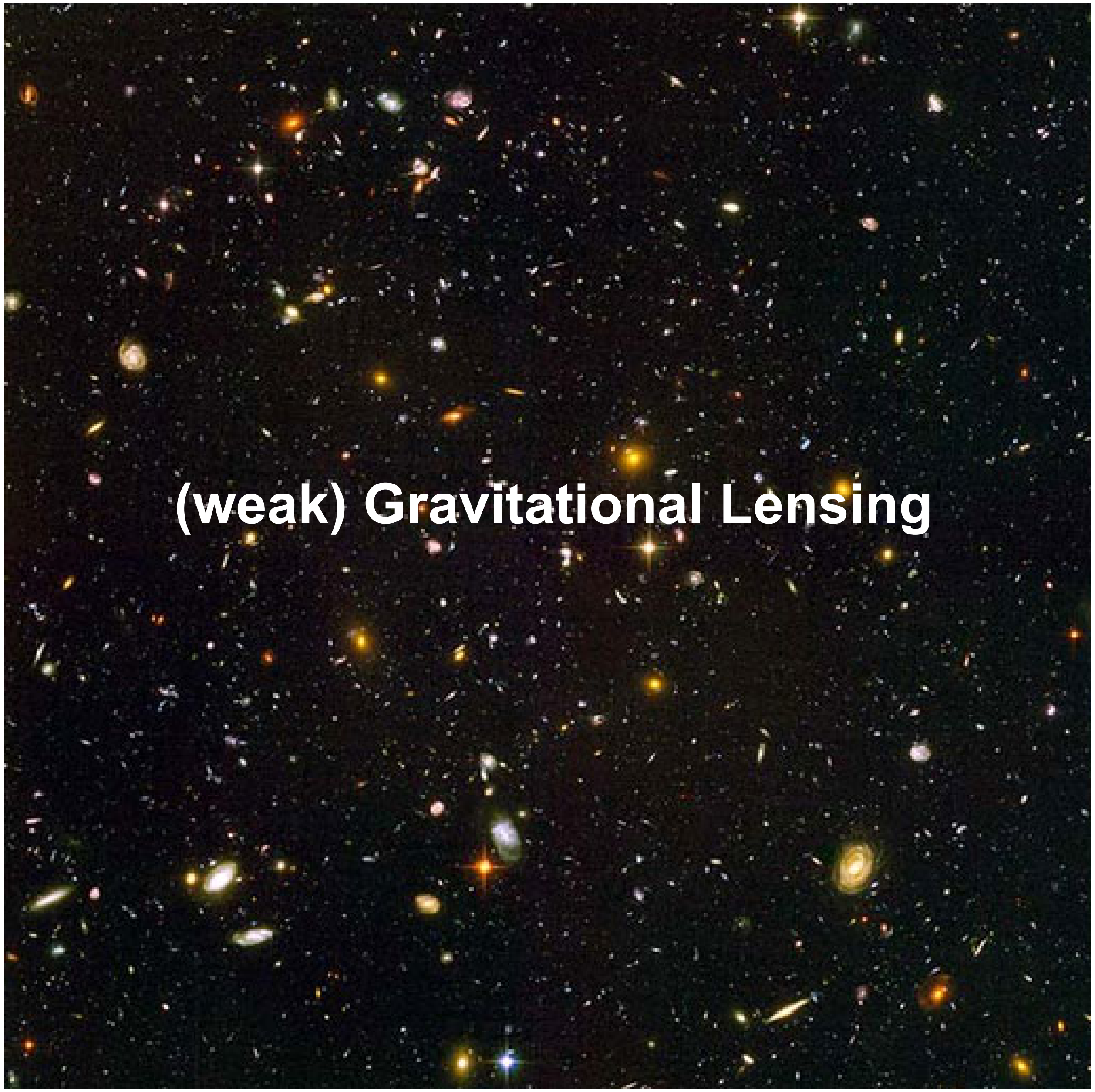
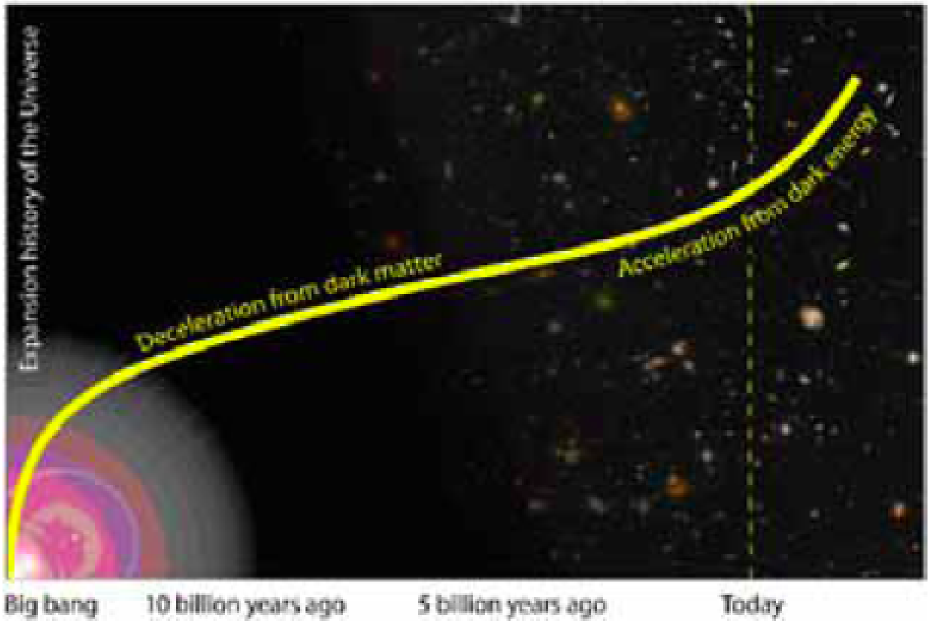
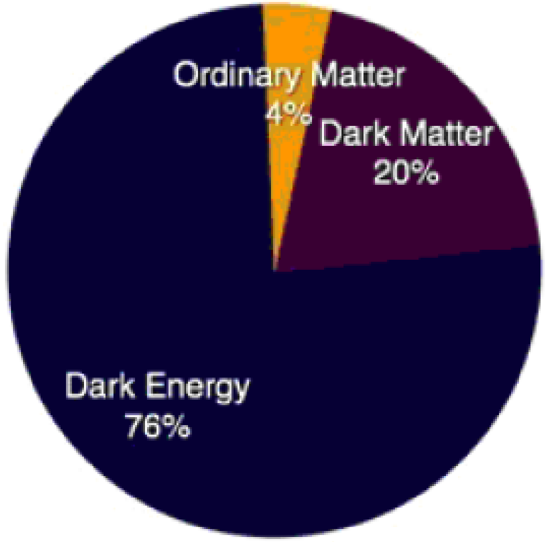


(weak) Gravitational Lensing

Thomas Kitching tdk@roe.ac.uk


Expansion history of the Universe
Deceleration from dark matter
Acceleration from dark energy
Big bang
10 billion years ago
5 billion years ago
Today


Ordinary Matter
4%
Dark Matter
20%
Dark Energy
76%

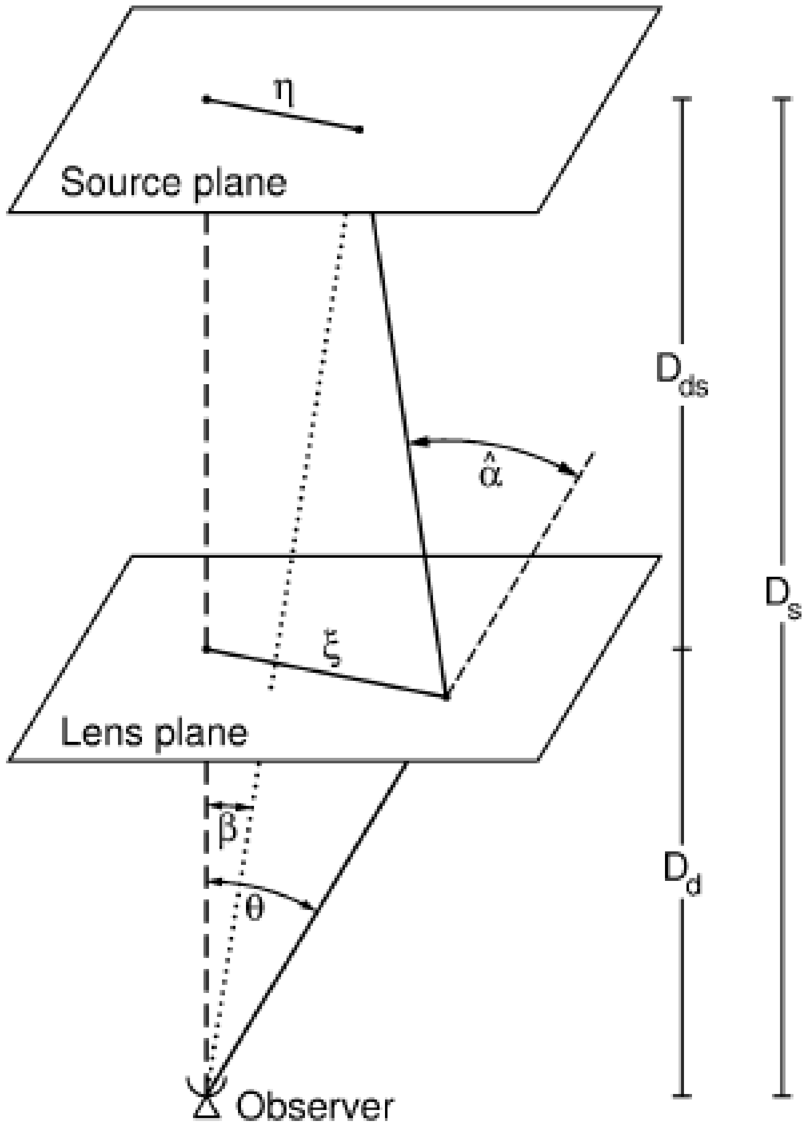
- What is weak lensing ?

- Why is it interesting ?

- How do we measure weak lensing (help!) ?

What is Gravitational Lensing?

- Propagating photons follow geodesics in space
- The geodesics are distorted from straight lines by the presence of massive objects
- Even in Newtonian Gravity photons would be deflected from straight line path



$$\beta = \theta - \hat{\alpha}\frac{D_{ds}}{D_s}$$
$$\beta = \theta - \alpha.$$

Relates True Position to Observed Position

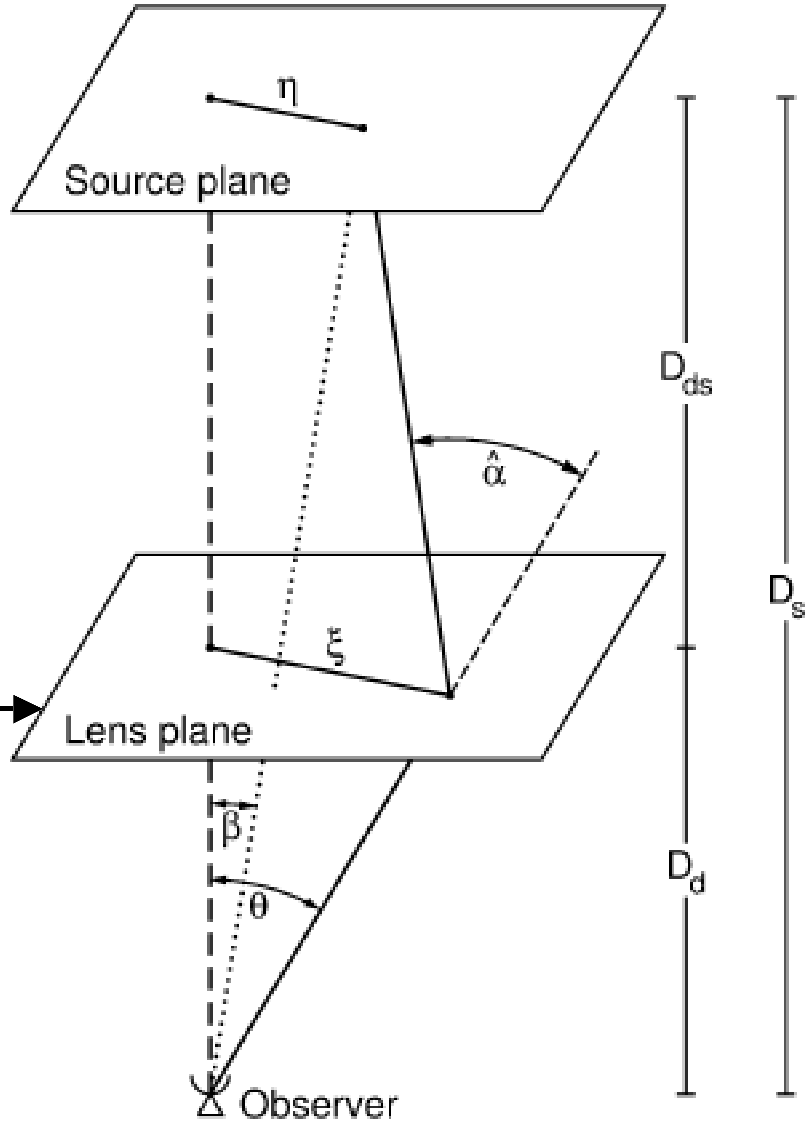
$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma(\boldsymbol{\theta})}{\Sigma_{\rm cr}}.$$

- Kappa > 1 = Strong Lensing
- Kappa << 1 = Weak Lensing

Thin Sheet Approximation :

