methods that generically the procedure *can* be iterated to give smooth manifolds.

Ian Porteous, building on earlier work of Thom, used geometrical methods to determine the cohomology classes of Σ^i in terms of Chern classes (or Stiefel-Whitney classes) of $f : V \to W$, that is the classes of V and the pullpacks by f of the classes of W. Already this yielded a number of results from classical algebraic geometry and the formula is known as the 'Thom-Porteous Formula'.

The calculations were extended by Felice Ronga, also in the Liverpool volume, to $\Sigma^{i,j}$.

These 'universal polynomials' are now known as Thom Polynomials and have become an independent industry with major contributions from Kleiman, Szücs, Rimanyi, Kazarian and others.

The third paper of Ian's in the Liverpool volumes,

Geometric differentiation—a Thomist view of differential geometry

was the start of a serious application of Thom's ideas in catastrophe/singularity theory to the study of surfaces and higher-dimensional manifolds by means of singularities of maps, and using the key idea of unfoldings.

Ian was at the forefront of these applications with articles such as 'The Normal Singularities of a Submanifold', *J.Differential Geometry* 1971, and articles in the *Arcata Symposium 1981*.

By ‘normal singularities’ of (say) a surface M in $\mathbf{R}^3$ is meant the singularities within the family of distance-squared functions

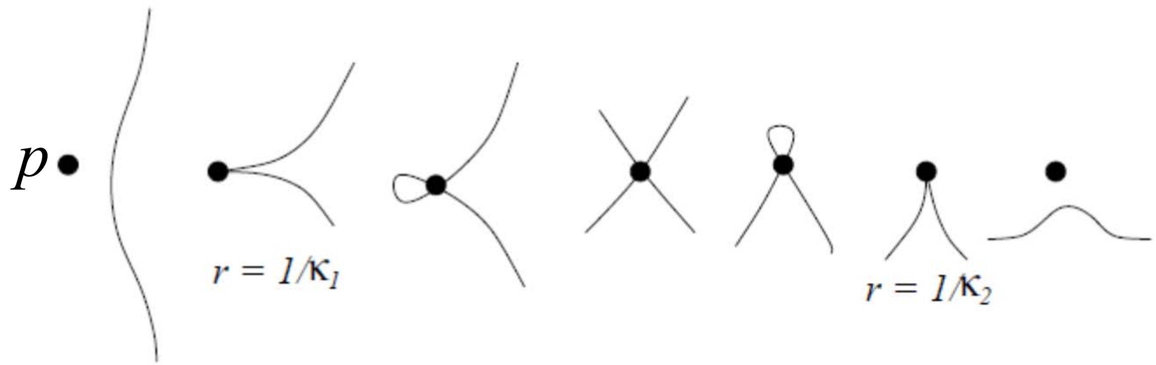
$$V : M \times \mathbf{R}^3 \to \mathbf{R},\ (p, \mathbf{x}) \mapsto ||p - \mathbf{x}||^2,$$

for example

$V_{\mathbf{x}}$ (that is V with $\mathbf{x}$ fixed) is singular at p when $\mathbf{x}$ lies on the normal to M at p.

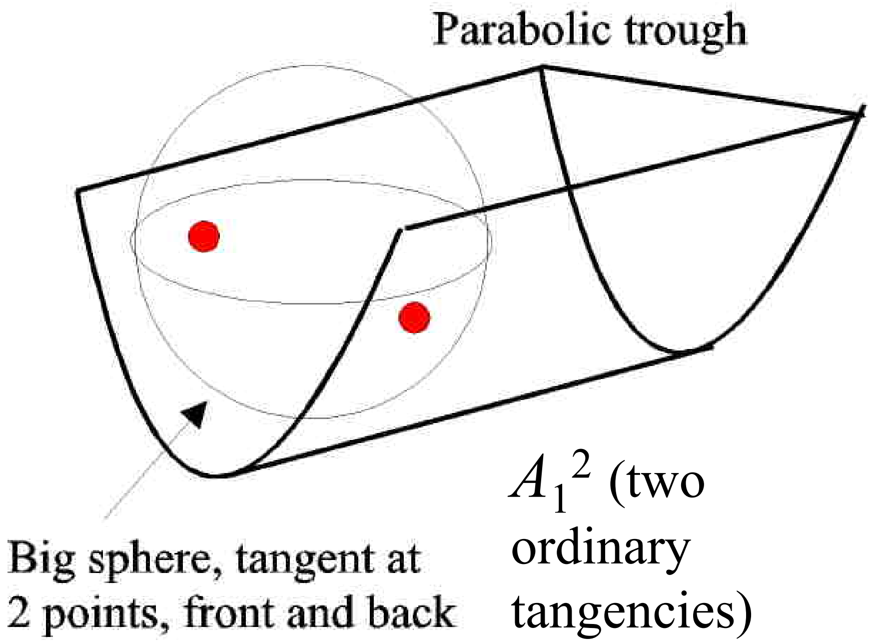
$V_{\mathbf{x}}$ has a degenerate ($A_{\geq 2}$) singularity means that $\mathbf{x}$ is the centre of a sphere of curvature at p, that is a sphere having higher contact with M at p.

