Chapter 8

JAMES JOSEPH SYLVESTER[1]

(1814-1897)

James Joseph Sylvester was born in London, on the 3d of September, 1814. He was by descent a Jew. His father was Abraham Joseph Sylvester, and the future mathematician was the youngest but one of seven children. He received his elementary education at two private schools in London, and his secondary education at the Royal Institution in Liverpool. At the age of twenty he entered St. John's College, Cambridge; and in the tripos examination he came out second wrangler. The senior wrangler of the year did not rise to any eminence; the fourth wrangler was George Green, celebrated for his contributions to mathematical physics; the fifth wrangler was Duncan F. Gregory, who subsequently wrote on the foundations of algebra. On account of his religion Sylvester could not sign the thirty-nine articles of the Church of England; and as a consequence he could neither receive the degree of Bachelor of Arts nor compete for the Smith's prizes, and as a further consequence he was not eligible for a fellowship. To obtain a degree he turned to the University of Dublin. After the theological tests for degrees had been abolished at the Universities of Oxford and Cambridge in 1872, the University of Cambridge granted him his well-earned degree of Bachelor of Arts and also that of Master of Arts.

On leaving Cambridge he at once commenced to write papers, and these were at first on applied mathematics. His first paper was entitled "An analytical development of Fresnel's optical theory of crystals," which was published in the *Philosophical Magazine*. Ere long he was appointed Professor of Physics in University College, London, thus becoming a colleague of De Morgan. At that time University College was almost the only institution of higher education in England in which theological distinctions were ignored. There was then no physical laboratory at University College, or indeed at the University of

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Cambridge; which was fortunate in the case of Sylvester, for he would have made a sorry experimenter. His was a sanguine and fiery temperament, lacking the patience necessary in physical manipulation. As it was, even in these pre-laboratory days he felt out of place, and was not long in accepting a chair of pure mathematics.

In 1841 he became professor of mathematics at the University of Virginia. In almost all notices of his life nothing is said about his career there; the truth is that after the short space of four years it came to a sudden and rather tragic termination. Among his students were two brothers, fully imbued with the Southern ideas about honor. One day Sylvester criticised the recitation of the younger brother in a wealth of diction which offended the young man's sense of honor; he sent word to the professor that he must apologize or be chastised. Sylvester did not apologize, but provided himself with a sword-cane; the young man provided himself with a heavy walking-stick. The brothers lay in wait for the professor; and when he came along the younger brother demanded an apology, almost immediately knocked off Sylvester's hat, and struck him a blow on the bare head with his heavy stick. Sylvester drew his sword-cane, and pierced the young man just over the heart; who fell back into his brother's arms, calling out "I am killed." A spectator, coming up, urged Sylvester away from the spot. Without waiting to pack his books the professor left for New York, and took the earliest possible passage for England. The student was not seriously hurt; fortunately the point of the sword had struck fair against a rib.

Sylvester, on his return to London, connected himself with a firm of actuaries, his ultimate aim being to qualify himself to practice conveyancing. He became a student of the Inner Temple in 1846, and was called to the bar in 1850. He chose the same profession as did Cayley; and in fact Cayley and Sylvester, while walking the law-courts, discoursed more on mathematics than on conveyancing. Cayley was full of the theory of invariants; and it was by his discourse that Sylvester was induced to take up the subject. These two men were life-long friends; but it is safe to say that the permanence of the friendship was due to Cayley's kind and patient disposition. Recognized as the leading mathematicians of their day in England, they were yet very different both in nature and talents.

Cayley was patient and equable; Sylvester, fiery and passionate. Cayley finished off a mathematical memoir with the same care as a legal instrument; Sylvester never wrote a paper without foot-notes, appendices, supplements; and the alterations and corrections in his proofs were such that the printers found their task well-nigh impossible. Cayley was well-read in contemporary mathematics, and did much useful work as referee for scientific societies; Sylvester read only what had an immediate bearing on his own researches, and did little, if any, work as a referee. Cayley was a man of sound sense, and of great service in University administration; Sylvester satisfied the popular idea of a mathematician as one lost in reflection, and high above mundane affairs. Cayley was modest and retiring; Sylvester, courageous and full of his own importance. But while Cayley's papers, almost all, have the stamp of pure logical mathematics, Sylvester's are full of human interest. Cayley was no orator and no poet;

Sylvester was an orator, and if not a poet, he at least prided himself on his poetry. It was not long before Cayley was provided with a chair at Cambridge, where he immediately married, and settled down to work as a mathematician in the midst of the most favorable environment. Sylvester was obliged to continue what he called "fighting the world" alone and unmarried.