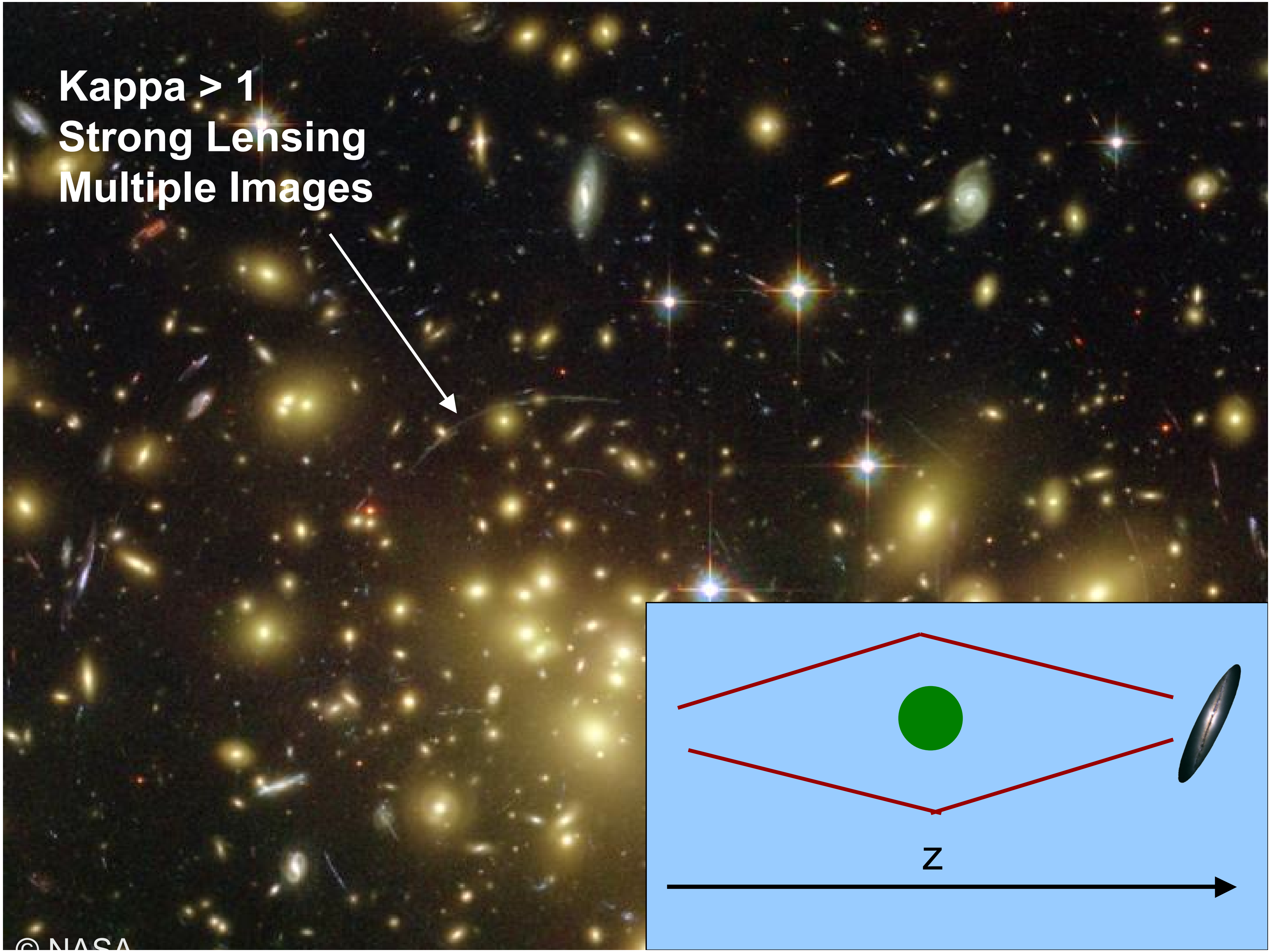
$$\Sigma(\boldsymbol{\xi}) = \int \rho(\boldsymbol{\xi}, z) dz$$

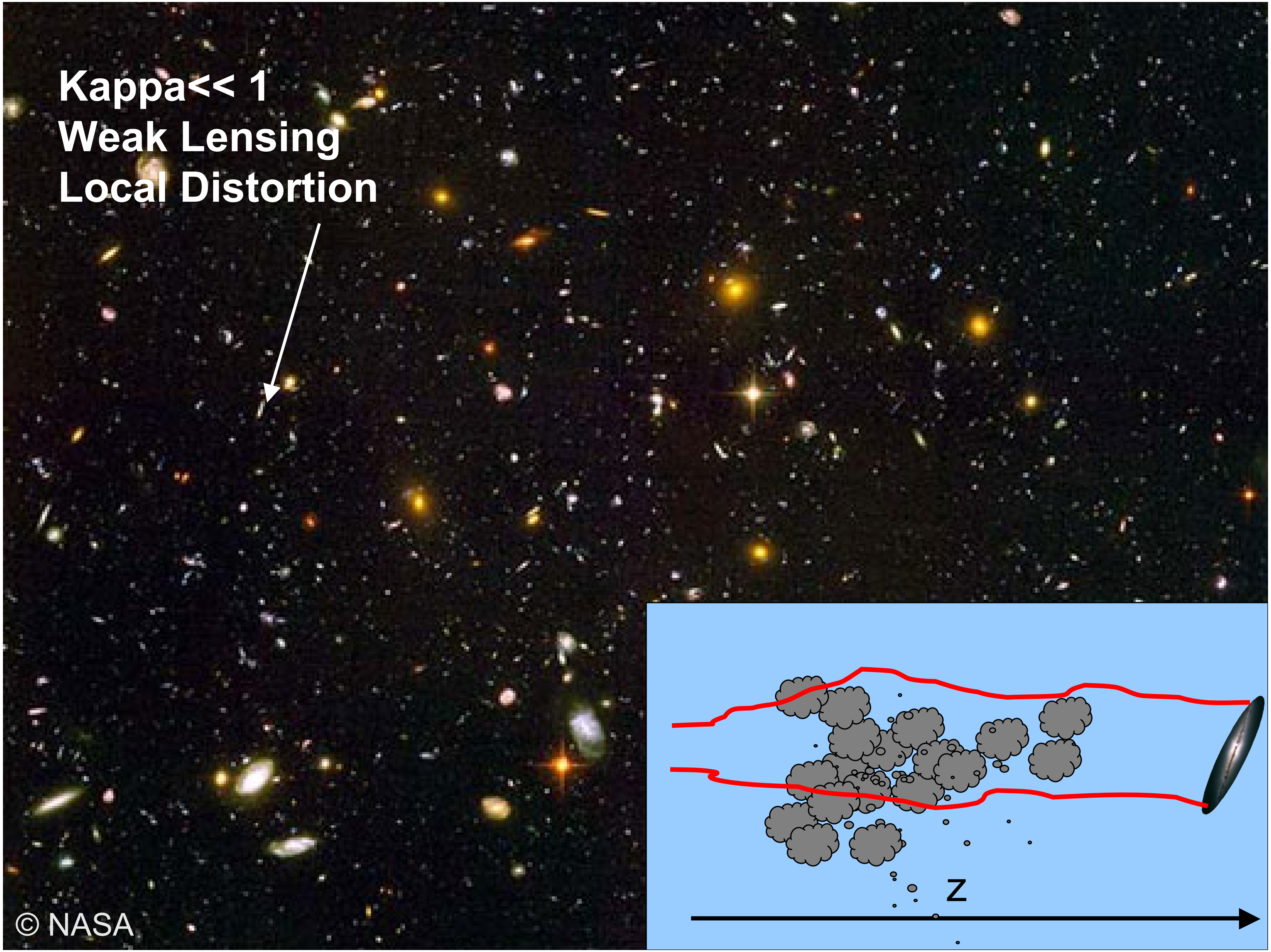
Surface Mass Distribution



$$\Sigma_{\rm cr} = \frac{c^2}{4\pi G}\frac{D_s}{D_d D_{ds}}.$$

$$\hat{\alpha} = \int r_i dt$$


Kappa > 1
Strong Lensing
Multiple Images
z
© NASA



Kappa<< 1
Weak Lensing
Local Distortion
z
© NASA

$$\hat{\alpha} = \int r_i dt$$

From GR

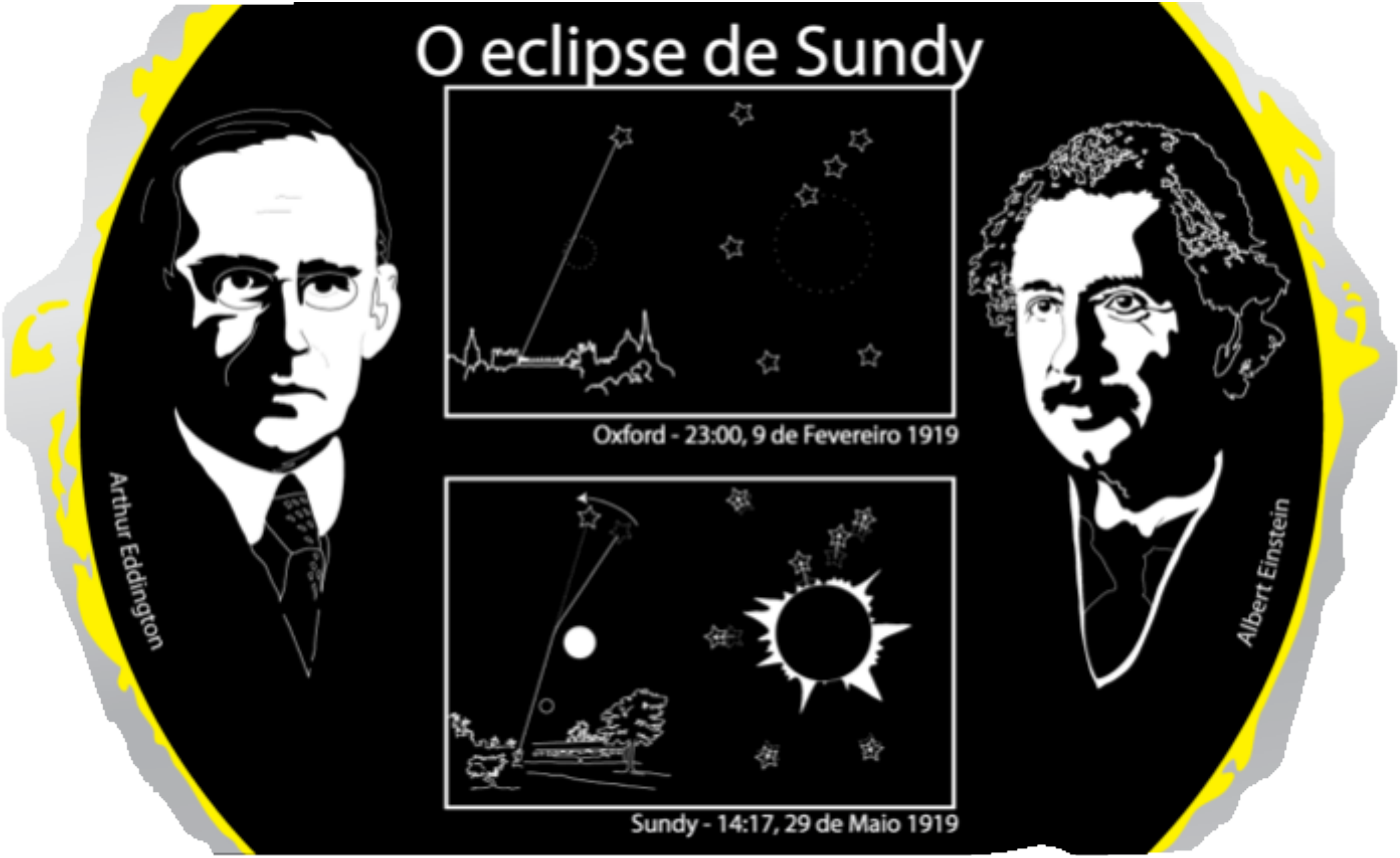
- Metric in the Weak Field Limit $ds^2 = -g_{\mu\nu}dx^\mu dx^\nu = (1+2\Phi)dt^2 - (1-2\Phi)\delta_{\alpha\beta}dx^\alpha dx^\beta$
- Newtonian Potential
- GR Equation of Motion $a^\mu = u^\mu_{;\nu}u^\nu = \dot{u}^\mu + \Gamma^\mu_{\nu\lambda}u^\lambda u^\nu = 0$
- Γ related to metric g $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}g^{\mu\eta}\left(g_{\nu\eta,\lambda} + g_{\lambda\eta,\nu} - g_{\nu\lambda,\eta}\right)$