There will be two such points $\mathbf{x}$ for all p except for umbilics, where they coincide. Thee two points sweep out the **focal surface** of M.

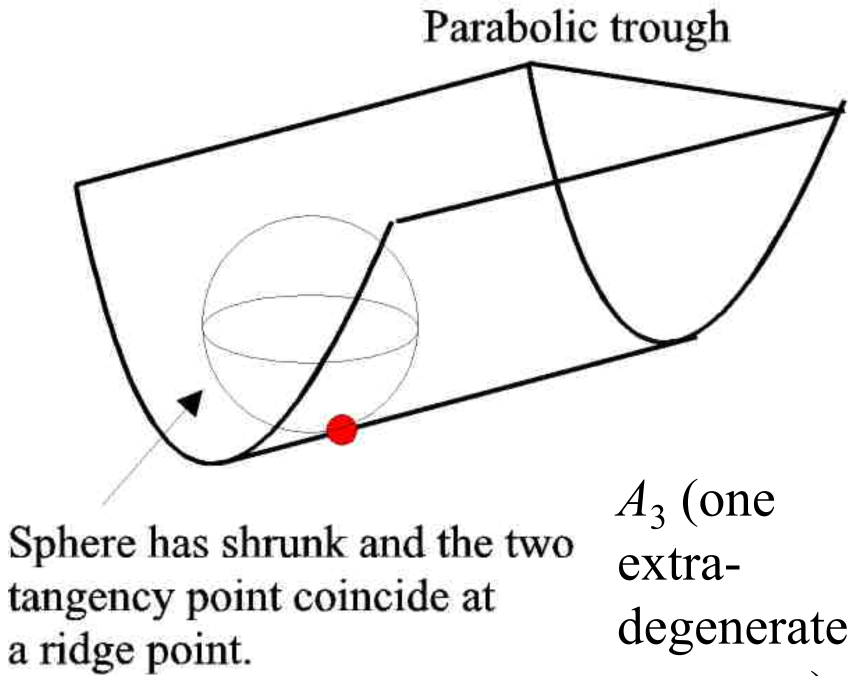


Intersection between *M* and a sphere tangent at *p* as the radius increases

Ridge points have spheres with A_3 contact. One way to visualise this is to think of spheres which are tangent in *two* places to M when these two places come into coincidence.



A_1^2 (two ordinary tangencies)



A_3 (one extra-degenerate tangency)

There are also beautiful definitions of ridge points in terms of the *extreme points of principal curvatures along the principal curves* of a surface.

Then ridges are of four types depending on maximum or minimum of either of the two principal curvatures along their lines of curvature.

The four types are sometimes called ***blue max, blue min, red max*** and ***red min.***

Blue and red ridges cross 'accidentally' at (Ian's term) ***purple flyovers.***

Umbilics play a special role: they have one or three ridges through them, and in a family of surfaces umbilics are 'born' on 'fertile ridges' (Ian's term again); other ridges are 'sterile'.

This picture uses more colours; the umbilics are at the black dots and they have one ridge through them. The other crossings are all flyovers.