There is an ancient foundation in London, named after its founder, Gresham College. In 1854 the lectureship of geometry fell vacant and Sylvester applied. The trustees requested him and I suppose also the other candidates, to deliver a probationary lecture; with the result that he was not appointed. The professorship of mathematics in the Royal Military Academy at Woolwich fell vacant; Sylvester was again unsuccessful; but the appointee died in the course of a year, and then Sylvester succeeded on a second application. This was in 1855, when he was 41 years old.

He was a professor at the Military Academy for fifteen years; and these years constitute the period of his greatest scientific activity. In addition to continuing his work on the theory of invariants, he was guided by it to take up one of the most difficult questions in the theory of numbers. Cayley had reduced the problem of the enumeration of invariants to that of the partition of numbers; Sylvester may be said to have revolutionized this part of mathematics by giving a complete analytical solution of the problem, which was in effect to enumerate the solutions in positive integers of the indeterminate equation:

$$ax + by + cz + \ldots + ld = m.$$

Thereafter he attacked the similar problem connected with two such simultaneous equations (known to Euler as the problem of the Virgins) and was partially and considerably successful. In June, 1859, he delivered a series of seven lectures on compound partition in general at King's College, London. The outlines of these lectures have been published by the Mathematical Society of London.

Five years later (1864) he contributed to the Royal Society of London what is considered his greatest mathematical achievement. Newton, in his lectures on algebra, which he called "Universal Arithmetic" gave a rule for calculating an inferior limit to the number of imaginary roots in an equation of any degree, but he did not give any demonstration or indication of the process by which he reached it. Many succeeding mathematicians such as Euler, Waring, Maclaurin, took up the problem of investigating the rule, but they were unable to establish either its truth or inadequacy. Sylvester in the paper quoted established the validity of the rule for algebraic equations as far as the fifth degree inclusive. Next year in a communication to the Mathematical Society of London, he fully established and generalized the rule. "I owed my success," he said, "chiefly to merging the theorem to be proved in one of greater scope and generality. In mathematical research, reversing the axiom of Euclid and controverting the proposition of Hesiod, it is a continual matter of experience, as I have found myself over and over again, that the whole is less than its part."

Two years later he succeeded De Morgan as president of the London Mathematical Society. He was the first mathematician to whom that Society awarded the Gold medal founded in honor of De Morgan. In 1869, when the British Association met in Exeter, Prof. Sylvester was president of the section of mathematics and physics. Most of the mathematicians who have occupied that position have experienced difficulty in finding a subject which should satisfy the two conditions of being first, cognate to their branch of science; secondly, interesting to an audience of general culture. Not so Sylvester. He took up certain views of the nature of mathematical science which Huxley the great biologist had just published in *Macmillan's Magazine* and the *Fortnightly Review.* He introduced his subject by saying that he was himself like a great party leader and orator in the House of Lords, who, when requested to make a speech at some religious or charitable, at-all-events non-political meeting declined the honor on the ground that he could not speak unless he saw an adversary before him. I shall now quote from the address, so that you may hear Sylvester's own words.

"In obedience," he said, "to a somewhat similar combative instinct, I set to myself the task of considering certain utterances of a most distinguished member of the Association, one whom I no less respect for his honesty and public spirit, than I admire for his genius and eloquence, but from whose opinions on a subject he has not studied I feel constrained to differ. I have no doubt that had my distinguished friend, the probable president-elect of the next meeting of the Association, applied his uncommon powers of reasoning, induction, comparison, observation and invention to the study of mathematical science, he would have become as great a mathematician as he is now a biologist; indeed he has given public evidence of his ability to grapple with the practical side of certain mathematical questions; but he has not made a study of mathematical science as such, and the eminence of his position, and the weight justly attaching to his name, render it only the more imperative that any assertion proceeding from such a quarter, which may appear to be erroneous, or so expressed as to be conducive to error should not remain unchallenged or be passed over in silence.