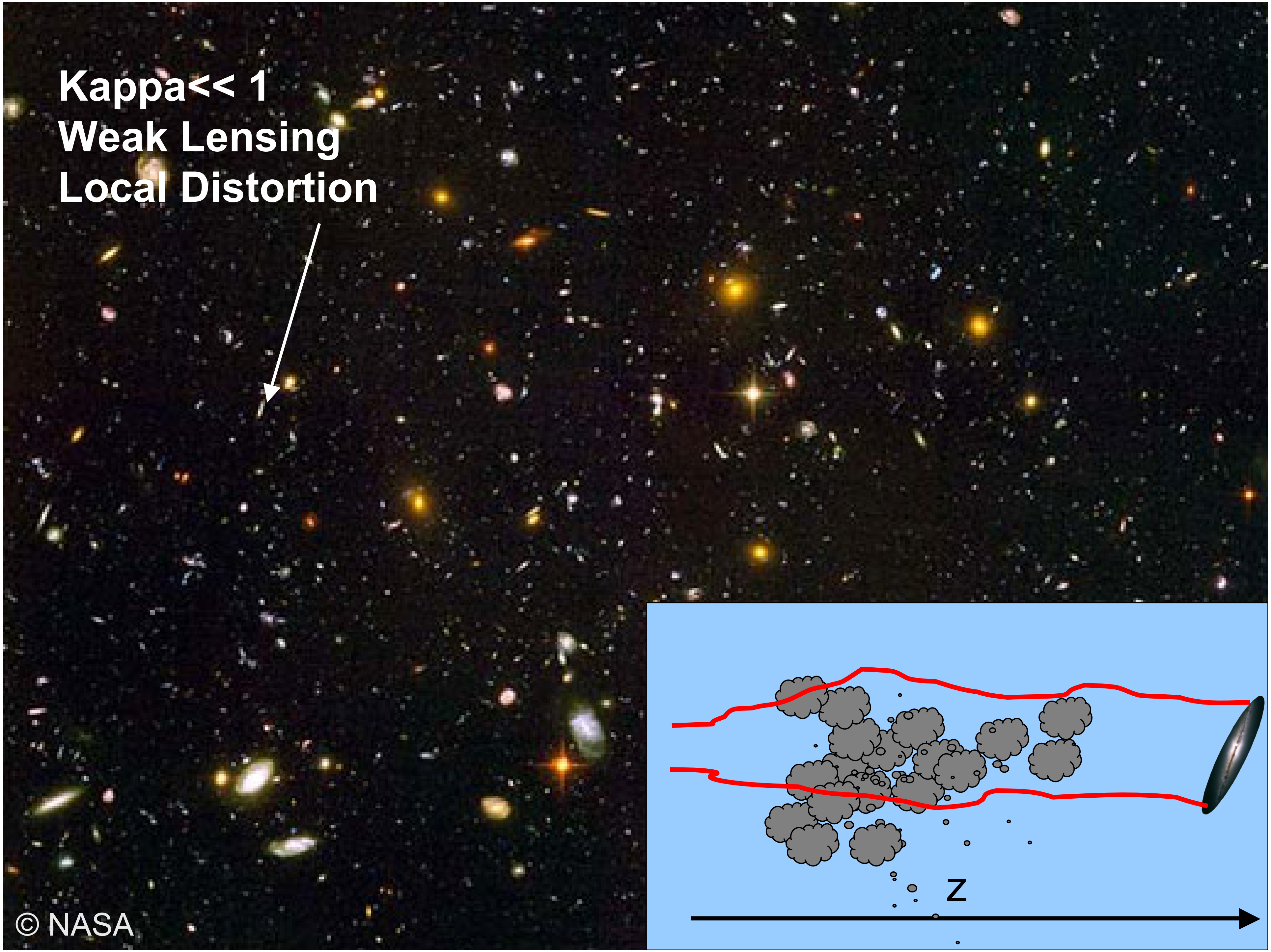
- Set time-changes in metric to be small $g_{i0}=0$. Spatial Part: $\dot{u}^i = \delta_{ij}\left[\frac{1}{2}g_{00,j} - g_{kj,0}u^k - \left(g_{kj,m} - \frac{1}{2}g_{km,j}\right)u^k u^m\right]$

- Sub metric : $\dot{u}^i = \left[2u^i\dot{\Phi} + \left((1+u^2)\delta_{ij} - 2u^i u^j\right)\nabla_j\Phi\right]$

- Newtonian Limit $u_i << c$: $\dot{u}_i = \nabla_i\Phi$
- Relativistic Limit $u_i \sim r_i c$: $\dot{r}_i = 2\left(\delta^K_{ij} - r_i r_j\right)\nabla_j\Phi = 2\nabla^\perp_j\Phi$

GR-Newton Factor of 2 Confirmed in 1919

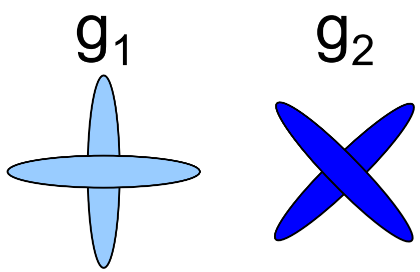


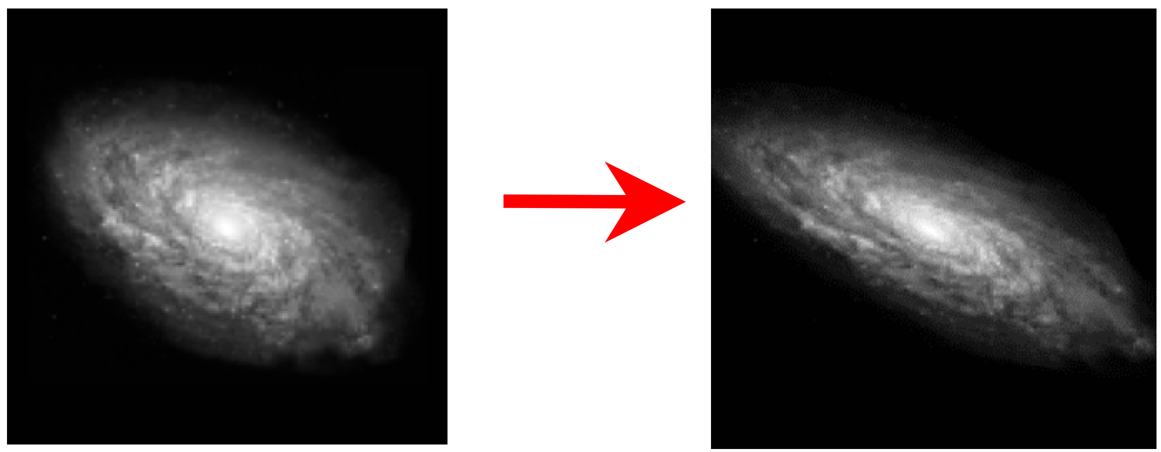

Kappa<< 1
Weak Lensing
Local Distortion
© NASA
Z

Weak Lensing Limit

- In the weak limit where deflection angles are small
- The distortion induced on a single galaxy images is a local conformal mapping
- Parameterised by a spin-2 quantity called **shear**

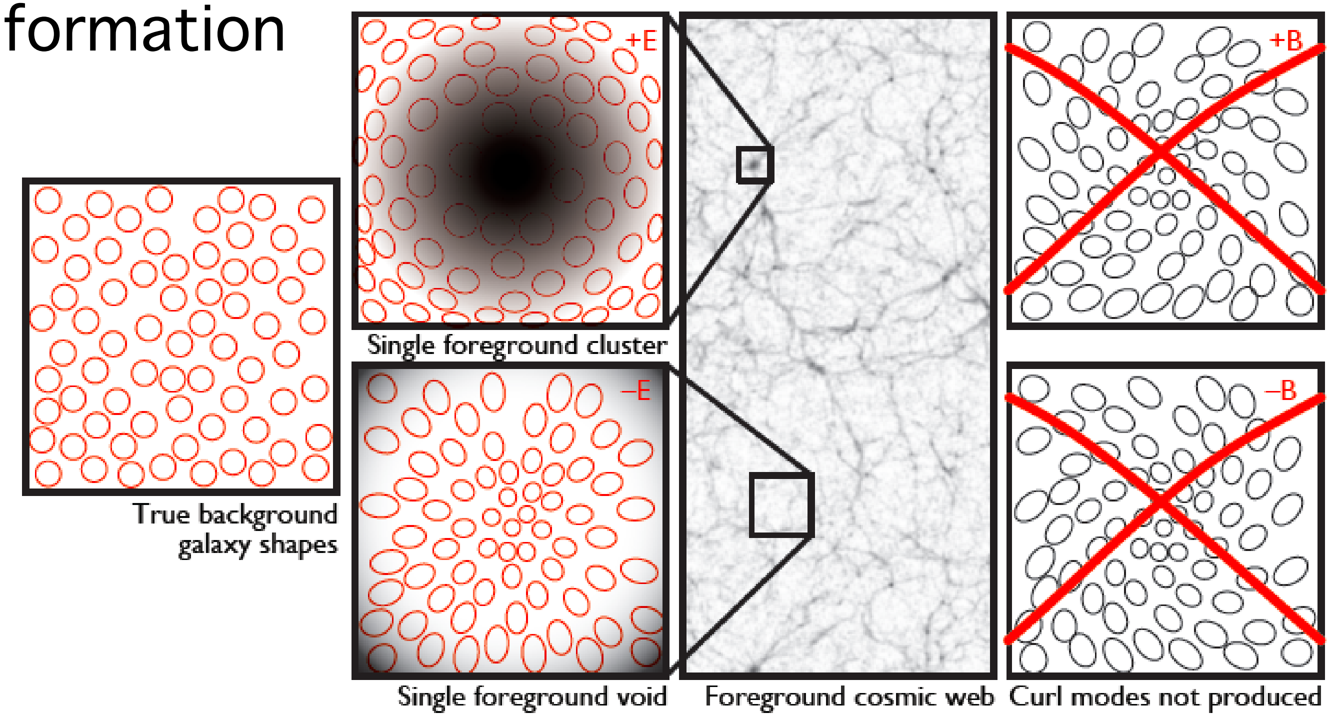
$$\begin{pmatrix} x_{\mathrm{u}} \\ y_{\mathrm{u}} \end{pmatrix} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix} \begin{pmatrix} x_{\mathrm{l}} \\ y_{\mathrm{l}} \end{pmatrix}$$



