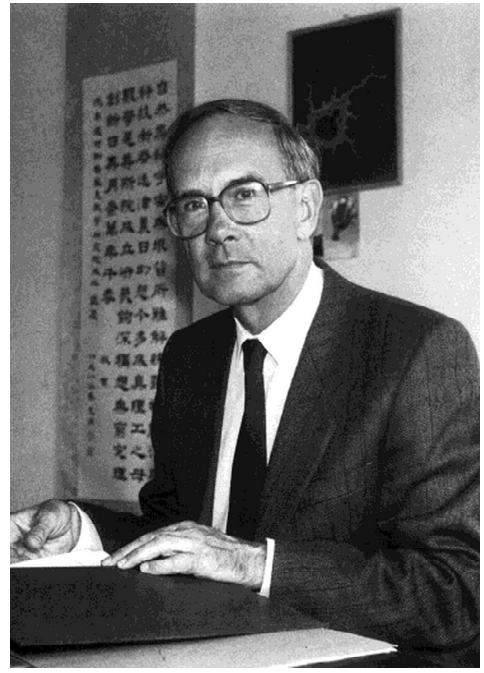
FRITZ HIRZEBRUCH (1927-2012)



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http://www.maths.ed.ac.uk/~aar
Oberwolfach, 29 May, 2012

Hirzebruch's influence, especially on surgery theory

- Hirzebruch worked in many areas of mathematics: singularities, topology, complex manifolds and algebraic geometry.
- Name lives on:
  - the Hirzebruch surfaces,
  - the Hirzebruch signature theorem,
  - the Hirzebruch $\mathcal{L}$-genus,
  - the Hirzebruch-Riemann-Roch theorem,
  - the Atiyah-Hirzebruch spectral sequence,
  - the Hirzebruch modular surfaces
  - ...
- His work had enormous influence, not least in surgery theory!

The Hirzebruch signature theorem

- The signature of a closed oriented $4k$-dimensional manifold M is defined by

$$\tau(M) \;=\; \text{signature}(H^{2k}(M), \text{intersection pairing}) \in \mathbb{Z}\ .$$

- **Theorem** (H.,1953) The signature of M is

$$\tau(M) \;=\; \langle \mathcal{L}_k(M), [M] \rangle \in \mathbb{Z} \subset \mathbb{Q}$$

with $[M] \in H_{4k}(M)$ the fundamental class, and $\mathcal{L}_*(M) \in H^{4*}(M;\mathbb{Q})$ the $\mathcal{L}$-genus, a $\mathbb{Q}$-coefficient polynomial in the Pontrjagin classes $p_i(\tau_M) \in H^{4i}(M)$.

- The coefficients in the $\mathcal{L}$-genus are determined explicitly by the Bernoulli numbers, starting with

$$\mathcal{L}_1(M) \;=\; p_1(M)/3 \in H^4(M;\mathbb{Q})\ .$$

- Princeton 1970 lecture of Hirzebruch:
The signature theorem: reminiscences and recreation

The Milnor exotic spheres

- Milnor discovered the exotic spheres in 1956 by observing that the Hirzebruch signature theorem failed for 3-connected 8-dimensional manifolds with non-empty boundary $(M, \partial M)$, i.e. that in general

$$\tau(M) - \langle \mathcal{L}_2(M), [M] \rangle \notin \mathbb{Z} \subset \mathbb{Q}$$

- Princeton 1996 lecture of Milnor: Classification of $(n-1)$-connected $2n$-dimensional manifolds and the discovery of the exotic spheres describes the discovery.
- The Hirzebruch signature theorem plays a central role in the 1962 surgery classification of exotic spheres by Kervaire and Milnor, giving the simply-connected $4k$-dimensional surgery obstruction.

Differentiable manifolds and quadratic forms

- Hirzebruch 1960 lecture
  Zur Theorie der Mannigfaltigkeiten
  gave the first E_8-plumbing construction of an exotic 7-sphere.
- 1962 book with Koh
  Differentiable manifolds and quadratic forms
  Still the best introduction to the relationship of manifolds and quadratic forms!
- Hirzebruch's 1967 Bourbaki seminar
  Singularities and exotic spheres
  describes the Brieskorn construction of exotic spheres as links of singularities, which was informed by Hirzebruch's work on the topology of singularities.

The Hirzebruch signature theorem in Browder-Novikov theory I.

- **Theorem** (B., 1962) Let X be a $4k$-dimensional Poincaré complex. For $k \geqslant 2$ and $\pi_1(X) = \{1\}$ X is homotopy equivalent to a closed $4k$-dimensional manifold if and only if there exists a j-plane vector bundle ν over X such that the fundamental class $[X] \in H_n(X) \cong H_{n+j}(T(\nu))$ is represented by a map $\rho : S^{n+j} \to T(\nu)$ such that the Hirzebruch signature formula holds

$$\tau(X) \;=\; \langle \mathcal{L}(-\nu), [X] \rangle \in \mathbb{Z} \; .$$

- This converse of the signature theorem proved in Browder's 1962 paper
  Homotopy types of differentiable manifolds

The Hirzebruch signature theorem in Browder-Novikov theory II.

- The Hirzebruch signature formula plays a similar role in Novikov's 1964 paper
  Homotopically equivalent smooth manifolds.
- The difference between a signature and the evaluation of the $\mathcal{L}$-genus as the surgery obstruction to making a homotopy equivalence of simply-connected $(4k-1)$-dimensional manifolds homotopic to a diffeomorphism.

Hirzebruch and the Novikov conjecture

- The 1969 Novikov conjecture started as a question about non-simply-connected analogues of the Hirzebruch signature theorem.
- See Volume I of the
  Proceedings of the 1993 Oberwolfach conference on Novikov conjectures, index theorems and rigidity
  for the background.

Hirzebruch in Edinburgh

- 1958, International Congress of Mathematicians, at which Hirzebruch was a plenary speaker.
- 2003, Hodge100 conference
- 2009, Atiyah80 conference
- Reminiscences of the Fifties
  Video of Hirzebruch lecture on Atiyah
- 2010, Honorary Fellow of the Royal Society of Edinburgh
- Aspects of quadratic forms in the work of Hirzebruch and Atiyah
  Slides of lectures given in 2010 in Edinburgh and Bonn by A.R.

Hirzebruch in Edinburgh, September, 2010

Hirzebruch-related links

- Max Planck Institute for Mathematics, Bonn
- Wikipedia Biography
- MacTutor Biography
- Simons Foundation Video
- Simons Foundation Photo Archive