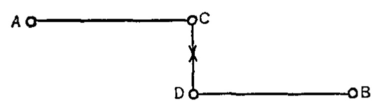
"Huxley says 'mathematical training is almost purely deductive. The mathematician starts with a few simple propositions, the proof of which is so obvious that they are called self-evident, and the rest of his work consists of subtle deductions from them. The teaching of languages at any rate as ordinarily practised, is of the same general nature—authority and tradition furnish the data, and the mental operations are deductive.' It would seem from the above somewhat singularly juxtaposed paragraphs, that according to Prof. Huxley, the business of the mathematical student is, from a limited number of propositions (bottled up and labelled ready for use) to deduce any required result by a process of the same general nature as a student of languages employs in declining and conjugating his nouns and verbs—that to make out a mathematical proposition and to construe or parse a sentence are equivalent or identical mental operations. Such an opinion scarcely seems to need serious refutation. The passage is taken from an article in *Macmillan's Magazine* for June last, entitled, 'Scientific Education—Notes of an after-dinner speech'; and I cannot but think would have been couched in more guarded terms by my distinguished friend, had his speech been made *before* dinner instead of *after*.

"The notion that mathematical truth rests on the narrow basis of a limited number of elementary propositions from which all others are to be derived by a process of logical inference and verbal deduction has been stated still more strongly and explicitly by the same eminent writer in an article of even date with the preceeding in the *Fortnightly Review*; where we are told that 'Mathematics is that study which knows nothing of observation, nothing of experiment, nothing of induction, nothing of causation.' I think no statement could have been made more opposite to the undoubted facts of the case, which are that mathematical analysis is constantly invoking the aid of new principles, new ideas and new methods not capable of being defined by any form of words, but springing direct from the inherent powers and activity of the human mind, and from continually renewed introspection of that inner world of thought of which the phenomena are as varied and require as close attention to discern as those of the outer physical world; that it is unceasingly calling forth the faculties of observation and comparison; that one of its principal weapons is induction; that is has frequent recourse to experimental trial and verification; and that it affords a boundless scope for the exercise of the highest efforts of imagination and invention."

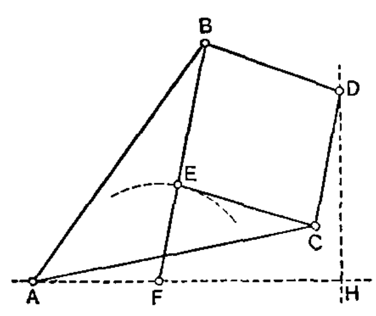
Huxley never replied; convinced or not, he had sufficient sagacity to see that he had ventured far beyond his depth. In the portion of the address quoted, Sylvester adds parenthetically a clause which expresses his theory of mathematical knowledge. He says that the inner world of thought in each individual man (which is the world of observation to the mathematician) may be conceived to stand in somewhat the same general relation of correspondence to the outer physical world as an object to the shadow projected from it. To him the mental order was more real than the world of sense, and the foundation of mathematical science was ideal, not experimental.

By this time Sylvester had received most of the high distinctions, both domestic and foreign, which are usually awarded to a mathematician of the first rank in his day. But a discontinuity was at hand. The War Office issued a regulation whereby officers of the army were obliged to retire on half pay on reaching the age of 55 years. Sylvester was a professor in a Military College; in a few months, on his reaching the prescribed age, he was retired on half pay. He felt that though no longer fit for the field he was still fit for the classroom. And he felt keenly the diminution in his income. It was about this time that he issued a small volume—the only book he ever published; not on mathematics, as you may suppose, but entitled *The Laws of Verse*. He must have prided himself a good deal on this composition, for one of his last letters in *Nature* is signed "J. J. Sylvester, author of The Laws of Verse." He made some excellent translations from Horace and from German poets; and like Sir W. R. Hamilton he was accustomed to express his feelings in sonnets.