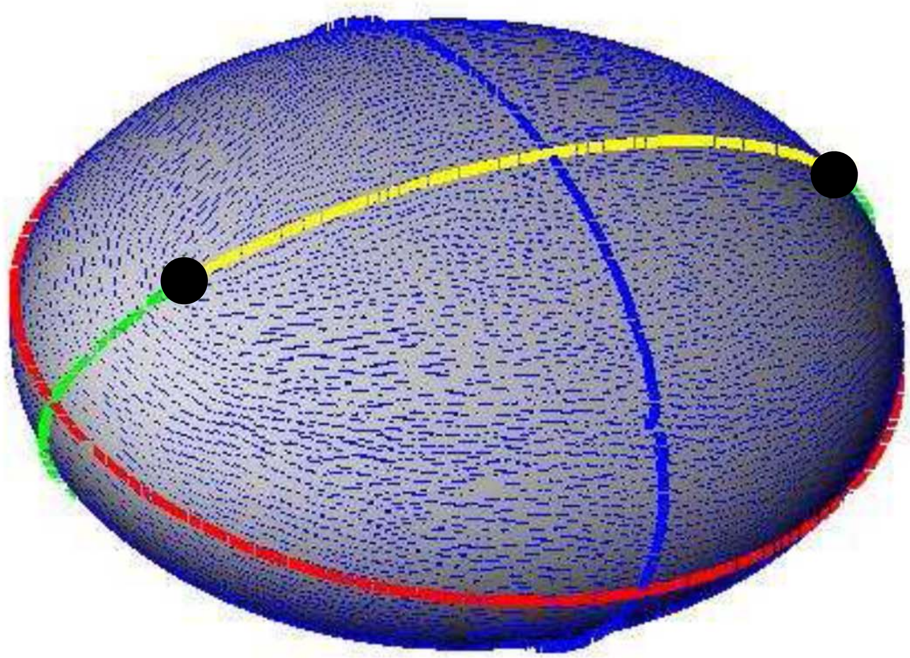
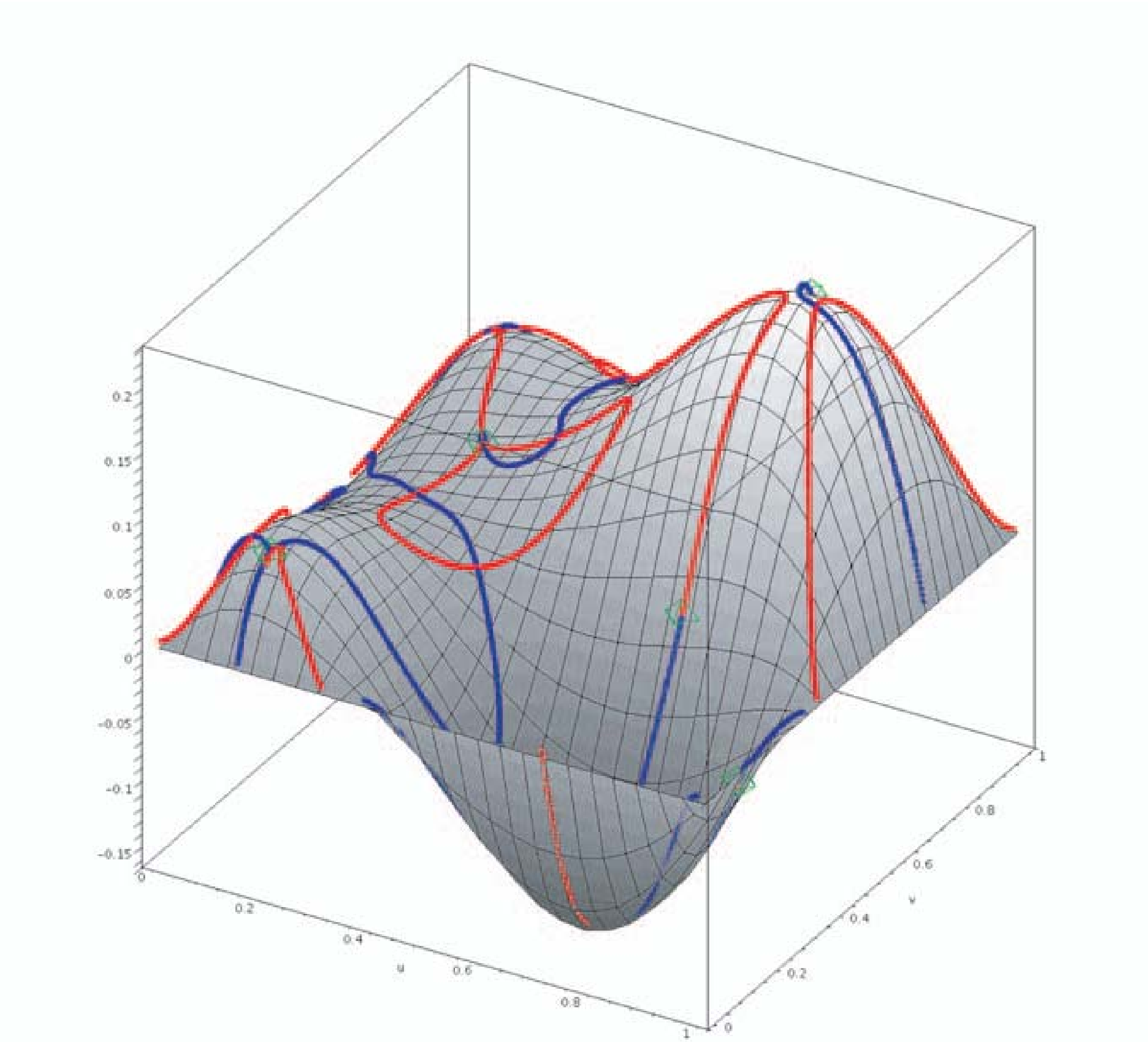