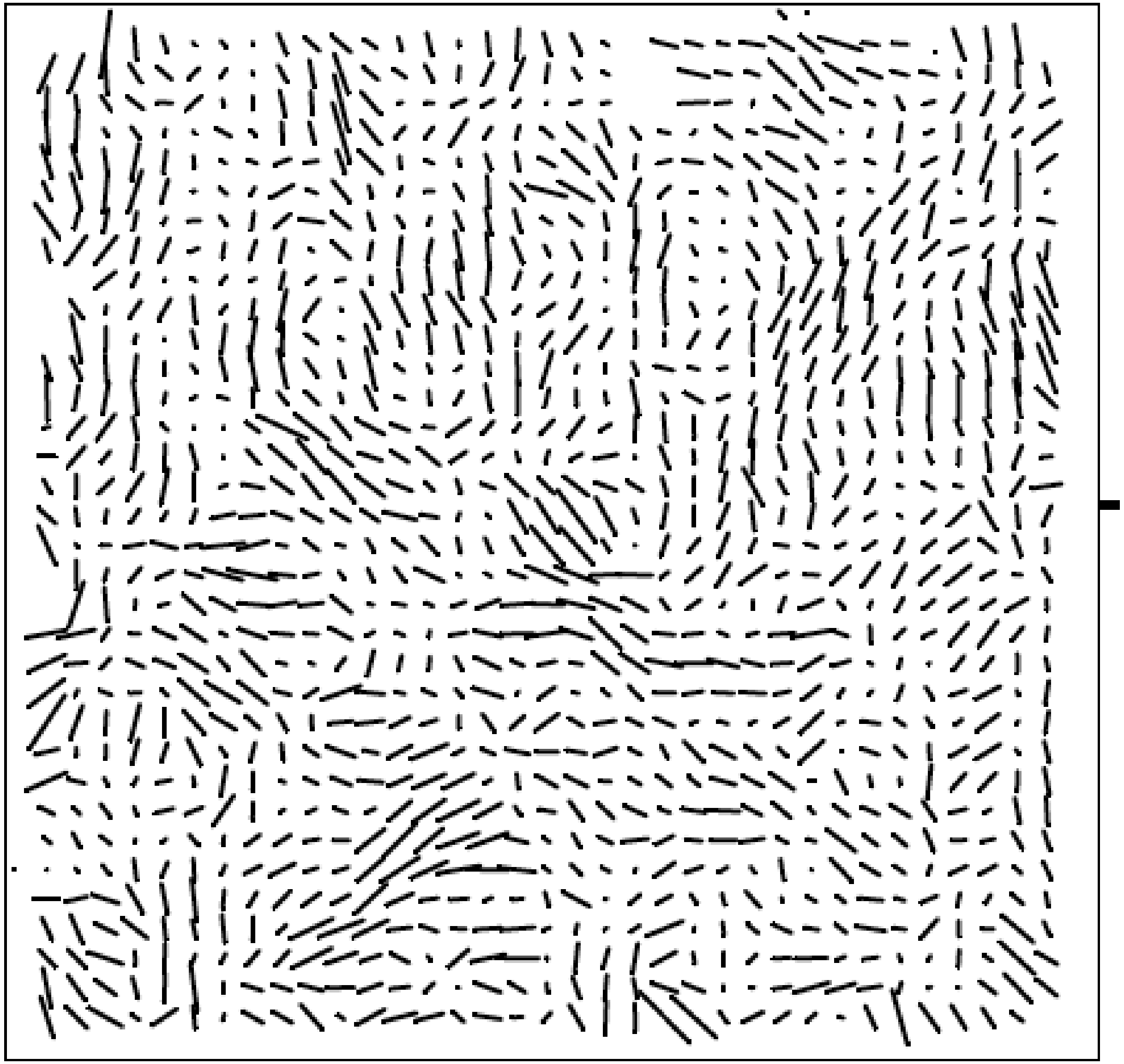


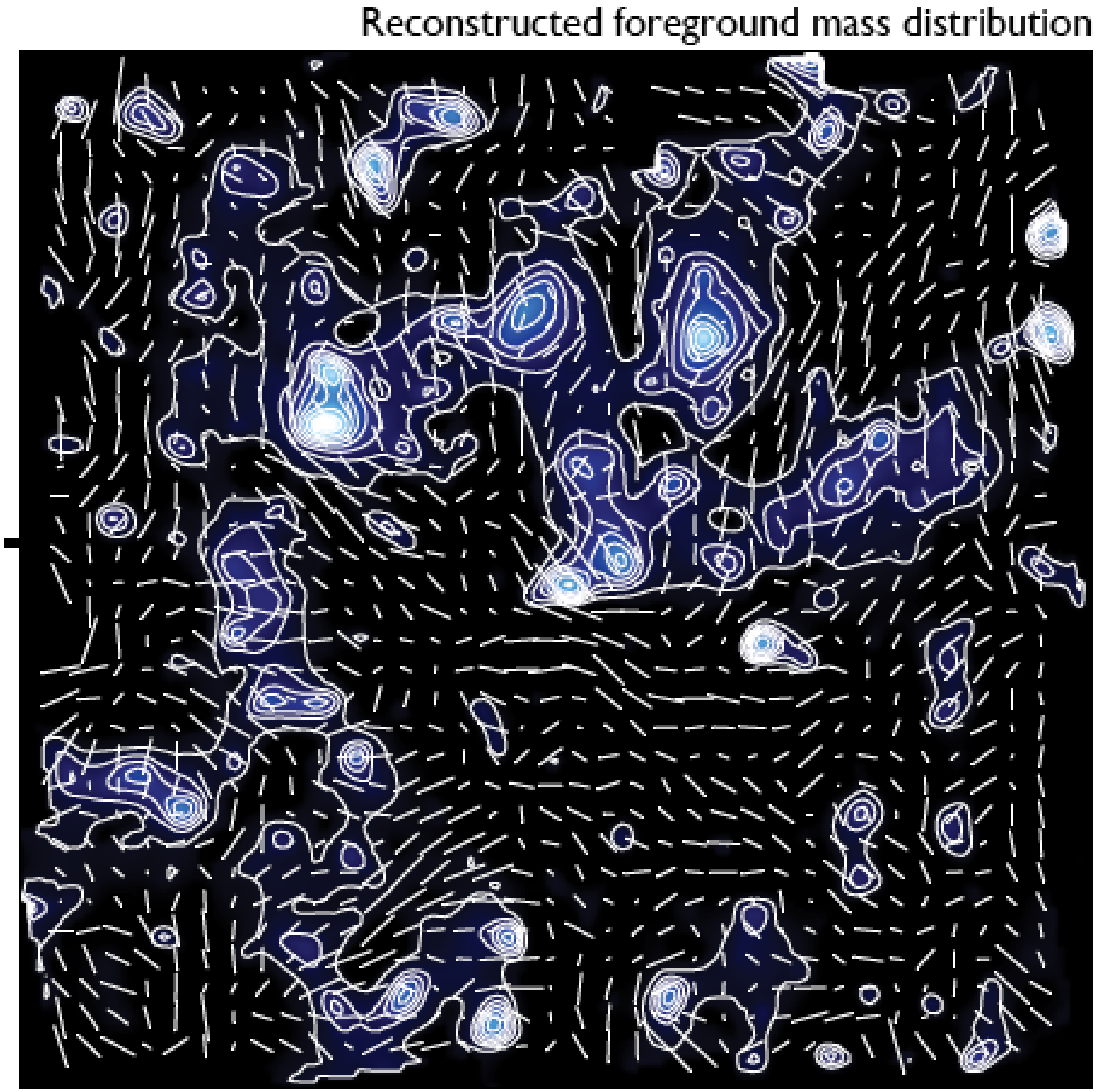
Observables

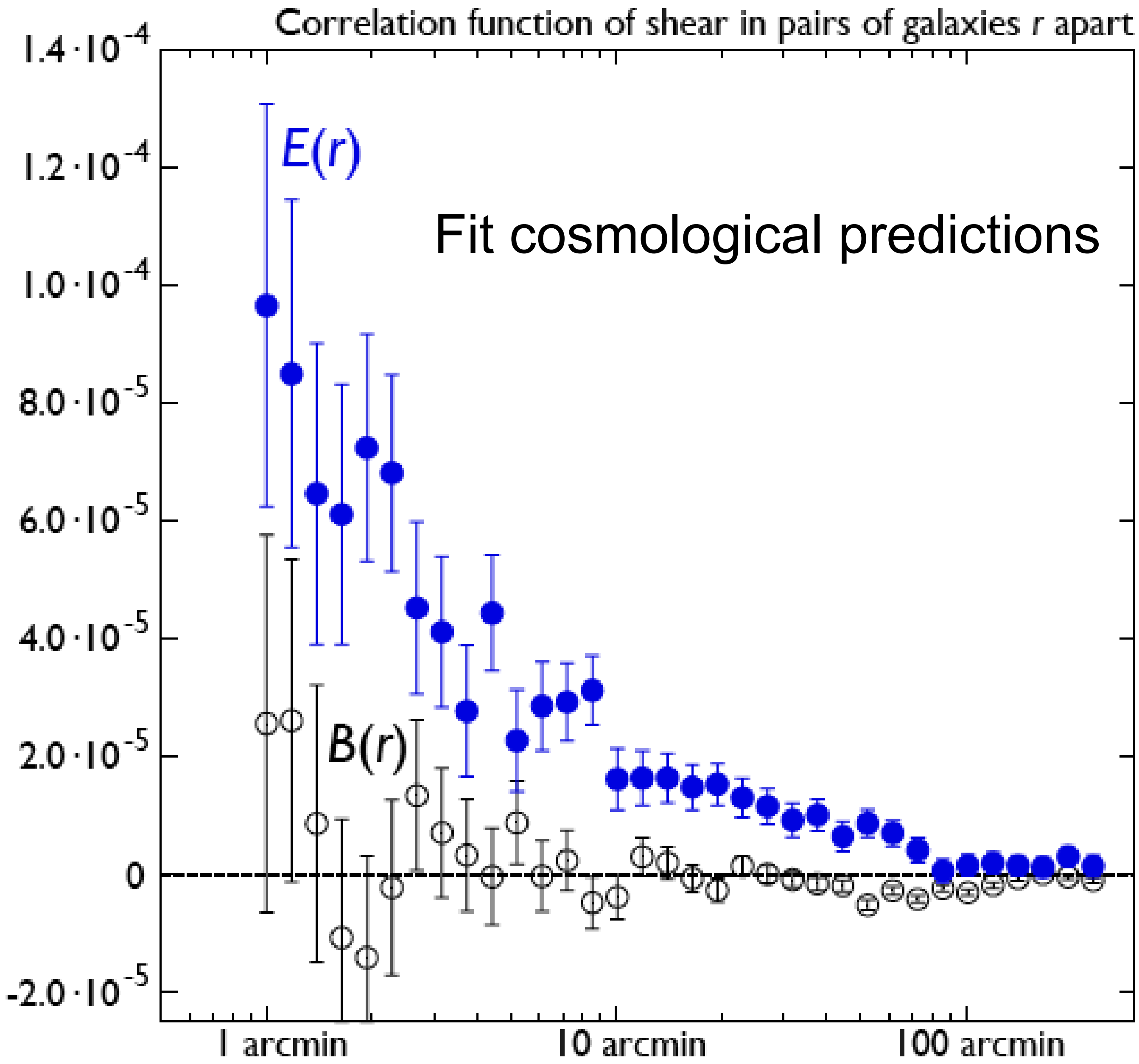
- Every Galaxy Experiences a weak shear from the cosmic web of large scale structure
  - Average Shear is zero - galaxies are randomly aligned
- Correlation function or Power Spectrum contains information



Direction and magnitude of mean shear (~100 galaxies per tick)

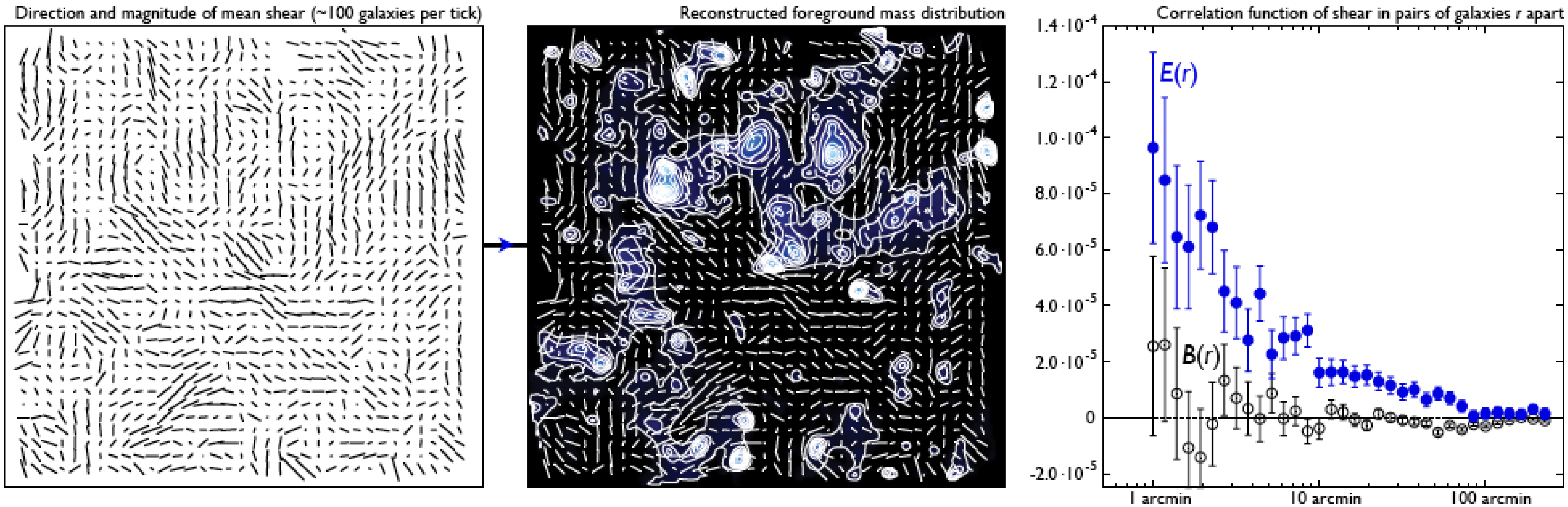



Reconstructed foreground mass distribution



Correlation function of shear in pairs of galaxies r apart
1.4·10⁻⁴
1.2·10⁻⁴
1.0·10⁻⁴
8.0·10⁻⁵
6.0·10⁻⁵
4.0·10⁻⁵
2.0·10⁻⁵
0
-2.0·10⁻⁵
E(r)
B(r)
Fit cosmological predictions
1 arcmin
10 arcmin
100 arcmin

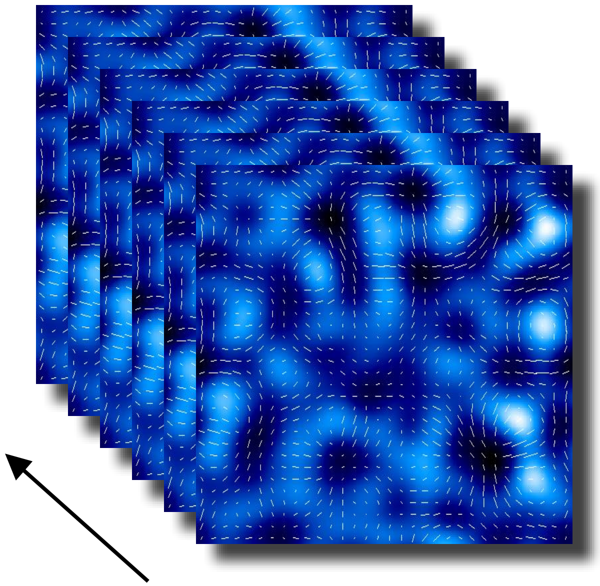
Variable Fields & Power Spectra



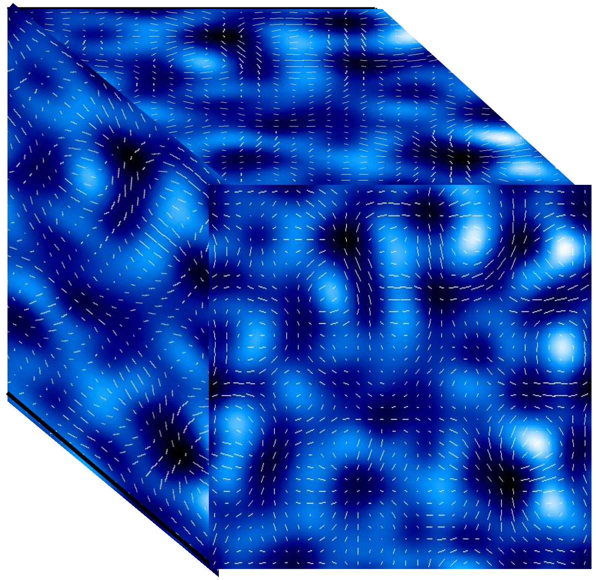
- Every galaxy have an associated shear
- Can use this shear to reconstruct the density and distribution of the lensing matter along the line of sight
- Can also take the correlation function which contains cosmological information

3D Weak Lensing

- Can combine the shear information with redshift information = 3D weak lensing



- Tomography
  - Bins in redshift
  - Can take the auto and cross correlation of the shear in each bin

