SMSTC (2013/14)
Geometry and Topology
INFORMATION SHEET
www.smstc.ac.uk

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Course overview:

This is an introductory course in Geometry and Topology intended both for students who did not specialise in these topics at undergraduate level and for specialists, i.e. graduate students in Geometry and Topology. In addition to setting regular assignments the lecturers will suggest readings (as much as possible available on the internet) to supplement the course material, to further fulfil the SMSTC mission of broadening in fundamental areas of mathematics.

Course lecturers

- Andrew Ranicki, University of Edinburgh, Stream Leader
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- Vanya Cheltsov, University of Edinburgh
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- Richard Hepworth, University of Aberdeen
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- Brendan Owens, University of Glasgow
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- Bernd Schroers, Heriot-Watt University
  bernd@ma.hw.ac.uk http://www.macs.hw.ac.uk/staff-directory/bernd-schroers.htm

Guest lecturers

- Jeremy Gray, Open University
  http://www.mathematics.open.ac.uk/People/jeremy.gray
- Etienne Ghys, ENS Lyon
  http://www.umpa.ens-lyon.fr/~ghys/

Prerequisites:

A student taking this course requires a working knowledge of metric spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions); vector calculus (differentiable map, Jacobian matrix and div/grad/curl).

Assessment

There will be 3 written assignments, each to be submitted electronically to the University of Edinburgh LEARN system. Login details will be supplied nearer the time.

- Assignment 1 is due on Friday of Week 8, December 6, 2013
- Assignment 2 is due on Friday of Week 11, January 17, 2014
- Assignment 3 is due on Friday of Week 16, February 21, 2014
- Assignment 4 is due on Friday of Week 20, March 21, 2014

Lectures

In Semester 1 there will be 10 lectures, at 1-3PM on consecutive Thursdays in Weeks 1-10, starting on October 10 and ending on December 19. The lectures will cover:

Part 1*: Elementary Topology** ***[Brendan Owens]

- Lecture 1 (BO), October 17: Metric and topological spaces. Continuous functions.
- Lecture 2 (BO), October 24: Compactness and connectedness. Products and quotient topologies.
- Lecture 3 (BO), October 31: Examples of spaces: graphs, manifolds, surfaces, matrix groups, projective spaces. Basic homotopy theory. The Hopf map.

Part 2*: Algebraic Topology** ***[Richard Hepworth, Vanya Cheltsov, Andrew Ranicki]

- Lecture 4, November 7 (RH) Homology I
- Lecture 5, November 14 (RH) Homology II.
- Lecture 6, November 21 (AR) The genus of a surface. The fundamental group. Covering spaces.
- Lecture 7, November 28 (VC) Algebraic curves and Riemann surfaces.
- Lecture 8, December 5 (RH) Homology III.
- Lecture 9, December 12 (AR) The Seifert-van Kampen theorem. The fundamental groups of surfaces.

Guest lecture

- Lecture 10, December 19, Jeremy Gray (Open University) Poincaré and topology.

There will be no assignments on Lecture 10.

In Semester 2, there will be 10 lectures, at 1-3PM on Thursdays in Weeks 11-20, starting on January 16 and ending on March 20. The lectures will cover:

Part 3*: Differential Geometry of surfaces** ***[Bernd Schroers]

- Lecture 11, January 16 (BS) Local manifolds in $\mathbb{R}^n$, the implicit function theorem, Sard's theorem.
- Lecture 12, January 23 (BS) Smooth curves and surfaces in $\mathbb{R}^3$.
- Lecture 13, January 30 (BS) Linear connections, Christoffel symbols, geodesics.
- Lecture 14, February 6 (BS) The Theorema Egregium. Curvature.

Part 4*: Introduction to manifolds** ***[Bernd Schroers, Andrew Ranicki]

- Lecture 15, February 13 (BS) The Gauss-Bonnet Theorem for surfaces.
- Lecture 16, February 20 (AR) Differentiable manifolds, tangent bundle, vector fields.
- Lecture 17, February 27 (AR) Differential forms, deRham complex, Poincaré Lemma.
- Lecture 18, March 6 (AR) deRham cohomology: examples. Integration of differential forms; Stokes's Theorem. Poincaré duality.
- Lecture 19, March 13 (AR) Thom isomorphism. Intersection pairing. Linking numbers. The Hopf invariant.

Guest lecture

- Lecture 20, March 20 (Etienne Ghys) Poincaré and geodesy

There will be no assignments on Lecture 20.

Course material and useful references

Lecture notes, slides texts and assignments are available from the SMSTC wiki http://www.smstc.ac.uk Throughout the course we make reference to several books or online resources, ordered according to their appearance in the course

- S.A. Morris, *Topology without tears*,http://uob-community.ballarat.edu.au/~smorris/topbook.pdf
- A. Hatcher, *Algebraic Topology*, CUP 2002. http://www.math.cornell.edu/~hatcher/AT/ATpage.html
- J. Gallier, *The Classification Theorem for Compact Surfaces And A Detour On Fractals*, http://front.math.ucdavis.edu/0805.0562
- F. Kirwan, *Complex algebraic curves*, LMS Student Text, CUP (1992)
- J. Dupont, *Differential Geometry*, Arhus notes (1993) http://www.maths.ed.ac.uk/~aar/papers/dupontnotes.pdf
- J. Lee, *Riemannian manifolds*, GTM 176, Springer (1997).
- R. Bott and L. Tu, *Differential forms in Algebraic Topology*, GTM Springer (1982).
- J. Milnor, *Topology from the Differentiable Viewpoint*, U.P. Virginia (1965).

Individual lecturers are happy to provide additional suggestions on request.