The break in his life appears to have discouraged Sylvester for the time being from engaging in any original research. But after three years a Russian mathematician named Tschebicheff, a professor in the University of Saint Petersburg, visiting Sylvester in London, drew his attention to the discovery by a Russian student named Lipkin, of a mechanism for drawing a perfect straight line. Mr. Lipkin received from the Russian Government a substantial award. It was found that the same discovery had been made several years before by M. Peaucellier, an officer in the French army, but failing to be recognized at its true value had dropped into oblivion. Sylvester introduced the subject into England in the form of an evening lecture before the Royal Institution, entitled "On recent discoveries in mechanical conversion of motion." The Royal Institution of London was founded to promote scientific research; its professors have been such men as Davy, Faraday, Tyndall, Dewar. It is not a teaching institution, but it provides for special courses of lectures in the afternoons and for Friday evening lectures by investigators of something new in science. The evening lectures are attended by fashionable audiences of ladies and gentlemen in full dress.



Euclid bases his *Elements* on two postulates; first, that a straight line can be drawn, second, that a circle can be described. It is sometimes expressed in this way; he postulates a ruler and compass. The latter contrivance is not difficult to construct, because it does not involve the use of a ruler or a compass in its own construction. But how is a ruler to be made straight, unless you already have a ruler by which to test it? The problem is to devise a mechanism which shall assume the second postulate only, and be able to satisfy the first. It is the mechanical problem of converting motion in a circle into motion in a straight line, without the use of any guide. James Watt, the inventor of the steam-engine, tackled the problem with all his might, but gave it up as impossible. However, he succeeded in finding a contrivance which solves the problem very approximately. Watt's parallelogram, employed in nearly every beam-engine, consists of three links; of which AC and BD are equal, and have fixed pivots at A and B respectively. The link CD is of such a length that AC and BD are parallel when horizontal. The tracing point is attached to the middle point of CD. When C and D move round their pivots, the tracing point describes a straight line very approximately, so long as the arc of displacement is small. The complete figure which would be described is the figure of 8, and the part utilized is near the point of contrary flexure.



A linkage giving a closer approximation to a straight line was also invented by the Russian mathematician before mentioned—Tschebicheff; it likewise made use of three links. But the linkage invented by Peaucellier and later by Lipkin had seven pieces. The arms AB and AC are of equal length, and have a fixed pivot at A. The links DB, BE, EC, CD are of equal length. EF is an arm connecting E with the fixed pivot F and is equal in length to the distance between A and F. It is readily shown by geometry that, as the point E describes a circle around the center F, the point D describes an exact straight line perpendicular to the line joining it and F. The exhibition of this contrivance at work was the climax of Sylvester's lecture.

In Sylvester's audience were two mathematicians, Hart and Kempe, who took up the subject for further investigation. Hart perceived that the contrivances of Watt and of Tschebicheff consisted of three links, whereas Peaucellier's consisted of seven. Accordingly he searched for a contrivance of five links which would enable a tracing point to describe a perfect straight line; and he succeeded in inventing it. Kempe was a London barrister whose specialty was ecclesiastical law. He and Sylvester worked up the theory of linkages together, and discovered among other things the skew pantograph. Kempe became so imbued with linkage that he contributed to the Royal Society of London a paper on the "Theory of Mathematical Form," in which he explains all reasoning by means of linkages.

About this time (1877) the Johns Hopkins University was organized at Baltimore, and Sylvester, at the age of 63, was appointed the first professor of mathematics. Of his work there as a teacher, one of his pupils, Dr. Fabian Franklin, thus spoke in an address delivered at a memorial meeting in that University: "The one thing which constantly marked Sylvester's lectures was enthusiastic love of the thing he was doing. He had in the fullest possible degree, to use the French phrase, the defect of this quality; for as he almost always spoke with enthusiastic ardor, so it was almost never possible for him to speak on matters incapable of evoking this ardor. In other words, the substance of his lectures had to consist largely of his own work, and, as a rule, of work hot from the forge. The consequence was that a continuous and systematic presentation of any extensive body of doctrine already completed was not to be expected from him. Any unsolved difficulty, any suggested extension, such as would have been passed by with a mention by other lecturers, became inevitably with him the occasion of a digression which was sure to consume many weeks, if indeed it did not take him away from the original object permanently. Nearly all of the important memoirs which he published, while in Baltimore, arose in this way. We who attended his lectures may be said to have seen these memoirs in the making. He would give us on the Friday the outcome of his grapplings with the enemy since the Tuesday lecture. Rarely can it have fallen to the lot of any class to follow so completely the workings of the mind of the master. Not only were all thus privileged to see 'the very pulse of the machine,' to learn the spring and motive of the successive steps that led to his results, but we were set aglow by the delight and admiration which, with perfect naïveté and with that luxuriance of language peculiar to him, Sylvester lavished upon these results. That in this enthusiastic admiration he sometimes lacked the sense of proportion cannot be denied. A result announced at one lecture and hailed with loud acclaim as a marvel of beauty was by no means sure of not being found before the next lecture to have been erroneous; but the Esther that supplanted this Vashti was quite certain to be found still more supremely beautiful. The fundamental thing, however, was not this occasional extravagance, but the deep and abiding feeling for truth and beauty which underlay it. No young man of generous mind could stand before that superb grey head and hear those expositions of high and dear-bought truths, testifying to a passionate devotion undimmed by years or by arduous labors, without carrying away that which ever after must give to the pursuit of truth a new and deeper significance in his mind."