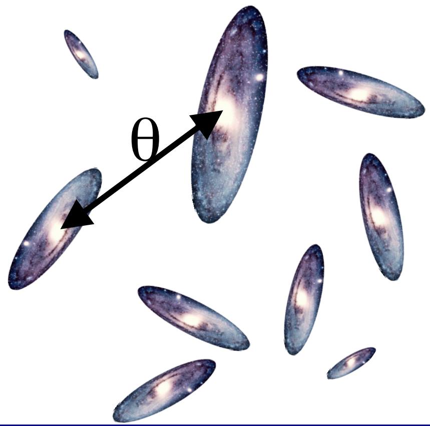


- 3D Cosmic Shear
  - Decompose into angular and radial modes

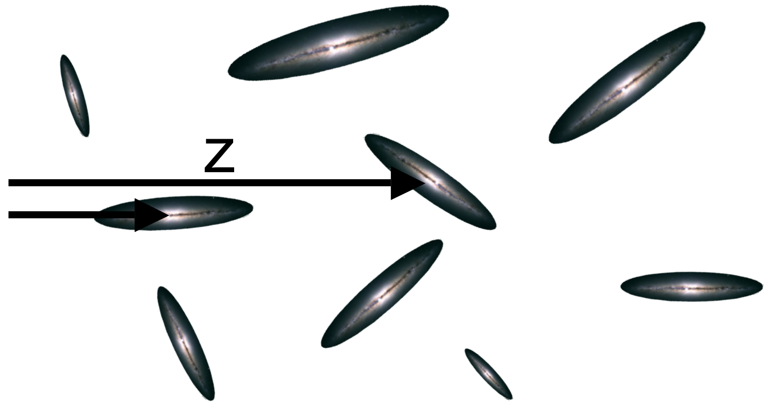
$$\hat{\gamma}(k,\boldsymbol{\ell}) = \sqrt{\frac{2}{\pi}} \sum_g \gamma(\mathbf{r}) k j_\ell(k r_g^0) \exp(-i\boldsymbol{\ell}.\boldsymbol{\theta}_g) W(r_g^0)$$

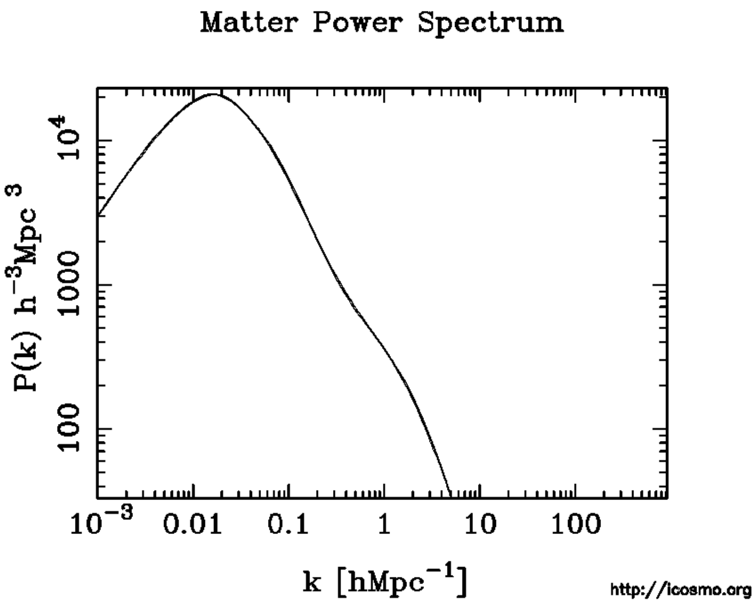
What can we observe?

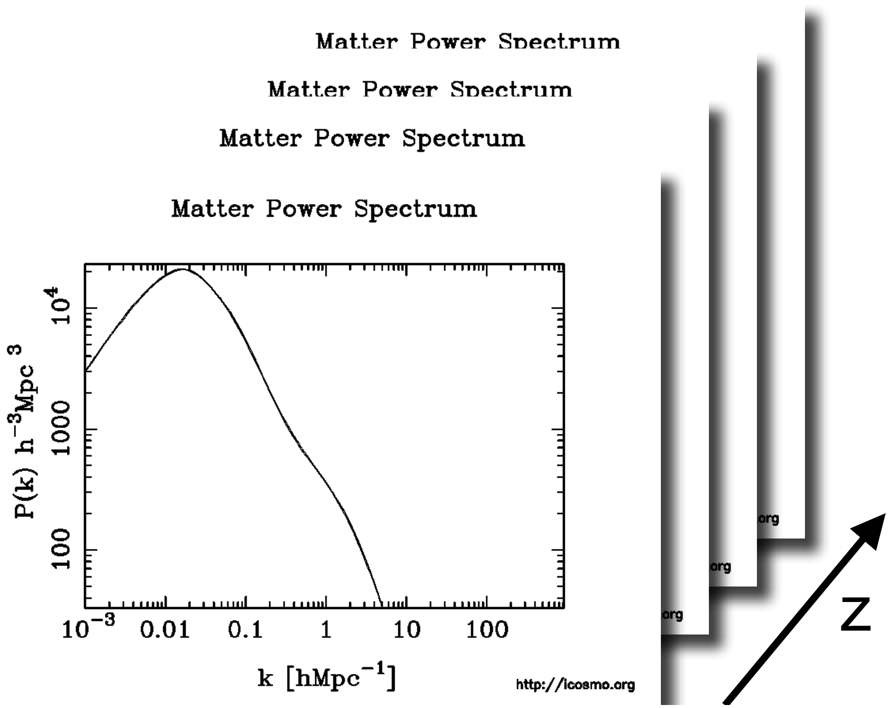
Angles



Wavelengths

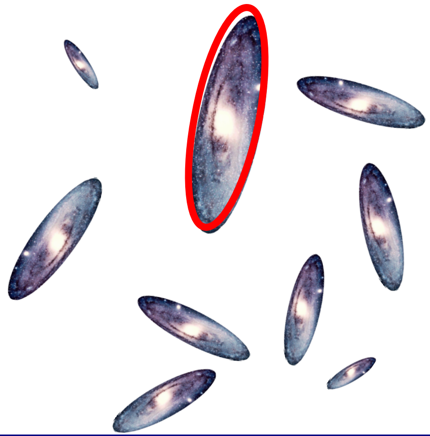


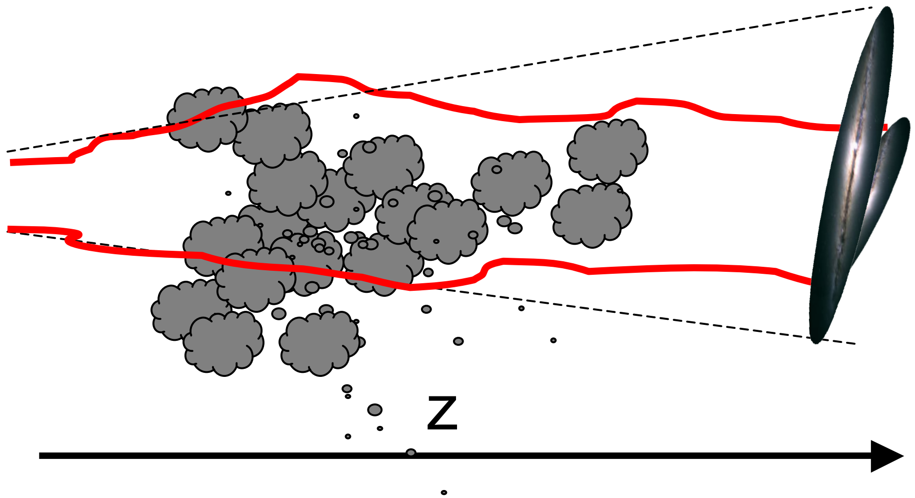


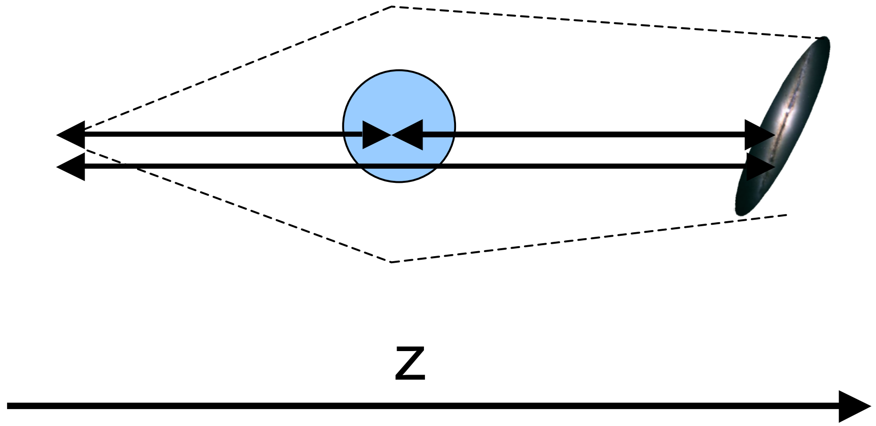


What can we observe?

Shapes!

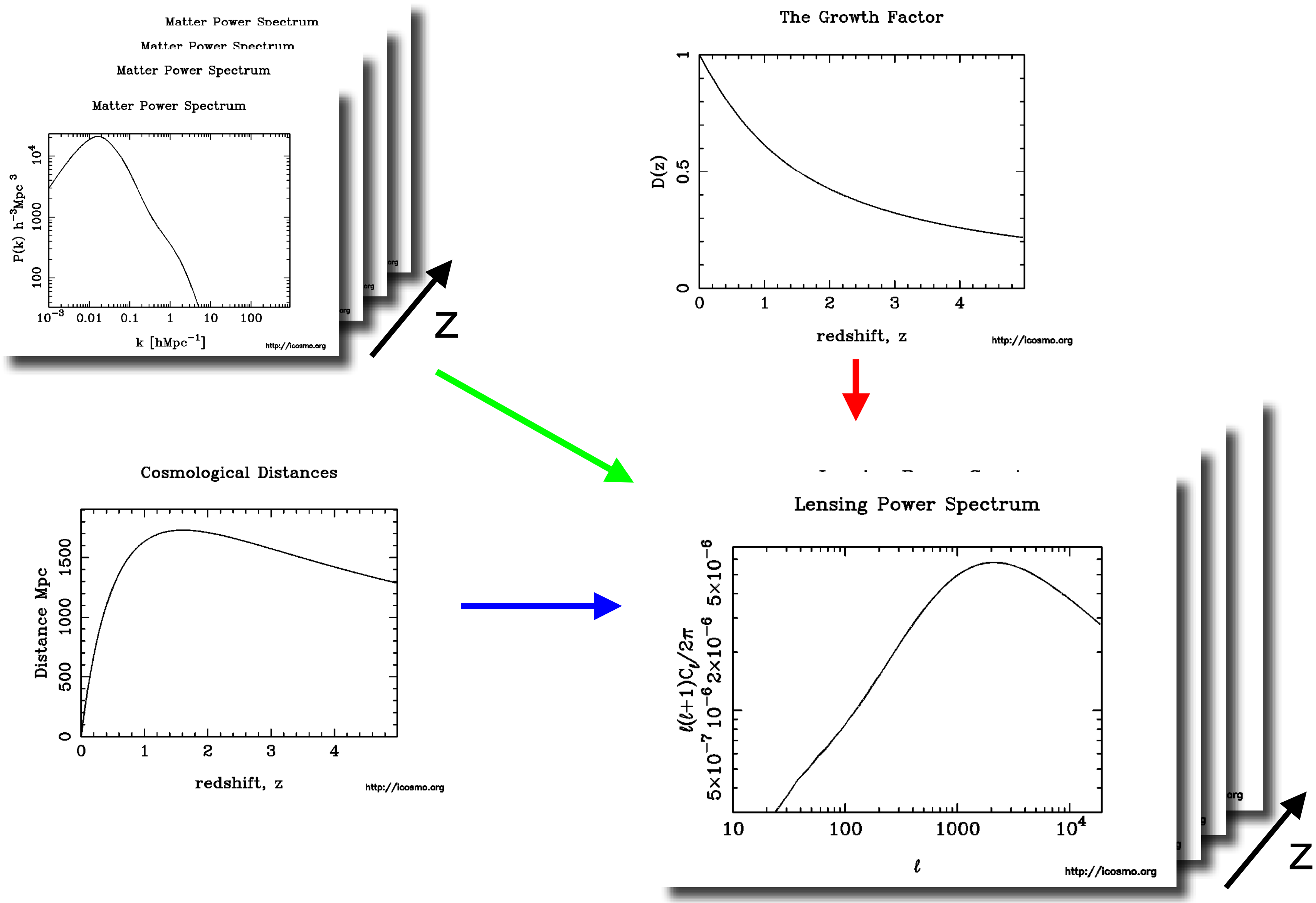






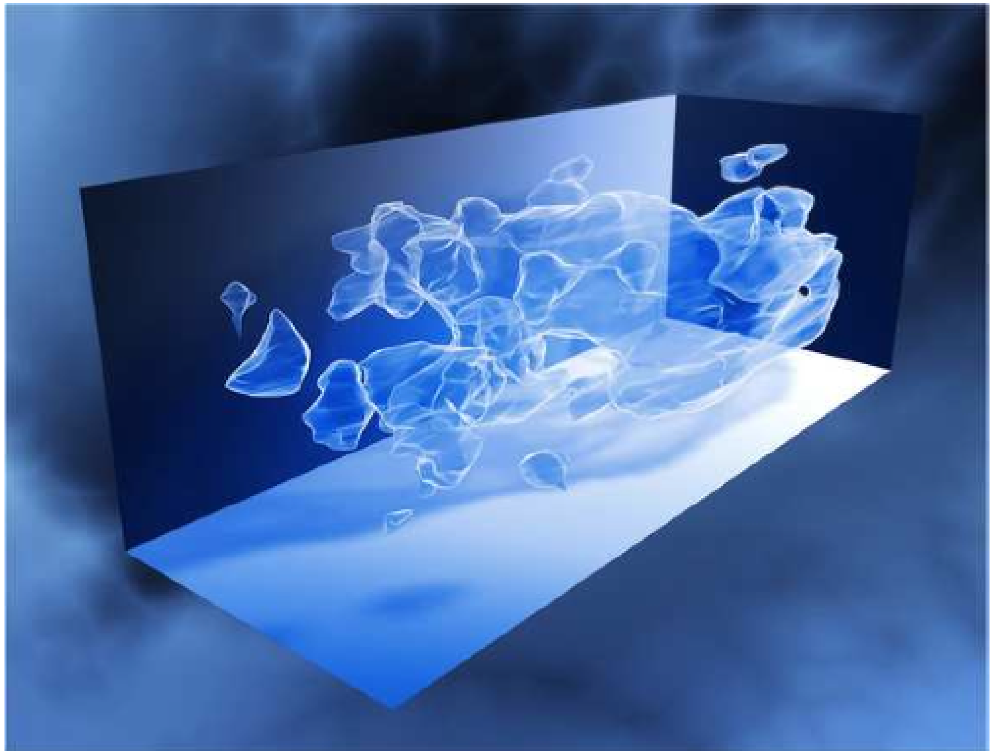
- Information on
  - Matter power spectrum
  - Angular Diameter Distance