Fig. 1: Umbilics, ridges, and principal blue foliation on the ellipsoid



Finding ridges on surfaces has become a major industry in computer aided design. This is from work of Cazals, Pouget and others at CNRS.

In 2007 (at age 77) Ian was external examiner of a French PhD thesis on ridge detection.

Ian was particularly interested in the relationship between ridges and umbilics and he produced many interesting examples such as ‘bumpy spheres’ which are of the form

$x^2 + y^2 + z^2 + \varepsilon C(x,y,z) = 1$, C a cubic form, $|\varepsilon| << 1$

not to mention ‘bumpy oranges’, ‘bumpy tennis-balls’ and other irregular surfaces.

(The general classification of evolving ridge curves had to wait for the formidable trio of Bruce-Giblin-Tari, around 1999, but modesty forbids....)

16.4 Bumpy spheres 279

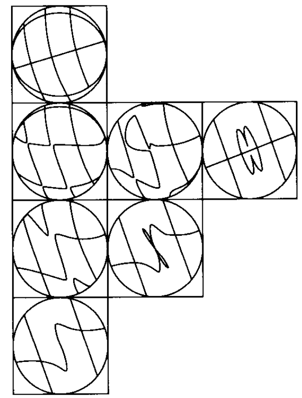


Figure 16.14

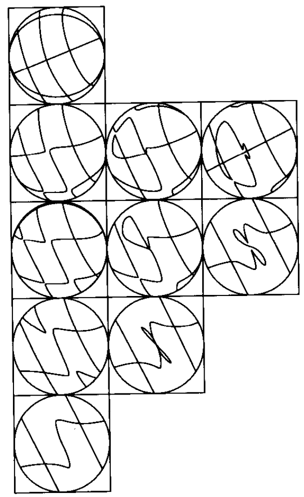
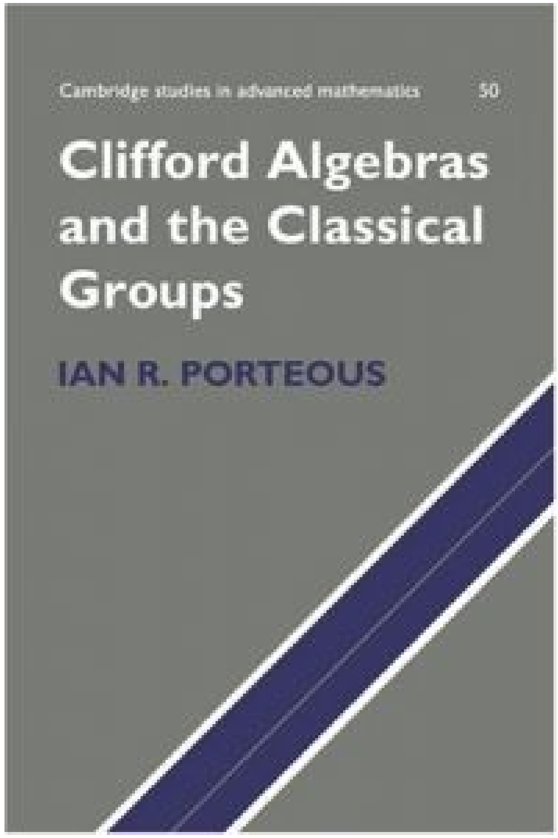