One of Sylvester's principal achievements at Baltimore was the founding of the *American Journal of Mathematics*, which, at his suggestion, took the quarto form. He aimed at establishing a mathematical journal in the English language, which should equal Liouville's *Journal* in France, or Crelle's *Journal* in Germany. Probably his best contribution to the *American Journal* consisted in his "Lectures on Universal Algebra"; which, however, were left unfinished, like a great many other projects of his.

Sylvester had that quality of absent-mindedness which is popularly supposed to be, if not the essence, at least an invariable accompaniment, of a distinguished mathematician. Many stories are related on this point, which, if not all true, are at least characteristic. Dr. Franklin describes an instance which actually happened in Baltimore. To illustrate a theory of versification contained in his book *The Laws of Verse*, Sylvester prepared a poem of 400 lines, all rhyming with the name Rosalĭnd or Rosalīnd; and it was announced that the professor would read the poem on a specified evening at a specified hour at the Peabody Institute. At the time appointed there was a large turn-out of ladies and gentlemen. Prof. Sylvester, as usual, had a number of footnotes appended to his production; and he announced that in order to save interruption in reading the poem itself, he would first read the footnotes. The reading of the footnotes suggested various digressions to his imagination; an hour had passed, still no poem; an hour and a half passed and the striking of the clock or the unrest of his audience reminded him of the promised poem. He was astonished to find how time had passed, excused all who had engagements, and proceeded to read the Rosalind poem.

In the summer of 1881 I visited London to see the Electrical Exhibition in the Crystal Palace—one of the earliest exhibitions devoted to electricity exclusively. I had made some investigations on the electric discharge, using a Holtz machine where De LaRue used a large battery of cells. Mr. De LaRue was Secretary of the Royal Institution; he gave me a ticket to a Friday evening discourse to be delivered by Mr. Spottiswoode, then president of the Royal Society, on the phenomena of the intensive discharge of electricity through gases; also an invitation to a dinner at his own house to be given prior to the lecture. Mr. Spottiswoode, the lecturer for the evening, was there; also Prof. Sylvester. He was a man rather under the average height, with long gray beard and a profusion of gray locks round his head surmounted by a great dome of forehead. He struck me as having the appearance of an artist or a poet rather than of an exact scientist. After dinner he conversed very eloquently with an elderly lady of title, while I conversed with her daughter. Then cabs were announced to take us to the Institution. Prof. Sylvester and I, being both bachelors, were put in a cab together. The professor, who had been so eloquent with the lady, said nothing; so I asked him how he liked his work at the Johns Hopkins University. "It is very pleasant work indeed," said he, "and the young men who study there are all so enthusiastic." We had not exhausted that subject before we reached our destination. We went up the stairway together, then Sylvester dived into the library to see the last number of *Comptes Rendus* (in which he published many of his results at that time) and I saw him no more. I have always thought it very doubtful whether he came out to hear Spottiswoode's lecture.