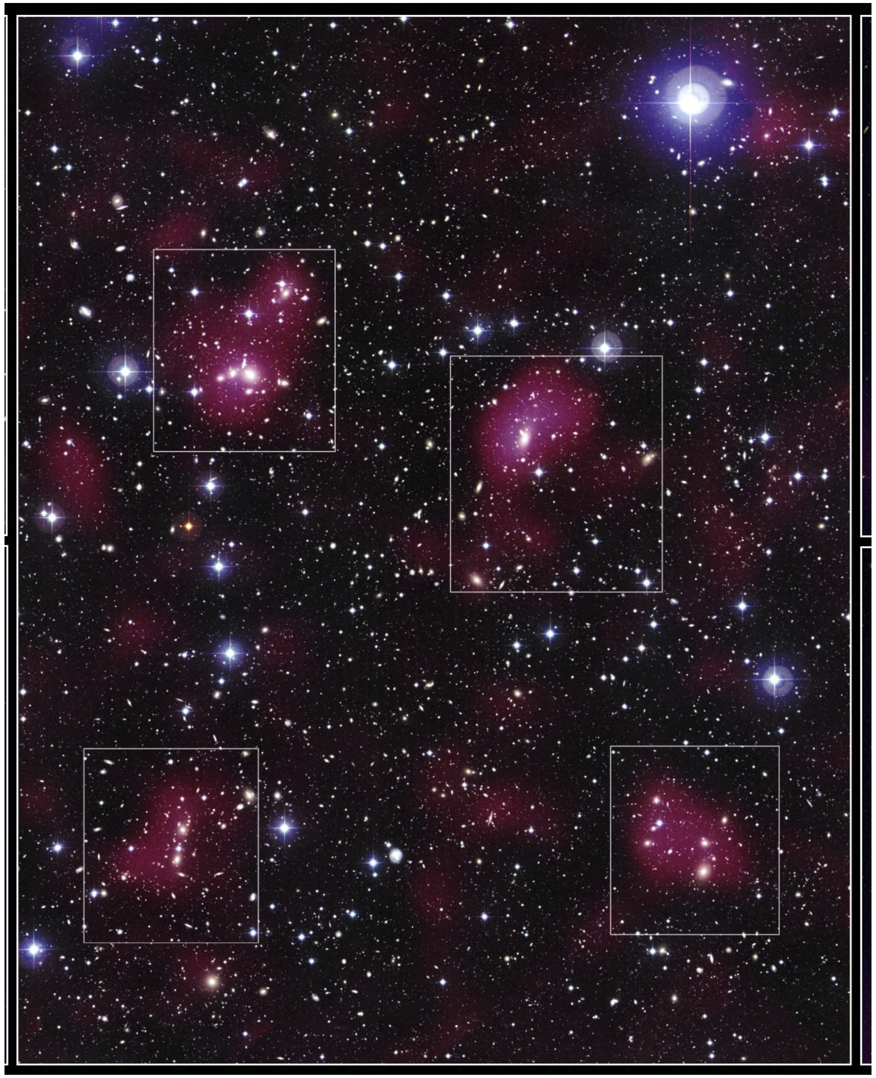
3D lensing


Matter Power Spectrum
Matter Power Spectrum
Matter Power Spectrum
Matter Power Spectrum
P(k) h^{-3}Mpc ^{3}
100
1000
10^4
10^{-3} 0.01 0.1 1 10 100
k [hMpc^{-1}]
http://lcosmo.org
z
The Growth Factor
D(z)
0 0.5 1
0 1 2 3 4
redshift, z
http://lcosmo.org
Cosmological Distances
Distance Mpc
0 500 1000 1500
0 1 2 3 4
redshift, z
http://lcosmo.org
Lensing Power Spectrum
$\ell(\ell+1)C_\ell/2\pi$
5×10^{-7} 10^{-6} 2×10^{-6} 5×10^{-6}
10 100 1000 10^4
ℓ
http://lcosmo.org
z

Why is this Important : Dark Matter

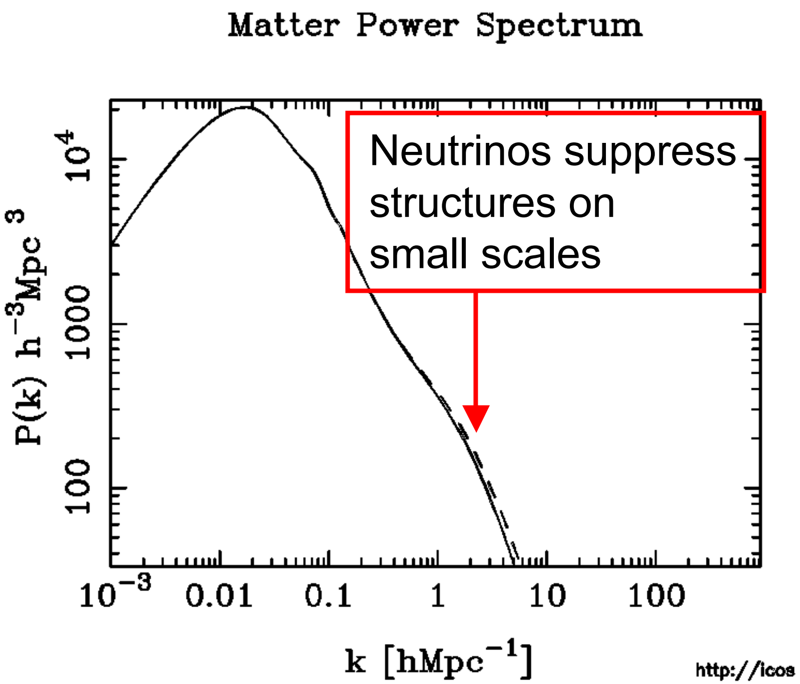
- Can map dark matter itself in 3D

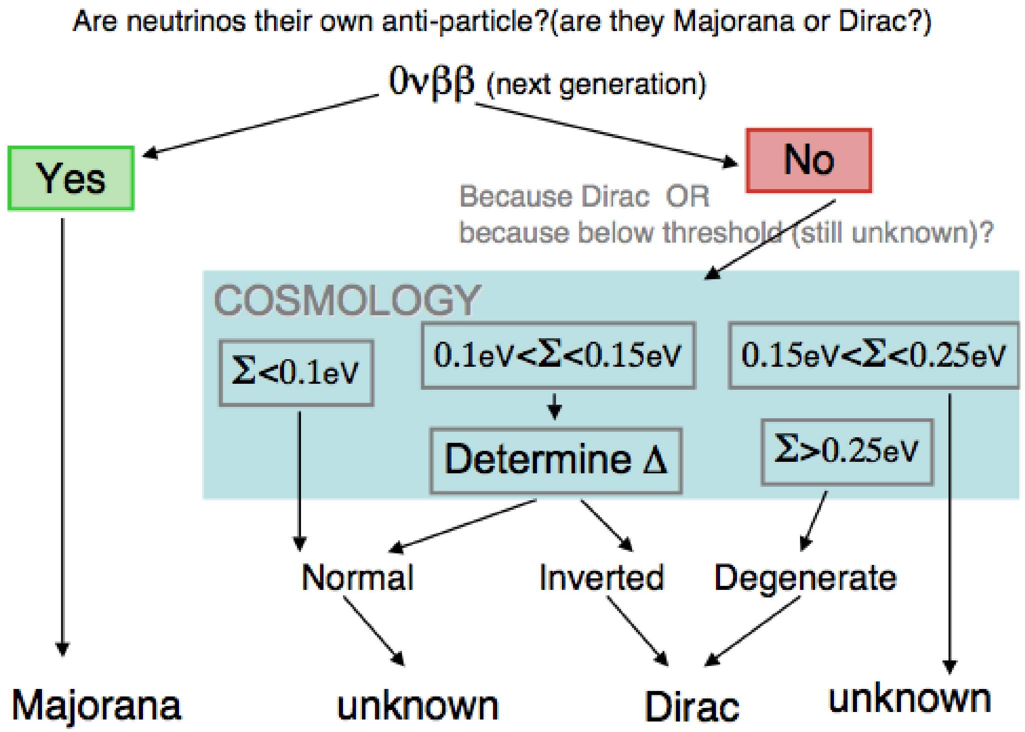




Why is it Important : Dark Matter

- Help to constrain dark matter particle mass
- One of the most sensitive probes of neutrino mass
- With near term experiments Δm_ν~0.03eV ΔN_ν~0.30 Hierachy

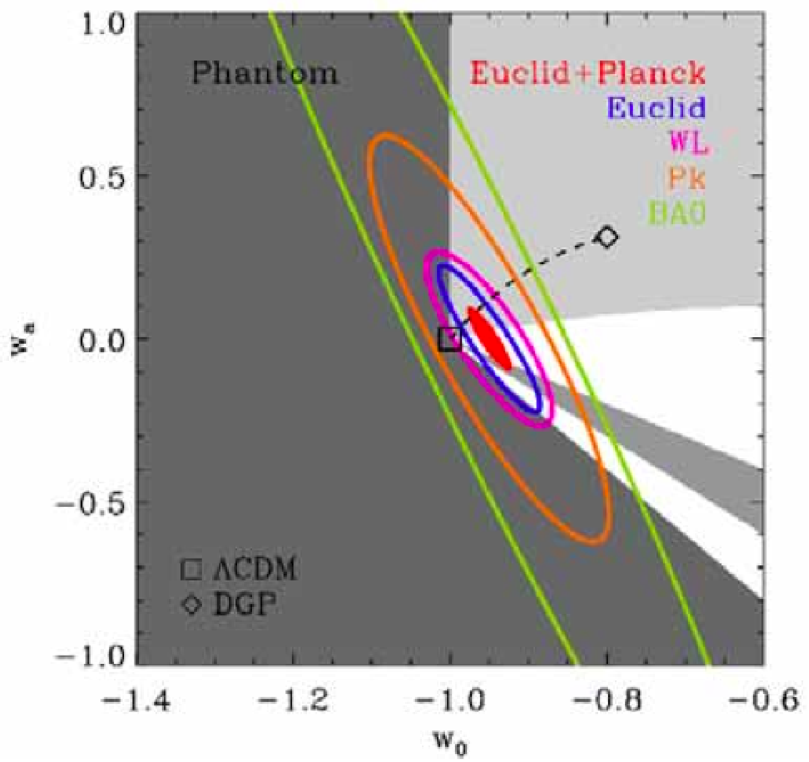




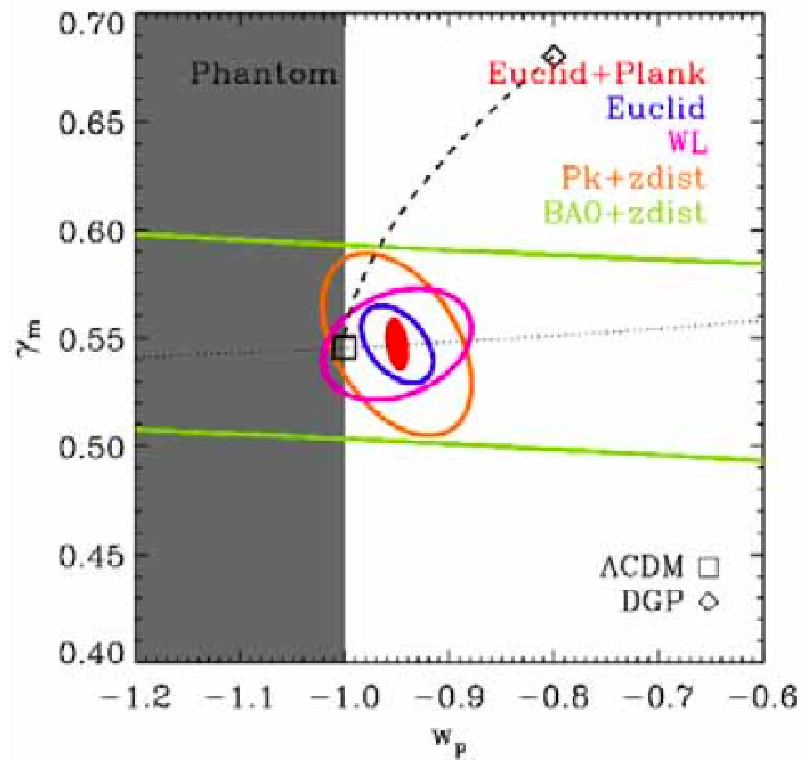
Why is it Important : Dark Energy

- Widely accepted to be **<u>the</u>** most promising method for determining the dark energy equation of state