Figure 16.15

Evolution of ridges on spheres becoming bumpier (Stellios Markatis, PhD thesis 1980)



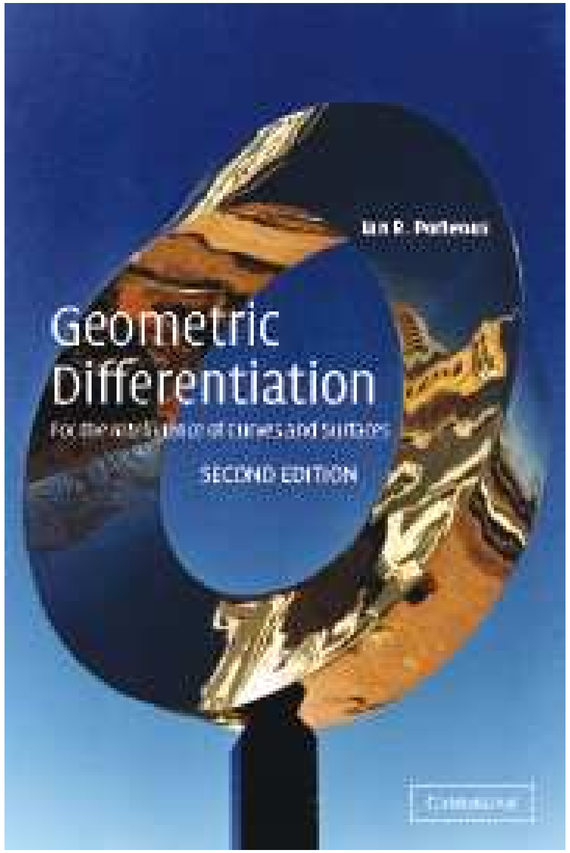
Ian also maintained an interest in geometrical algebra, particularly Clifford algebras. One chapter of this book (1995) evolved from a talk he gave on the group of conformal transformations of a real vector space furnished with a quadratic form

$$-x_1^2 - \ldots - x_p^2 + x_{p+1}^2 + \ldots + x_{p+q}^2$$

at a Banach Center conference in 1994.

He described this group in terms of the Clifford algebra associated with this quadratic form.

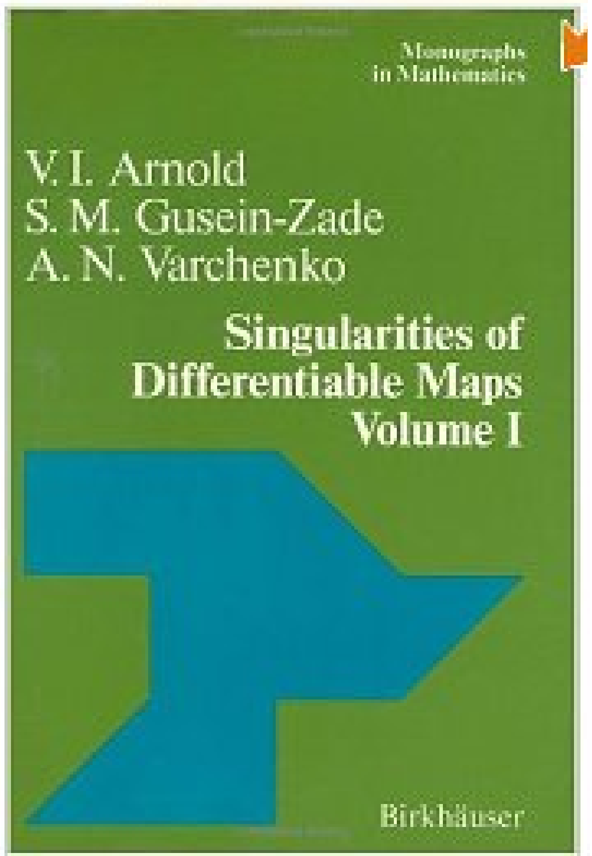
But it was the differential geometry of surfaces, via singularity theory of functions and maps, which became Ian's life work in mathematics and he published a book about his approach to the subject in 1994 (second edition 2001)