We have seen that H. J. S. Smith, the Savilian professor of Geometry at Oxford, died in 1883. Sylvester's friends urged his appointment, with the result that he was elected. After two years he delivered his inaugural lecture; of which the subject was differential invariants, termed by him reciprocants. An elementary reciprocant is $\frac{d^2y}{dx^2}$, for if $\frac{d^2y}{dx^2} = 0$ then $\frac{d^2x}{dy^2} = 0$. He looked upon this as the "grub" form, and developed from it the "chrysalis"

$$\begin{vmatrix} \frac{d^2\phi}{dx^2} & \frac{d^2\phi}{dxdy} & \frac{d\phi}{dx}, \\ \frac{d^2\phi}{dxdy} & \frac{d^2\phi}{dy^2} & \frac{d\phi}{dy}, \\ \frac{d\phi}{dx} & \frac{d\phi}{dy} & . \end{vmatrix}$$

and the "imago"

$$\begin{vmatrix} \frac{d^2\Phi}{dx^2} & \frac{d^2\Phi}{dxdy} & \frac{d^2\Phi}{dxdr}, \\ \frac{d^2\Phi}{dxdy} & \frac{d^2\Phi}{dy^2} & \frac{d^2\Phi}{dydr}, \\ \frac{d^2\Phi}{dxdr} & \frac{d^2\Phi}{dydr} & \frac{d^2\Phi}{dr^2}. \end{vmatrix}$$

You will observe that the chrysalis expression is unsymmetrical; the place of a ninth term is vacant. It moved Sylvester's poetic imagination, and into his inaugural lecture he interjected the following sonnet:

To a Missing Member of a Family Group of Terms in an Algebraical Formula:

Lone and discarded one! divorced by fate,
Far from thy wished-for fellows—whither art flown?
Where lingerest thou in thy bereaved estate,
Like some lost star, or buried meteor stone?
Thou minds't me much of that presumptuous one,
Who loth, aught less than greatest, to be great,
From Heaven's immensity fell headlong down
To live forlorn, self-centred, desolate:
Or who, new Heraklid, hard exile bore,
Now buoyed by hope, now stretched on rack of fear,
Till throned Astræa, wafting to his ear
Words of dim portent through the Atlantic roar,

> Bade him "the sanctuary of the Muse revere
> And strew with flame the dust of Isis' shore."

This inaugural lecture was the beginning of his last great contribution to mathematics, and the subsequent lectures of that year were devoted to his researches in that line. Smith and Sylvester were akin in devoting attention to the theory of numbers, and also in being eloquent speakers. But in other respects the Oxonians found a great difference. Smith had been a painstaking tutor; Sylvester could lecture only on his own researches, which were not popular in a place so wholly given over to examinations. Smith was an incessantly active man of affairs; Sylvester became the subject of melancholy and complained that he had no friends.

In 1872 a deputy professor was appointed. Sylvester removed to London, and lived mostly at the Athenæum Club. He was now 78 years of age, and suffered from partial loss of sight and memory. He was subject to melancholy, and his condition was indeed "forlorn and desolate." His nearest relatives were nieces, but he did not wish to ask their assistance. One day, meeting a mathematical friend who had a home in London, he complained of the fare at the Club, and asked his friend to help him find suitable private apartments where he could have better cooking. They drove about from place to place for a whole afternoon, but none suited Sylvester. It grew late: Sylvester said, "You have a pleasant home: take me there," and this was done. Arrived, he appointed one daughter his reader and another daughter his amanuensis. "Now," said he, "I feel comfortably installed; don't let my relatives know where I am." The fire of his temper had not dimmed with age, and it required all the Christian fortitude of the ladies to stand his exactions. Eventually, notice had to be sent to his nieces to come and take charge of him. He died on the 15th of March, 1897, in the 83d year of his age, and was buried in the Jewish cemetery at Dalston.

As a theist, Sylvester did not approve of the destructive attitude of such men as Clifford, in matters of religion. In the early days of his career he suffered much from the disabilities attached to his faith, and they were the prime cause of so much "fighting the world." He was, in all probability, a greater mathematical genius than Cayley; but the environment in which he lived for some years was so much less favorable that he was not able to accomplish an equal amount of solid work. Sylvester's portrait adorns St. John's College, Cambridge. A memorial fund of £1500 has been placed in the charge of the Royal Society of London, from the proceeds of which a medal and about £100 in money is awarded triennially for work done in pure mathematics. The first award has been made to M. Henri Poincaré of Paris, a mathematician for whom Sylvester had a high professional and personal regard.