Predicted errors for an all-Sky Hubble-type telescope *Euclid*



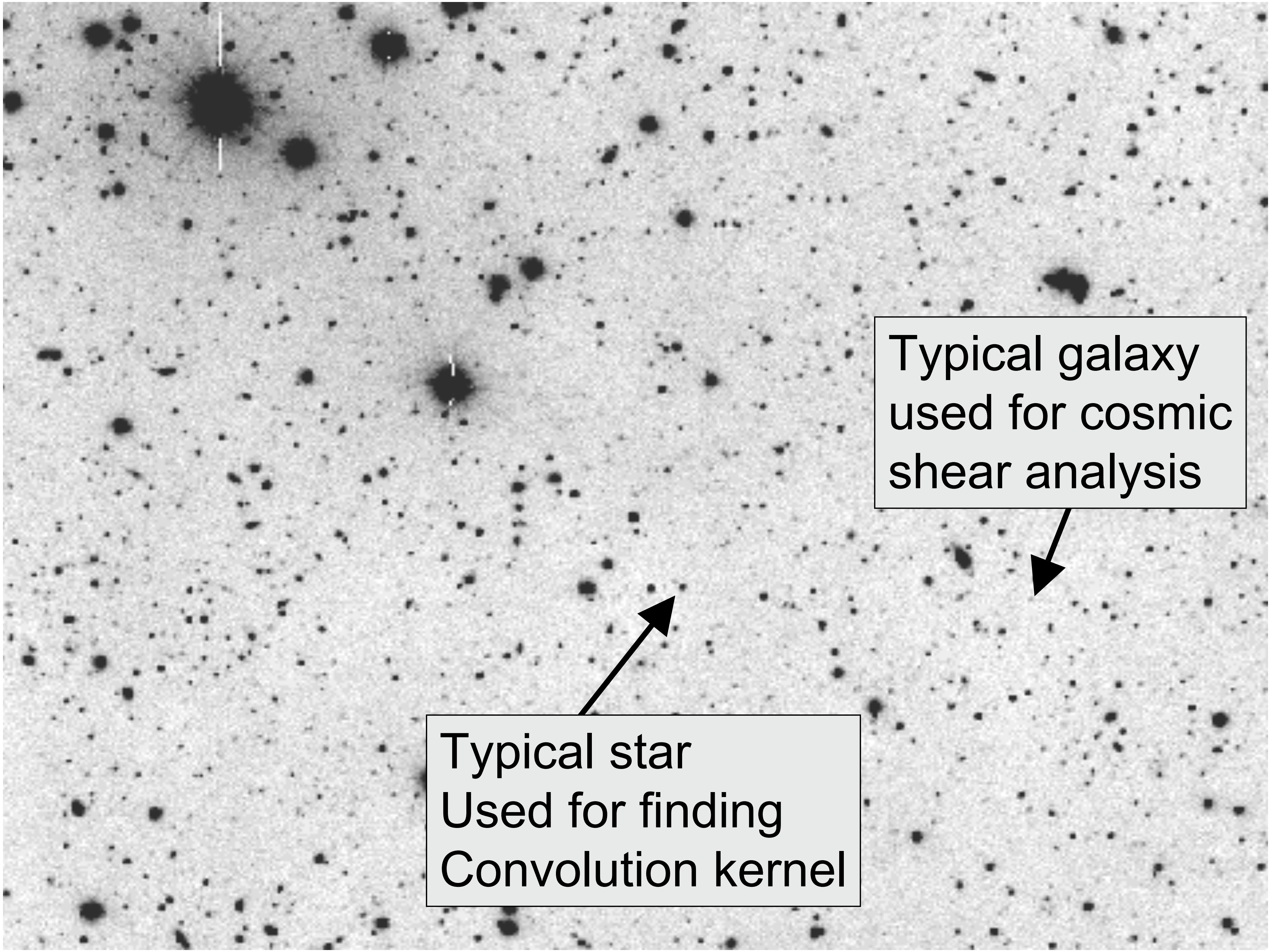
Dark Fluid



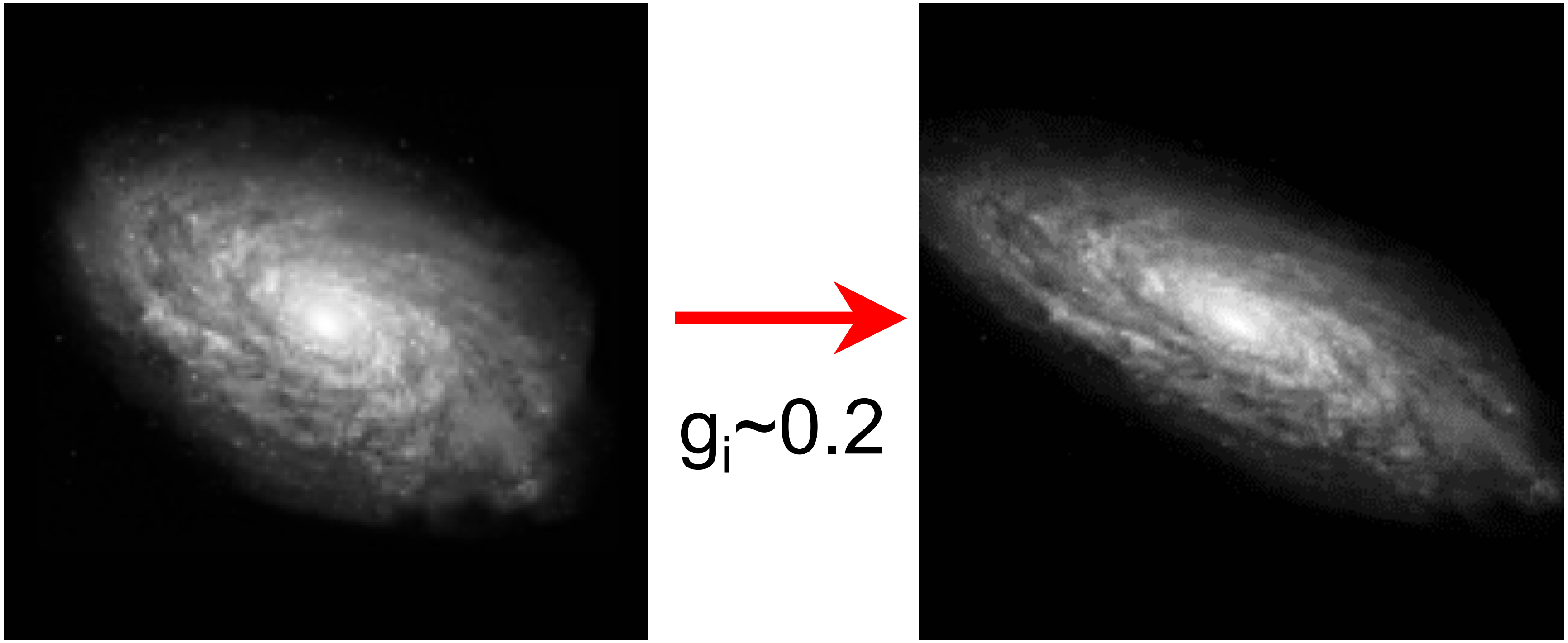
Dark Gravity

Measuring Shapes (more difficult than you might think!)

- How do we extract shear information from astronomical images?
- Have noise, convolutions effects


Typical galaxy
used for cosmic
shear analysis
Typical star
Used for finding
Convolution kernel

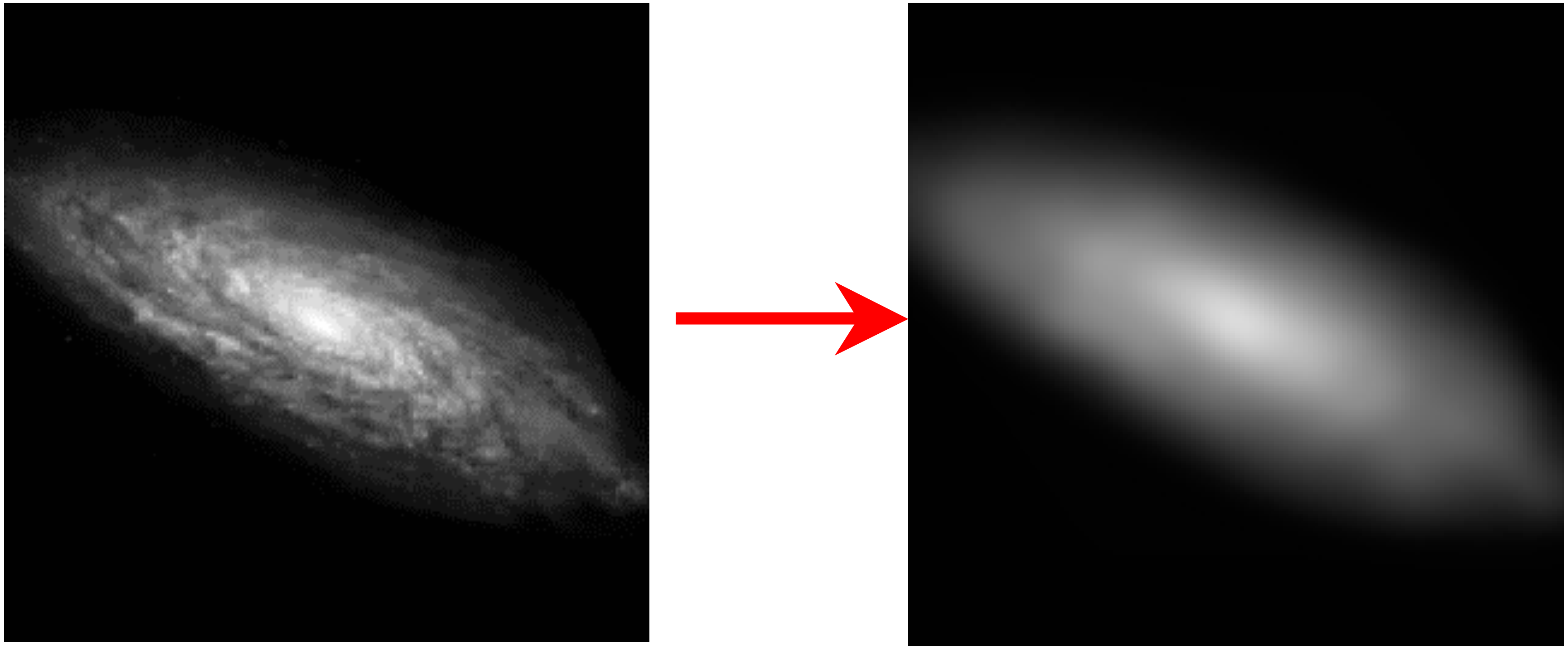
Cosmic Lensing



$$\begin{pmatrix} x_u \\ y_u \end{pmatrix} = \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix} \begin{pmatrix} x_l \\ y_l \end{pmatrix}$$