The cover illustrates Ian's interest in the sculpture of John Robinson—Ian was instrumental in bringing such an exhibition to Liverpool at the same time as a national travelling Maths Roadshow

'Eternity' from John Robinson's website; it's a (Zeeman) umbilic bracelet but generated by an equilateral triangle instead of a three-cusped curve.

Ian also translated many articles on singularities from Russian, as well as revising an earlier translation of Volume l of the standard work *Singularities of Differentiable Maps*.



Ian's students, such as James Montaldi, Brian Bellew, Alex Flegman, Stellios Markatis and Mike Puddephat, worked on singularities and their applications to geometry.

Ridge curves on surfaces in $\mathbf{R}^3$ (and their applications in computational geometry) fitted well with his leisure-time activity of mountain walking

which he shared with younger friends and colleagues. The photo was taken in the Scottish Highlands in 2009

Ian was a Liverpool City Councillor for the Liberal Party from 1974 to 1978 and closely involved with the Education Committee.

During this period and later Ian devoted a huge amount of energy and time to work with schools. A seminal development was

MEM (Mathematical Education on Merseyside), founded in 1977 and a Charity since 1986.

Ian was President of MEM from 1985 until his death.

I feel slightly worried by that orbit round the cube; let’s add another arc....

MEM, its **Challenge Competitions**, **Masterclasses** and 3-times a year **Orbiter** still thrive (well, they are chronically short of money). This is the top quarter of the current issue. Ian has been closely associated with the Challenge competitions for the last 30 years.

Please display or pass on to a colleague in your Mathematics Department

MATHEMATICAL EDUCATION ON MERSEYSIDE

ORBITER

2011
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MEM Orbiter can be downloaded free of charge from http://www.maths.liv.ac.uk/~mem

MEM Masterclasses 2011 The twenty-first series of Masterclasses will take place in the Autumn Term on five Saturdays starting in October 2011. Full details can be found at www.maths.liv.ac.uk/~mem/

If you wish to nominate two pupils from your school who are at present in Y7 or Y8 you should contact us now: Chris Marchant marchant@liv.ac.uk

Derek Haylock

What's really important in primary school mathematics?

A twilight seminar for students, teachers and mentors, suitable for primary and secondary colleagues

4:30PM Friday 20th May 2011

Liverpool Hope University
Eden Building

Matt Parker

The Stand-Up Mathematician

5:00PM Wednesday 5th October 2011

University of Liverpool

Full details in September Orbiter and also at www.livmathssoc.org.uk/

Ian Porteous

Liverpool Mathematical Society (LivMS)

5:30PM Thursday 12th May 2011

Mathematics Club

founded
in 1999

Ian gave a presentation to the Maths Club (prepared jointly with me) on 29 January 2011, the day before he died

Duads, synthemes and totals

To find out what this is about,
go to the Liverpool University
Maths Club website!

Then there is the Liverpool Mathematical Society

which celebrated its centenary in 1999 with the production of the

Liverpool Mathematical Society

L M S

Founded 1899

FUNMATHS

ROADSHOW

www.funmathsroadshow.com

This has become a national institution in the UK, taken into hundreds of schools to encourage pupils of all abilities to have a go at problems and activities which are fun but are real mathematics.

This was Ian's ruling passion for the past several years and its nationwide success is due to his personal input and the enthusiastic team which he helped to build.

Once in this team, it is difficult to escape!

(I speak from experience)

Two pictures of Ian working with children, typical of the last many years

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