Real data: $g_i \sim 0.03$

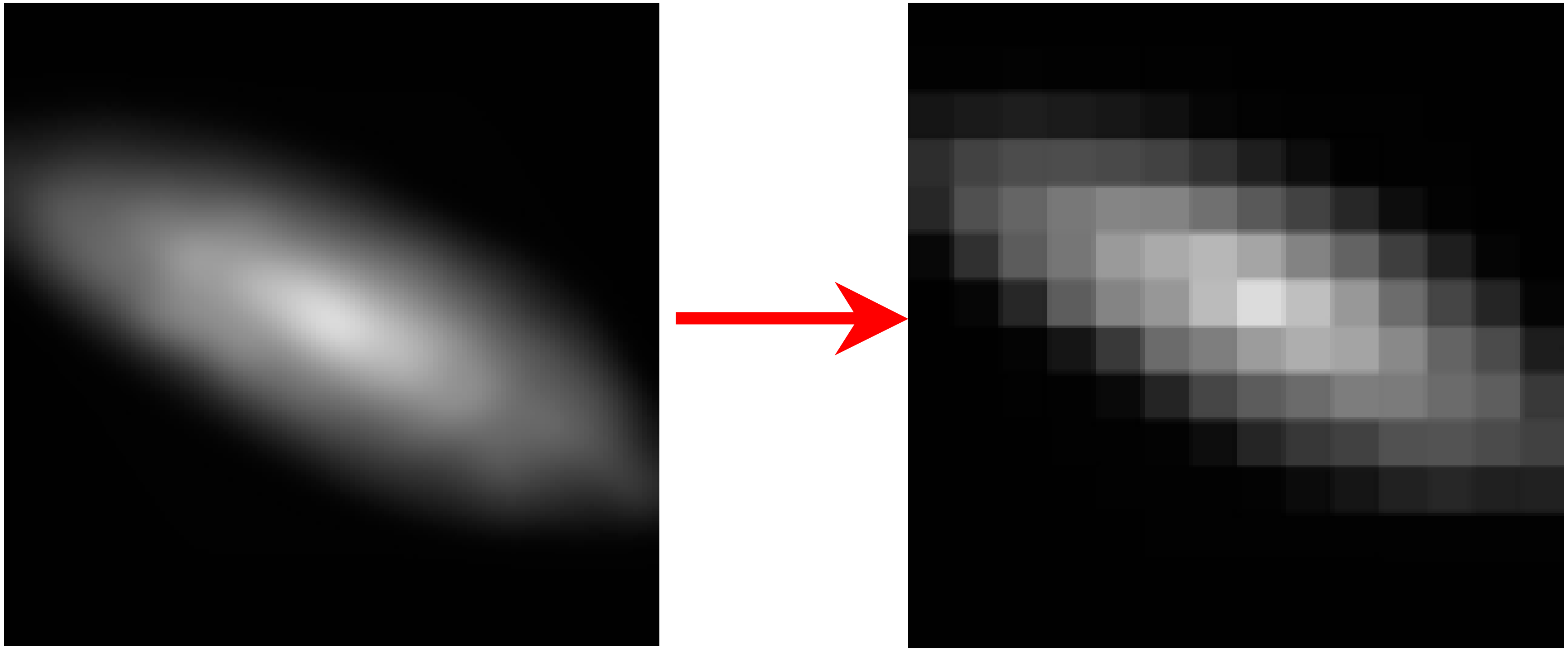
Atmosphere and Telescope



Convolution with kernel

Real data: Kernel size ~ Galaxy size

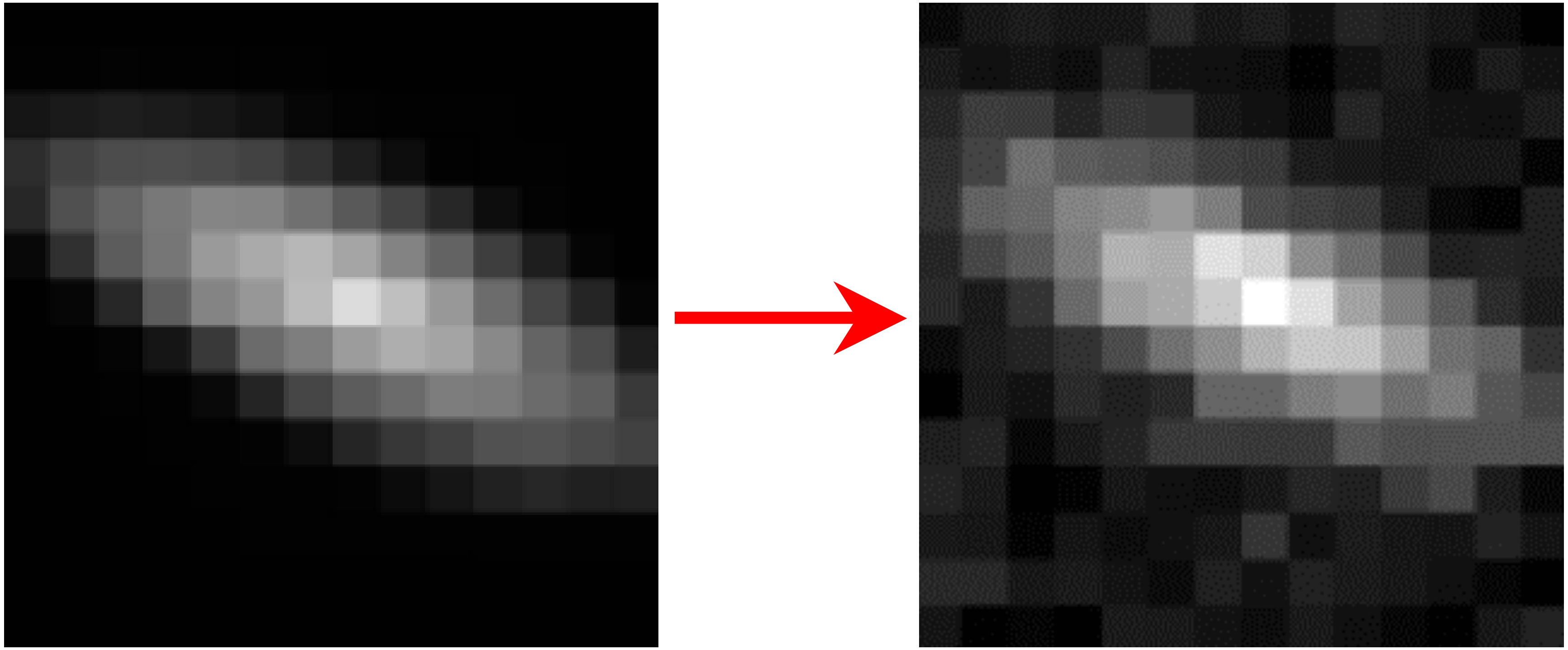
Pixelisation



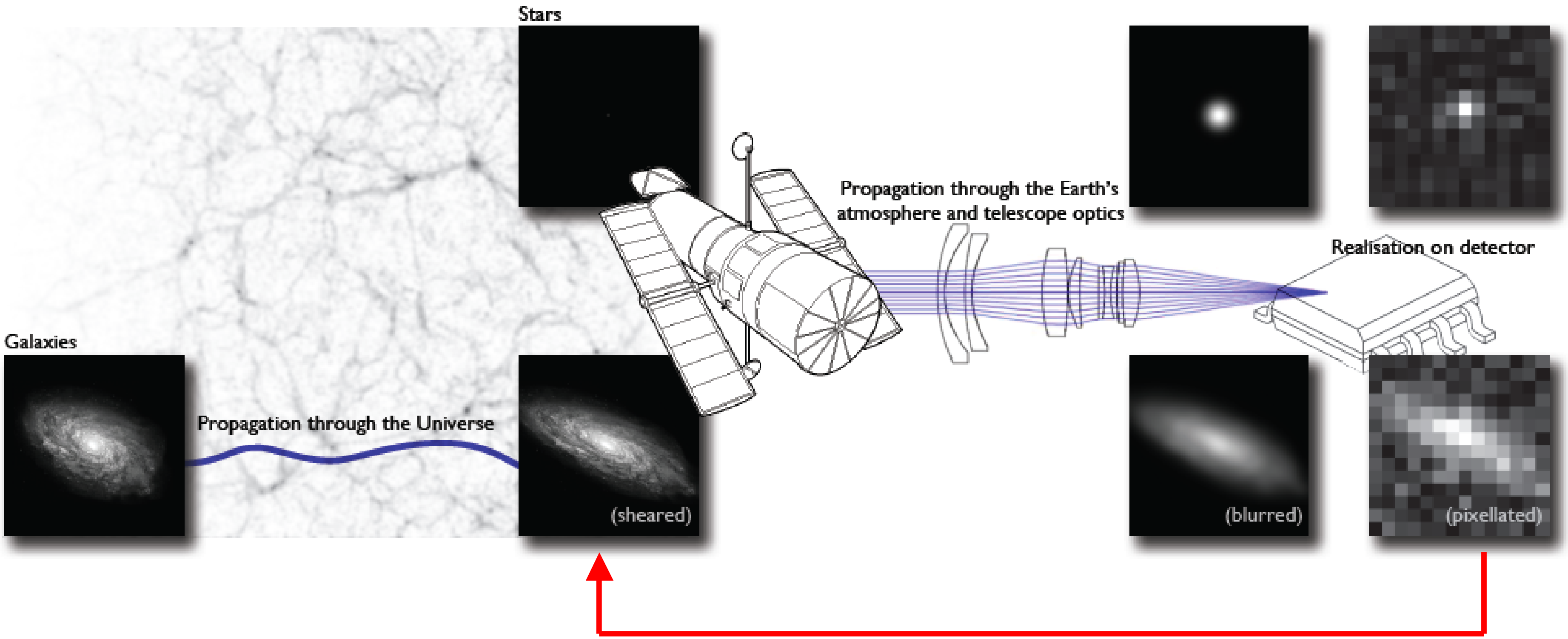
Sum light in each square

Real data: Pixel size ~ Kernel size /2

Noise



Mostly Poisson. Some Gaussian and bad pixels.
Uncertainty on total light ~ 5 per cent



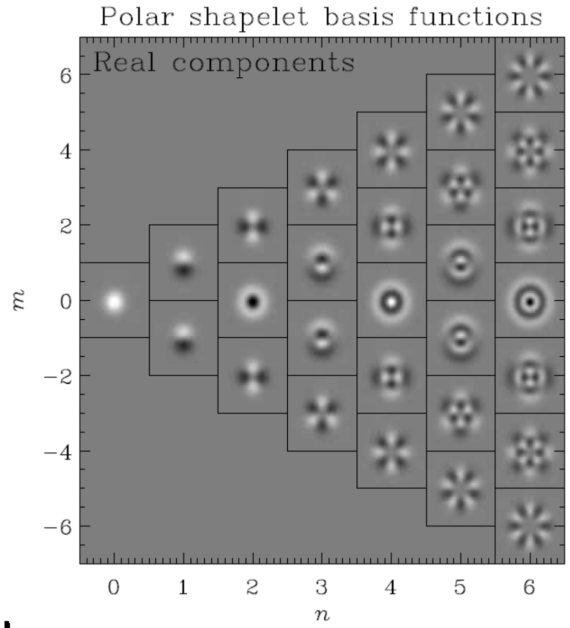
Need to measure shear to 10^{-3}

Current Methods

- Measuring Moments

$$Q_{ij} \equiv \int d^2x x_i x_j w(x) I(\mathbf{x})$$

- Fitting Shapelets



- Bayesian Fitting of Simpler Models

Good enough for current data.

Not good enough future all-sky experiments

Simulation Challenges

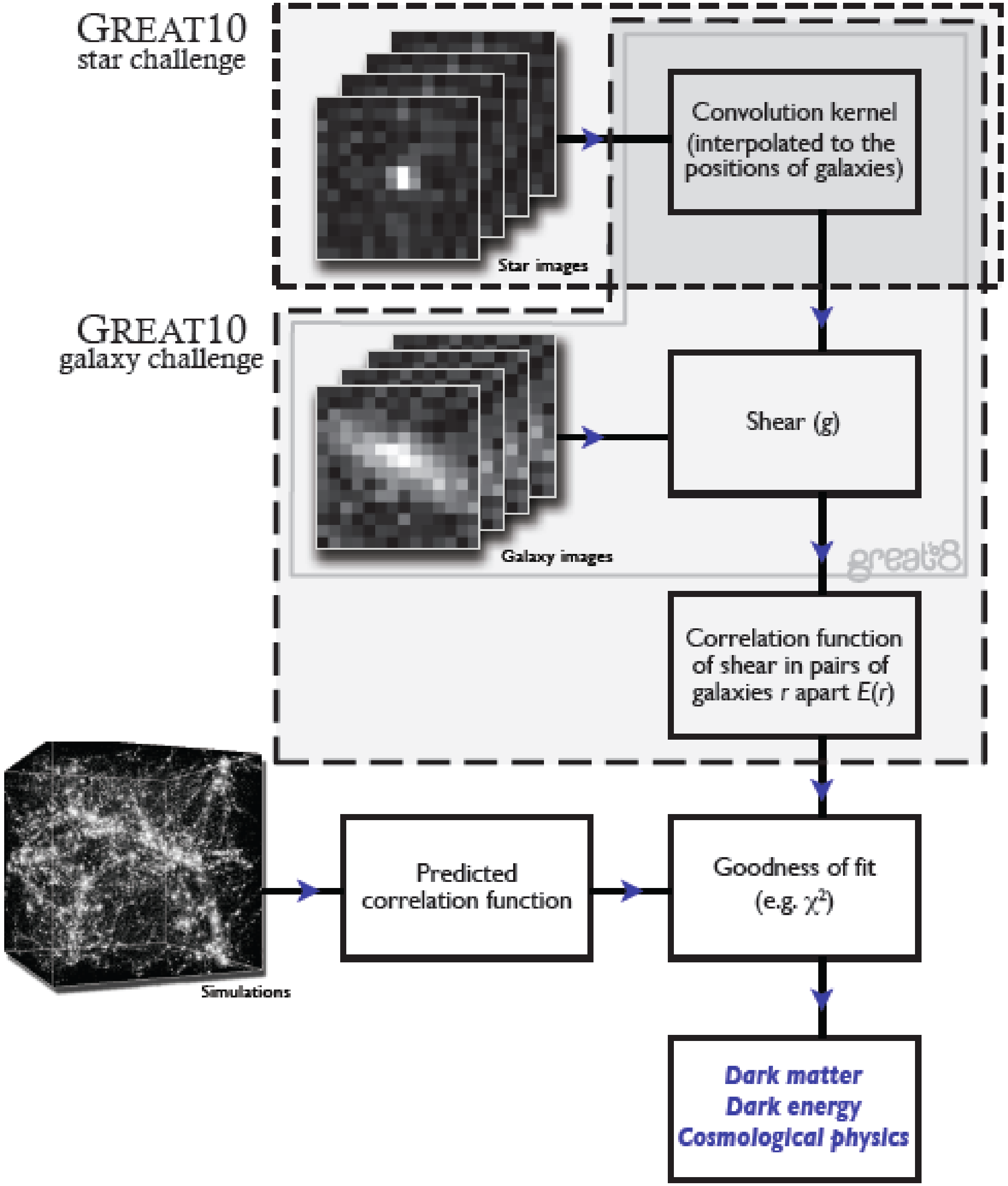
http://www.greatchallenges.info/

Active

Coming Soon : Mid 2010

Legacy Data

GRavitational lEnsing Accuracy Testing 2010 (GREAT10)



Why GREAT10?

- Shape measurement
  - There should exist an optimal method(s) to measure shear to the required accuracy
  - Statistical inference and image processing problem
  - Bring in people from outside astronomy

- It will be a 2010 PASCAL challenge
  - EU network of computational learning community
  - Winning a PASCAL challenge is prestigious
- Hope 0.03% error on shear
  - A fresh influx of ideas
  - Get people excited about the most powerful probe in cosmology

Conclusions

- Gravitational Lensing
  - Very Simple - Just Gravity and Geodesics
- Can be used to help us understand
  - Dark Matter
  - Particle Physics (neutrinos)
  - Dark Energy
  - Modified Gravity
- Measurement is challenging
  - GREAT10 will launch in Late 2010

http://www.greatchallenges.info/

GREAT10
GRavitational lEnsing Accuracy Testing